Getting Started:

Arithmetic, Algebra, and Computing

Arithmetic is Computing

- Fixed, pre-defined rules for ***primitive operators***:

$$2 + 3 = 5$$

$$4 \times 2 = 8$$

$$\cos(0) = 1$$

Arithmetic is Computing

- Fixed, pre-defined rules for ***primitive operators***:

$$2 + 3 \rightarrow 5$$

$$4 \times 2 \rightarrow 8$$

$$\cos(0) \rightarrow 1$$

- Rules for combining other rules:
  - ○ Evaluate sub-expressions first

$$4 \times (2 + 3) \rightarrow 4 \times 5 \rightarrow 20$$

  - ○ Precedence determines subexpressions:

$$4 + 2 \times 3 \rightarrow 4 + 6 \rightarrow 10$$

Algebra as Computing

- ○ Definition:

$$f(x) = \cos(x) + 2$$

- ○ Expression:

$$f(0) \rightarrow \cos(0) + 2 \rightarrow 1 + 2 \rightarrow 3$$

- First step uses the ***substitution*** rule for functions

Scheme Notation

- Put all operators at the front
- Start every operation with an open parenthesis
- Put a close parenthesis after the last argument
- Never add extra parentheses

Old	New
1 + 2	`(+ 1 2)`
4 + 2 × 3	`(+ 4 (* 2 3))`
cos(0) + 1	`(+ (cos 0) 1)`

Scheme Notation

- Use the keyword **`define`** instead of =
- Put **`define`** at the front, and group with parentheses
- Move open parenthesis from after function name to before

Old	New
f(x) = cos(x) + 2	`(define (f x) (+ (cos x) 2))`

- Move open parenthesis in function calls

Old	New
f(0)	`(f 0)`
f(2+3)	`(f (+ 2 3))`

Evaluation is the Same as Before

```
(define (f x) (+ (cos x) 2))

(f 0)
```

Evaluation is the Same as Before

```
(define (f x) (+ (cos x) 2))

(f 0)
 →  (+ (cos 0) 2)
```

Evaluation is the Same as Before

```
(define (f x) (+ (cos x) 2))

(f 0)
 → (+ (cos 0) 2)
 → (+ 1 2)
```

Evaluation is the Same as Before

```
(define (f x) (+ (cos x) 2))

(f 0)
 → (+ (cos 0) 2)
 → (+ 1 2)
 → 3
```

Beyond Numbers: Booleans

Numbers are not the only kind of values:

Old	New
$1 < 2$ → true	`(< 1 2)` → `true`
$1 > 2$ → true	`(> 1 2)` → `false`
$1 > 2$ → true	`(> 1 2)` → `false`
$2 \geq 2$ → true	`(>= 1 2)` → `true`

Beyond Numbers: Booleans

Old	New
true and false	`(and true false)`
true or false	`(or true false)`
$1 < 2$ and $2 > 3$	`(and (< 1 2) (> 2 3))`
$1 \leq 0$ and $1 = 1$	`(or (<= 1 0) (= 1 1))`
$1 \neq 0$	`(not (= 1 0))`

Beyond Numbers: Symbols

(symbol=? 'apple 'apple) → true

(symbol=? 'apple 'banana) → false

Beyond Numbers: Images

(filled-rect 35 35 'red) → 🟥

(filled-circle 25 25 'blue) → 🔵

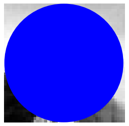
(image+ 🟥 🔵) →

(offset-image+ 🟥 5 5 🔵) →

(image=? (image+ 🟥 🔵))
→ (image=?)
→ true

Programming with Images

```
(define (anonymize i)
  (offset-image+
   i
   0 0
   (filled-circle (image-width i)
                  (image-height i)
                  'blue)))
```

(anonymize) → ... → 

Conditionals

Conditionals in Algebra

General format of conditionals in algebra:

$$\left\{ \begin{array}{ll} \textit{answer} & \textit{question} \\ \quad\ldots & \\ \textit{answer} & \textit{question} \end{array} \right.$$

Example:

$$\text{abs}(x) = \left\{ \begin{array}{ll} x & \text{if } x > 0 \\ -x & \text{otherwise} \end{array} \right.$$

abs(10) = 10

abs(-7) = 7

Conditionals

General syntax of **cond** in Scheme:

```
(cond
  [question answer]
  ...
  [question answer])
```

- Any number of **cond** lines
- Each line has one *question* expression and one *answer* expression

```
(define (abs x)
  (cond
    [(> x 0) x]
    [else (- x)]))
(abs 10) "should be" 10
(abs -7) "should be" 7
```

Completing max-image

- Use **cond** to complete `max-image`

```
(define (max-image a b)
  (cond
    [(bigger-image? a b) a]
    [else b]))
```

Evaluation Rules for cond

First question is literally `true` or `else`

```
(cond
  [true answer]
                       → answer
  ...
  [question answer])
```

- Keep only the first answer

Example:

```
(* 1 (cond          → (* 1 0) → 0
       [true 0]))
```

Evaluation Rules for cond

First question is literally `false`

```
(cond                            (cond
  [false answer]                   [question answer]
  [question answer]     →          ...
  ...                              [question answer])
  [question answer])
```

- Throw away the first line

Example:

```
(+ 1 (cond            → (+ 1 (cond
       [false 1]                 [true 17]))
       [true 17]))

                          → (+ 1 17) → 18
```

Evaluation Rules for cond

First question isn't a value, yet

```
(cond                            (cond
  [question answer]     →          [nextques answer]
  ...                              ...
  [question answer])               [question answer])
```

where *question* → *nextques*

- Evaluate first question as sub-expression

Example:

```
(+ 1 (cond            → (+ 1 (cond
       [(< 1 2) 5]               [true 5]
       [else 8]))                [else 8]))

                          → (+ 1 5) → 6
```

Evaluation Rules for cond

Only queston is false answers

```
(cond                →   error: all questions false
 [false 10])
```

Finding Images

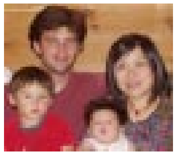
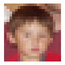
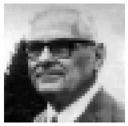
(image-inside?) → true

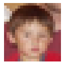
(image-inside?) → false

Image Tests in Conditionals

Now we can combine such operators with **cond**:

```
; detect-person : image image image -> image
; Returns a or b, depending on which is in i
(define (detect-person i a b)
  (cond
    [(image-inside? i a) a]
    [(image-inside? i b) b]))
```

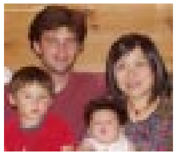
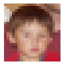
(detect-person   )

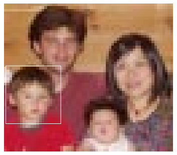
"should be" 

Compound Data

Finding and Adjusting Images

Suppose we want to write **frame-person**:

(frame-person  )

"should be" 

Need an operator that reports *where* an image exists

Finding an Image Position

~~find-image : image image -> num num~~

Must return a single value

Correct contract:

```
find-image : image image -> posn
```

- A **posn** is a ***compound value***

Positions

- A **posn** is

  `(make-posn X Y)`

  where **X** is a **num** and **Y** is a **num**

Examples:

`(make-posn 1 2)`

`(make-posn 17 0)`

A **posn** is a value, just like a number, symbol, or image

posn-x and posn-y

The **posn-x** and **posn-y** operators extract numbers from a **posn**:

`(posn-x (make-posn 1 2))` → `1`

`(posn-y (make-posn 1 2))` → `2`

- General evaluation rules for any **X** and **Y**:

  `(posn-x (make-posn X Y))` → `X`

  `(posn-y (make-posn X Y))` → `Y`

Positions and Values

Is `(make-posn 100 200)` a value?

Yes.

A **posn** is

`(make-posn X Y)`

where **X** is a **num** and **Y** is a **num**

Positions and Values

Is `(make-posn (+ 1 2) 200)` a value?

No. `(+ 1 2)` is not a **num**, yet.

- Two more evaluation rules:

  `(make-posn X Y)` → `(make-posn Z Y)`
  when **X** → **Z**

  `(make-posn X Y)` → `(make-posn X Z)`
  when **Y** → **Z**

Example:

`(make-posn (+ 1 2) 200)` → `(make-posn 3 200)`

Posn Examples

```
(make-posn (+ 1 2) (+ 3 4))

(posn-x (make-posn (+ 1 2) (+ 3 4)))

; pixels-from-corner : posn -> num
(define (pixels-from-corner p)
  (+ (posn-x p) (posn-y p)))
(pixels-from-corner (make-posn 1 2))

; flip : posn -> posn
(define (flip p)
  (make-posn (posn-y p) (posn-x p)))
(flip (make-posn 1 2))
```

Copy

Programmer-Defined Compound Data

Other Kinds of Data

Suppose we want to represent snakes:

- name
- weight
- favorite food

What kind of data is appropriate?

Not **num**, **bool**, **sym**, **image**, or **posn**...

Data Definitions and define-struct

Here's what we'd like:

A **snake** is
`(make-snake sym num sym)`

But **make-snake** is not built into DrScheme

We can tell DrScheme about **snake**:

```
(define-struct snake (name weight food))
```

Creates the following:

- **make-snake**
- **snake-name**
- **snake-weight**
- **snake-food**

Data Definitions and define-struct

Here’s what we’d like:

A **snake** is

```
(make-snake sym num sym)
```

But **make-snake** is not built into DrScheme

We can tell DrScheme about **snake**:

```
(define-struct snake (name weight food))
```

Creates the following:

```
(snake-name (make-snake X Y Z))  →  X
(snake-weight (make-snake X Y Z))  →  Y
(snake-food (make-snake X Y Z))  →  Z
```