Processes:

$$
\begin{array}{lcl}
P, Q, R & ::= & A\langle a_1, \ldots, a_n\rangle \\
 & | & M \\
 & | & P|P \\
 & | & \textsf{new}\ a\ P \\
M, N, L & ::= & \alpha.P \\
 & | & \alpha.P + M \\
A & ::= & \text{a defined process identifier} \\
a, b & ::= & \text{an action identifier, without or without bar} \\
\alpha & ::= & a \\
 & | & \tau
\end{array}
$$

Process definitions:

$$A(a_1, \ldots, a_n) \stackrel{\text{def}}{=} P$$

where the free variables of P are $a_1, \ldots, a_n$

Congruence, $\equiv$:

- $M + N \equiv N + M$, $(M + N) + L \equiv M + (N + L)$
- $P|Q \equiv Q|P$, $(P|Q)|R \equiv P|(Q|R)$
- $\textsf{new}\ a\ (P|Q) \equiv P|(\textsf{new}\ a\ Q)$ if a is not free in P
- $\textsf{new}\ a\ \textsf{new}\ b\ P \equiv \textsf{new}\ b\ \textsf{new}\ a\ P$
- $\textsf{new}\ a\ P \equiv \textsf{new}\ b\ P[a \leftarrow b]$
- $A\langle a_1, \ldots, a_n\rangle \equiv P[b_1 \leftarrow a_1, \ldots, b_n \leftarrow a_n]$ if $A(b_1, \ldots, b_n) \stackrel{\text{def}}{=} P$
- ... and the compatible closure of the above.

Reactions:

$$\tau.P + M \rightarrow P$$

$$(a.P + M)|(\overline{a}.Q + N) \rightarrow P|Q$$

$$\frac{P \rightarrow P'}{P|Q \rightarrow P'|Q}$$

$$\frac{P \rightarrow P'}{\textsf{new}\ a\ P \rightarrow \textsf{new}\ a\ P'}$$

$$\frac{P \rightarrow P'}{Q \rightarrow Q'} \text{ if } P \equiv Q \text{ and } P' \equiv Q'$$

Lottery:

$$
\begin{array}{l}
A(a, b, c) \stackrel{\text{def}}{=} \overline{a}.C\langle a, b, c\rangle \\
C(a, b, c) \stackrel{\text{def}}{=} \tau.B\langle a, b, c\rangle + c.A\langle a, b, c\rangle \\
B(a, b, c) \stackrel{\text{def}}{=} b.C\langle a, b, c\rangle
\end{array}
$$

Three-Event Machine:

$$C\langle a_1, b_1, a_2\rangle|A\langle a_2, b_2, a_3\rangle|A\langle a_3, b_3, a_1\rangle$$

Homework: Show reactions on the three-event machine so that it ends up blocked at b_3 (i.e., to make progrees, the machine would have to be combined with another process that it blocked at $\overline{b}_3$).