

Quiz

- **Question #1:** What is the value of the following expression?

$$\{ + 1 2 \}$$

- **Wrong answer: 0**
- **Wrong answer: 42**
- **Answer: 3**

Quiz

- **Question #2:** What is the value of the following expression?

{ + fun 17 8 }

- **Wrong answer: error**
- **Answer:** Trick question! { + fun 17 8 } is not an expression

Language Grammar for Quiz

```
<MFAE> ::= <num>
         | true
         | false
         | { + <MFAE> <MFAE> }
         | { - <MFAE> <MFAE> }
         | { = <MFAE> <MFAE> }
         | <id>
         | { fun { <id>* } <MFAE> }
         | { <MFAE> <MFAE>* }
         | { if <MFAE> <MFAE> <MFAE> }
```

Quiz

- **Question #3:** Is the following an expression?

`{{fun {} 1} 7}`

- **Wrong answer: No**
- **Answer: Yes** (according to our grammar)

Quiz

- **Question #4:** What is the value of the following expression?

`{{fun {} 1} 7}`

- **Answer: 1** (according to some interpreters)
- But no *real* language would accept `{{fun {} 1} 7}`
- Let's agree to call `{{fun {} 1} 7}` an ***ill-formed expression*** because `{fun {} 1}` should be used with only zero arguments
- Let's agree to never evaluate ill-formed expressions

Quiz

- **Question #5:** What is the value of the following expression?

```
{{fun {} 1} 7}
```

- **Answer: None** - the expression is ill-formed

Quiz

- **Question #6:** Is the following a well-formed expression?

{+ {fun {} 1} 8}

- **Answer: Yes**

Quiz

- **Question #7:** What is the value of the following expression?

`{+ {fun {} 1} 8}`

- **Answer: None** - it produces an error:

+: expects a numV, given a closureV

- Let's agree that a **fun** expression cannot be inside a **+** form

Quiz

- **Question #8:** Is the following a well-formed expression?

{ + { fun { } 1 } 8 }

- **Answer: No**

Quiz

- **Question #9:** Is the following a well-formed expression?

{+ {{fun {x} x} 7} 5}

- **Answer:** Depends on what we meant by *inside* in our most recent agreement
 - *Anywhere inside* - **No**
 - *Immediately inside* - **Yes**
- Since our interpreter produces **12**, and since that result makes sense, let's agree on *immediately inside*

Quiz

- **Question #10:** Is the following a well-formed expression?

`{+ {{fun {x} x} {fun {y} y}} 5}`

- **Answer:** **Yes**, but we don't want it to be!

Quiz

- **Question #11:** Is it possible to define *well-formed* (as a decidable property) so that we reject all expressions that produce errors?
- **Answer: Yes:** reject *all* expressions!

Quiz

- **Question #12:** Is it possible to define *well-formed* (as a decidable property) so that we reject *only* expressions that produce errors?

- **Answer: No**

`{+ 1 {if ... 1 {fun {x} x}}}`

- If we always knew whether `...` produces true or false, we could solve the halting problem

Types

- Solution to our dilemma
 - In the process of rejecting expressions that are certainly bad, also reject some expressions that are good

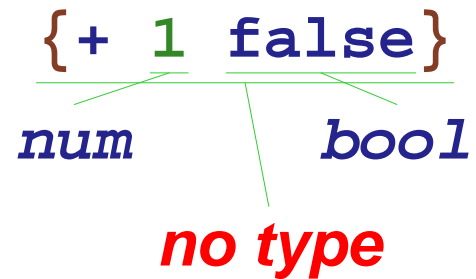
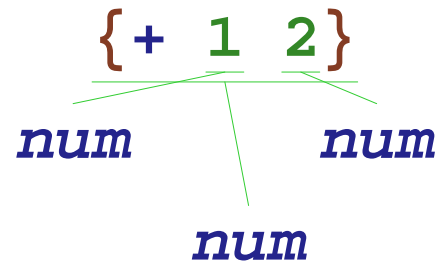
```
{+ 1 {if {prime? 131101} 1 {fun {x} x}}}
```

- Overall strategy:
 - Assign a **type** to each expression *without evaluating*
 - Compute the type of a complex expression based on the types of its subexpressions

Types

`1 : num`

`true : bool`



Type Rules

$\langle \text{num} \rangle : \text{num}$

$\text{true} : \text{bool}$

$\text{false} : \text{bool}$

$$\frac{\langle \text{MFAE} \rangle_1 : \text{num} \quad \langle \text{MFAE} \rangle_2 : \text{num}}{\{ + \langle \text{MFAE} \rangle_1 \langle \text{MFAE} \rangle_2 \} : \text{num}}$$

$1 : \text{num}$

$\text{true} : \text{bool}$

$$\frac{1 : \text{num} \quad 2 : \text{num}}{\{ + 1 2 \} : \text{num}}$$
$$\frac{1 : \text{num} \quad \text{false} : \text{bool}}{\{ + 1 \text{false} \} : \text{no type}}$$

Type Rules

$\langle \text{num} \rangle : \text{num}$

$\text{true} : \text{bool}$

$\text{false} : \text{bool}$

$\langle \text{MFAE} \rangle_1 : \text{num}$

$\langle \text{MFAE} \rangle_2 : \text{num}$

$\{ + \langle \text{MFAE} \rangle_1 \langle \text{MFAE} \rangle_2 \} : \text{num}$

$1 : \text{num}$

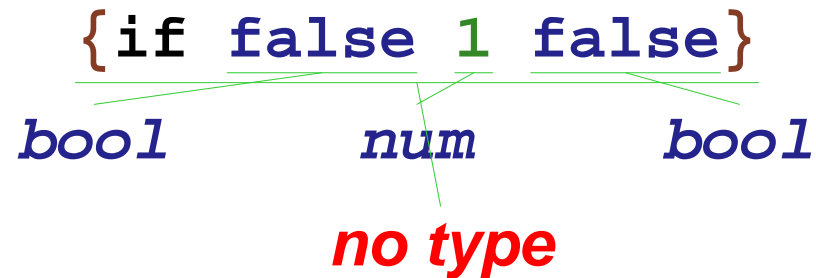
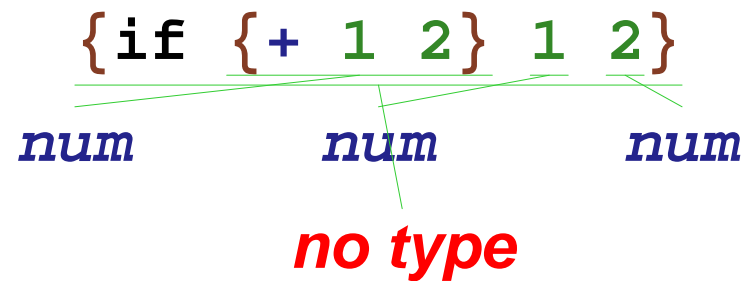
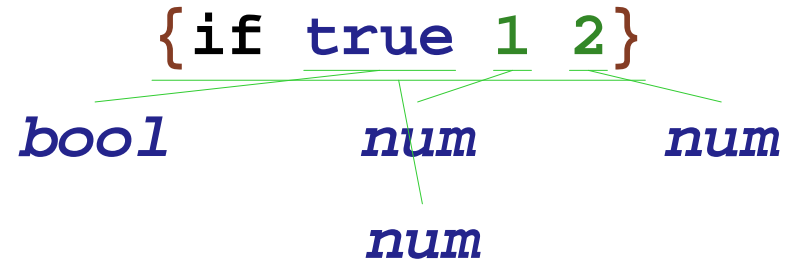
$2 : \text{num}$

$\{ + 1 2 \} : \text{num}$

$3 : \text{num}$

$\{ + \{ + 1 2 \} 3 \} : \text{num}$

Types: Conditionals



Conditional Type Rules

$$\frac{\langle \text{MFAE} \rangle_1 : \text{bool} \quad \langle \text{MFAE} \rangle_2 : \langle \text{type} \rangle_0 \quad \langle \text{MFAE} \rangle_3 : \langle \text{type} \rangle_0}{\{\text{if } \langle \text{MFAE} \rangle_1 \ \langle \text{MFAE} \rangle_2 \ \langle \text{MFAE} \rangle_3\} : \langle \text{type} \rangle_0}$$
$$\frac{\text{true} : \text{bool} \quad 1 : \text{num} \quad 2 : \text{num}}{\{\text{if true } 1 \ 2\} : \text{num}}$$
$$\frac{\{+ \ 1 \ 2\} : \text{num} \quad 1 : \text{num} \quad 2 : \text{num}}{\{\text{if } \{+ \ 1 \ 2\} \ 1 \ 2\} : \text{no type}}$$
$$\frac{\text{false} : \text{bool} \quad 1 : \text{num} \quad \text{false} : \text{bool}}{\{\text{if false } 1 \ \text{false}\} : \text{no type}}$$

Types: Variables and Functions

x : *no type*

`{fun {x : bool} x}`

bool

$(bool \rightarrow bool)$

`{fun {x : bool} {if x 1 2}}`

bool

num

num

num

$(bool \rightarrow num)$

Variable and Function Type Rules

$$[\dots \langle \text{id} \rangle \leftarrow \tau \dots] \vdash \langle \text{id} \rangle : \tau$$
$$\frac{\Gamma [\langle \text{id} \rangle \leftarrow \tau_1] \vdash \mathbf{e} : \tau_2}{\Gamma \vdash \{ \text{fun } \{ \langle \text{id} \rangle : \tau_1 \} \mathbf{e} \} : (\tau_1 \rightarrow \tau_2)}$$

Abbreviations: $\tau = \langle \text{type} \rangle$ $\mathbf{e} = \langle \text{MFAE} \rangle$ $\Gamma = \langle \text{env} \rangle$

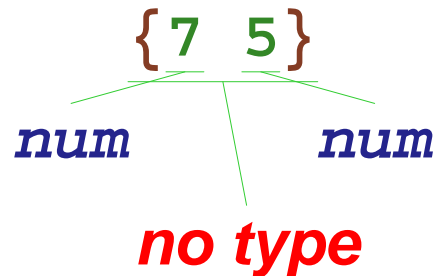
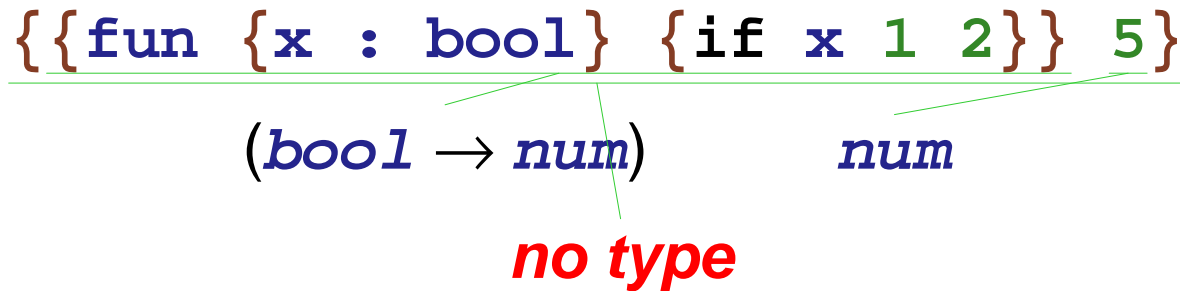
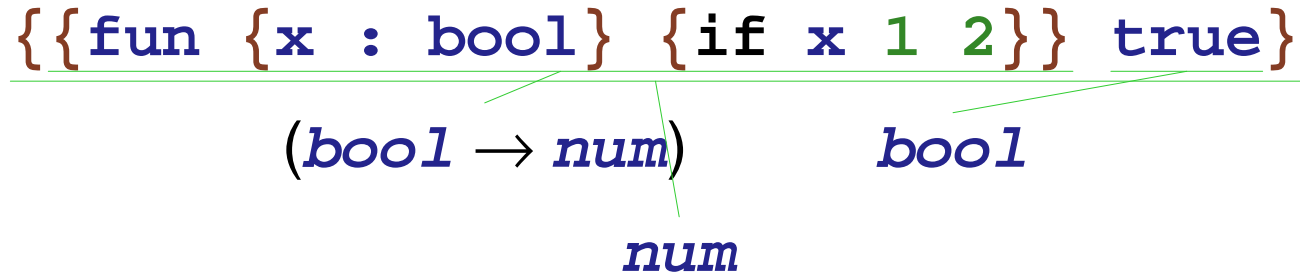
Variable and Function Type Rules

$$[\dots \langle \text{id} \rangle \leftarrow \tau \dots] \vdash \langle \text{id} \rangle : \tau$$
$$\frac{\Gamma [\langle \text{id} \rangle \leftarrow \tau_1] \vdash e : \tau_2}{\Gamma \vdash \{ \text{fun } \{ \langle \text{id} \rangle : \tau_1 \} e \} : (\tau_1 \rightarrow \tau_2)}$$
$$\emptyset \vdash x : \text{no type}$$
$$\frac{[x \leftarrow \text{bool}] \vdash x : \text{bool}}{\emptyset \vdash \{ \text{fun } \{ x : \text{bool} \} x \} : (\text{bool} \rightarrow \text{bool})}$$
$$\frac{\begin{array}{l} [x \leftarrow \text{bool}] \vdash x : \text{bool} \quad [x \leftarrow \text{bool}] \vdash 1 : \text{num} \quad [x \leftarrow \text{bool}] \vdash 2 : \text{num} \\ [x \leftarrow \text{bool}] \vdash \{ \text{if } x \ 1 \ 2 \} : \text{num} \end{array}}{\emptyset \vdash \{ \text{fun } \{ x : \text{bool} \} \{ \text{if } x \ 1 \ 2 \} \} : (\text{bool} \rightarrow \text{num})}$$

Revised Rules

$$\Gamma \vdash \langle \text{num} \rangle : \text{num}$$
$$\Gamma \vdash \text{true} : \text{bool}$$
$$\Gamma \vdash \text{false} : \text{bool}$$
$$\frac{\Gamma \vdash \mathbf{e}_1 : \text{num} \quad \Gamma \vdash \mathbf{e}_2 : \text{num}}{\Gamma \vdash \{ + \mathbf{e}_1 \ \mathbf{e}_2 \} : \text{num}}$$
$$\frac{\Gamma \vdash \mathbf{e}_1 : \text{bool} \quad \Gamma \vdash \mathbf{e}_2 : \tau_0 \quad \Gamma \vdash \mathbf{e}_3 : \tau_0}{\Gamma \vdash \{ \mathbf{if} \ \mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3 \} : \tau_0}$$

Types: Function Calls



Function Call Type Rule

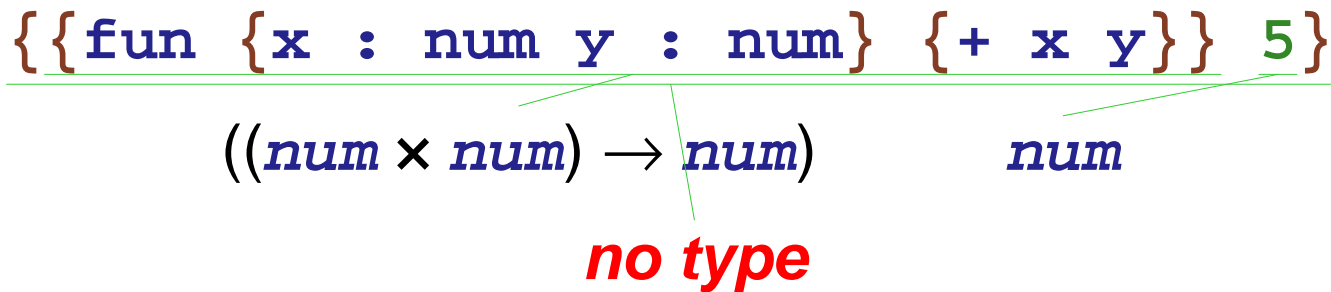
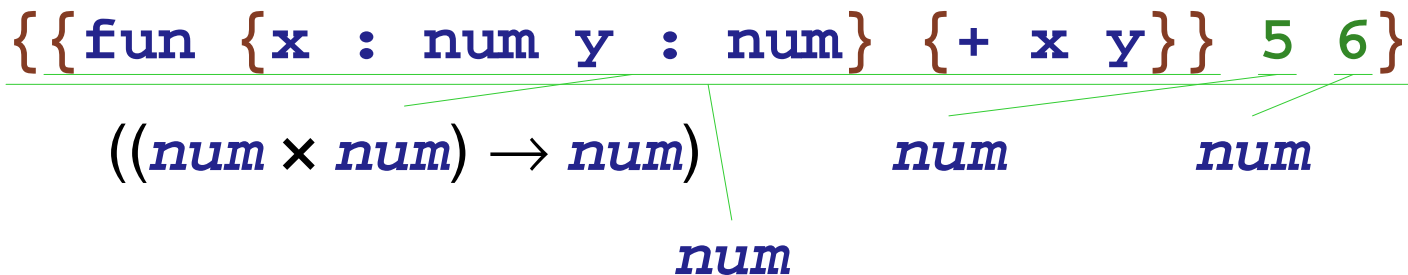
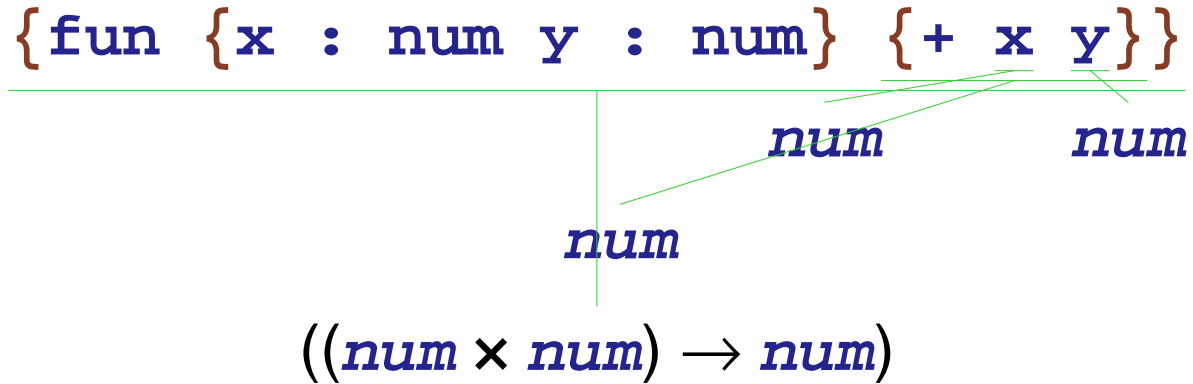
$$\frac{\Gamma \vdash e_1 : (\tau_2 \rightarrow \tau_3) \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \{e_1 e_2\} : \tau_3}$$

$$\frac{\emptyset \vdash \{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} : (\text{bool} \rightarrow \text{num}) \quad \emptyset \vdash \text{true} : \text{bool}}{\emptyset \vdash \{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \text{true}\} : \text{num}}$$

$$\frac{\emptyset \vdash \{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} : (\text{bool} \rightarrow \text{num}) \quad \emptyset \vdash 5 : \text{num}}{\emptyset \vdash \{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} 5\} : \text{no type}}$$

$$\frac{\emptyset \vdash 7 : \text{num} \quad \emptyset \vdash 5 : \text{num}}{\emptyset \vdash \{7 \ 5\} : \text{no type}}$$

Types: Multiple Arguments



Revised Function and Call Rules

$$\frac{\Gamma[\langle \text{id} \rangle_1 \leftarrow \tau_1 \dots \langle \text{id} \rangle_n \leftarrow \tau_n] \vdash e : \tau_0}{\Gamma \vdash \{ \text{fun } \{ \langle \text{id} \rangle_1 : \tau_1 \dots \langle \text{id} \rangle_n : \tau_n \} e \} : ((\tau_1 \times \dots \tau_n) \rightarrow \tau_0)}$$

$$\frac{\Gamma \vdash e_0 : ((\tau_1 \times \dots \tau_n) \rightarrow \tau_0) \quad \Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash \{ e_0 e_1 \dots e_n \} : \tau_0}$$