Current Book Language

<expr> ::= <num>
::= **true** | **false**
::= <id>
::= <prim> ({ <expr> }*(,))
::= **proc** ({ <tyexpr> <id> }*(,)) <expr>
::= (<expr> <expr>*)
::= **if** <expr> **then** <expr> **else** <expr>
::= **let** { <id> = <expr> }* **in** <expr>
::= **letrec** { <tyexpr> <id>({ <tyexpr> <id> }*(,))
= <expr> }*
in <expr>

<tyexpr> ::= **int**
::= **bool**
::= (<tyexpr> **->** <tyexpr>)

Types versus Type Expressions

<tyexpr>		<type>
int	expands to	`int`
bool	expands to	`bool`
(**bool -> int**)	expands to	(`bool` $\rightarrow$ `int`)
	etc.	

Datatype for types:

```
(define-datatype
 type type?
 (atomic-type (name symbol?))
 (proc-type (arg-types (list-of type?))
            (result-type type?)))

(define int-type (atomic-type 'int))
(define bool-type (atomic-type 'bool))
```

Implementing a Type Checker

```
;; type-of-expression : expr tenv -> type
;; signals an error if no type for exp
;;
(define (type-of-expression exp tenv)
  (cases expression exp
    (lit-exp ...)
    (true-exp ...)
    (false-exp ...)
    (var-exp ...)
    (primapp-exp ...)
    (proc-exp ...)
    (app-exp ...)
    (if-exp ...)
    (let-exp ...)
    (letrec-exp ...)))
```

Implementation: lit-exp case

- Example:

5

- The rule from previous lecture:

$E \vdash$ <num> : `int`

- In Scheme:

```
(lit-exp () int-type)
```

Implementation: true-exp and false-exp case

- Example:

true

- The rule from previous lecture:

$$E \vdash \langle\text{bool}\rangle : \texttt{bool}$$

- In Scheme:

```
(true-exp () bool-type)
(false-exp () bool-type)
```

Implementation: var-exp case

- Example:

... x ...

- The rule from previous lecture:

$$\{ \ldots \langle\text{id}\rangle : T \ldots \} \vdash \langle\text{id}\rangle : T$$

- In Scheme:

```
(var-exp (id) (apply-tenv tenv id))
;; where apply-tenv signals an error
;; if id is not in tenv
```

Implementation: if-exp case

if true then 5 **else** +(1,2)

- The rule from previous lecture:

$$\frac{E \vdash e_1 : \texttt{bool} \qquad E \vdash e_2 : T_0 \qquad E \vdash e_3 : T_0}{E \vdash \textbf{if } e_1 \textbf{ then } e_2 \textbf{ else } e_3 : T_0}$$

- In Scheme:

```
(if-exp (test-exp then-exp else-exp)
  (let ((test-type (type-of-expr test-exp tenv))
        (then-type (type-of-expr then-exp tenv))
        (else-type (type-of-expr else-exp tenv)))
    ;; succeeds or signals an error:
    (check-equal-type! test-type bool-type)
    (check-equal-type! then-type else-type)
    then-type)
```

Implementation: proc-exp case

proc(**int x**, **bool y**)**if y then x else** 0

- The rule from previous lecture:

$$\frac{\{ \langle\text{id}\rangle_1 : T_1, \ldots \langle\text{id}\rangle_n : T_n \} + E \vdash e : T_0}{E \vdash \textbf{proc}(T_1\ \langle\text{id}\rangle_1, \ldots\ T_n\ \langle\text{id}\rangle_n) e : (T_1 \times \ldots\ T_n \rightarrow T_0)}$$

- In Scheme:

```
(proc-exp (texps ids body)
  (let* ((arg-tys (expand-tyexprs texps))
         (new-tenv (extend-tenv ids arg-tys tenv))
         (res-type (type-of-expr body new-tenv)))
    (proc-type arg-types res-type)))
```

Implementation: app-exp case

(**proc**(**int x**, **int y**)+(**x**,**y**) 6 7)

- The rule from previous lecture:

$$\frac{E \vdash e_0 : (T_1 \times \ldots T_n \to T_0) \qquad E \vdash e_1 : T_1 \qquad \ldots \qquad E \vdash e_n : T_n}{E \vdash (e_0\ e_1 \ldots e_n) : T_0}$$

- In Scheme:

```
(app-exp (rator rands)
   (type-of-application
     (type-of-expression rator tenv)
     (types-of-expressions rands tenv)))
```

Implementation: app-exp case

```
(define (type-of-application rator-ty rand-tys)
  (cases type rator-ty
    (proc-type (arg-tys result-ty)
      (if (= (length arg-tys) (length rand-tys))
          (begin
            (check-equal-types! rand-tys arg-tys)
            result-ty)
           (error 'wrong-arg-count)))
    (else (error 'not-a-proc))))
```

Implementation: primapp-exp case

+(1, 2)

- The rule from previous lecture:

$$\frac{E \vdash e_1 : \textbf{num} \qquad E \vdash e_1 : \textbf{num}}{E \vdash +(e_1, e_2) : \textbf{num}}$$

- In Scheme (completely different):

```
(primapp-exp (prim rands)
  (type-of-application
   (type-of-primitive prim)
   (types-of-expressions rands tenv))
```

Implementation: primapp-exp case

```
(define (type-of-primitive prim)
   (cases primitive prim
     (add-prim ()
        (proc-type (list int-type int-type)
                   int-type))
     ...))
```

Implementation: let-exp case

let x = 5
f = **proc**(**int y**)**false**
in (**f x**)

- In Scheme:

```
(let-exp (ids rands body)
  (let* ((rand-tys (types-of-exprs rands tenv))
         (body-tenv (extend-tenv ids rand-tys
                                    tenv)))
    (type-of-expression body body-tenv)))
```

Implementation: letrec-exp case

letrec int f(**int x**) = (**g** +(**x**,1) **false**)
int g(**int y**, **bool b**) = **if b then** (**f y**) **else y**
in (**g** 10 **true**)

- In Scheme:

```
(letrec-exp (res-texps proc-ids texpss idss bodies
                body)
 (let*((arg-tyss (expand-tyexprss texpss))
       (res-tys (expand-tyexprs res-texps))
       (proc-tys (map proc-type arg-tyss res-tys)))
       (new-tenv (extend-tenv proc-ids proc-tys
                               tenv)))
   ...)
```

Implementation: letrec-exp case

letrec int f(**int x**) = (**g** +(**x**,1) **false**)
int g(**int y**, **bool b**) = **if b then** (**f y**) **else y**
in (**g** 10 **true**)

- In Scheme:

```
(letrec-exp (res-texps proc-ids texpss idss bodies
               body)
 ...
   (for-each
     (lambda (ids arg-tys body res-ty)
        (check-equal-type! res-ty
           (type-of-expr body
              (extend-tenv ids arg-tys new-tenv))))
     idss arg-tyss bodies res-tys)
   (type-of-expression body new-tenv))
```

Type-Checking Expressions

- What is the type of the following expression?

proc(**x**)+(**x**,1)

- **Answer:** Yet another trick question; it's not an expression in our typed language, because the argument type is missing
- But it seems like the answer *should* be (`int` → `int`)

Type Inference

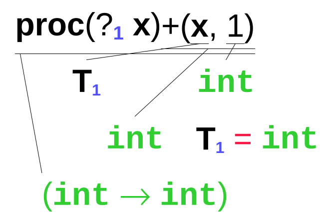
- ***Type inference*** is the process of inserting type annotations where the programmer omits them

- We'll use explicit question marks, to make it clear where types are omitted

proc (? **x**)+(**x**,1)

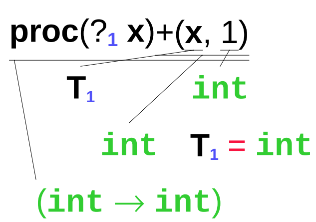
<tyexpr>	::=	**int**
	::=	**bool**
	::=	(<tyexpr> **->** <tyexpr>)
	::=	?

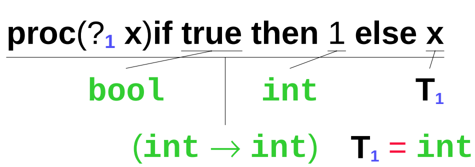
Type Inference



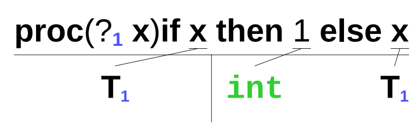
- Create a new type variable for each ?
- Change type comparison to install type equivalences

Type Inference





Type Inference: Impossible Cases



no type: T_1 can't be both `bool` and `int`

Type Inference: Many Cases

proc(?$_1$ **y**)**y**

T_1

$(T_1 \rightarrow T_1)$

- Sometimes, more than one type works
  - (`int` → `int`)
  - (`bool` → `bool`)
  - ((`int` → `bool`) → (`int` → `bool`))

so the type checker leaves variables in the reported type

Type Inference: Function Calls

(**proc**(?$_1$ **y**)**y** **proc**(?$_2$ **x**)+(**x**, 1))

$(T_1 \rightarrow T_1)$ (`int` → `int`)

(`int` → `int`)

T_1 = (`int` → `int`)

Type Inference: Function Calls

proc(?$_1$ **y**)(**y** 7)

T_1 `int`

T_2 T_1 = (`int` → T_2)

((`int` → T_2) → T_2)

- In general, create a new type variable record for the result of a function call

Type Inference: Cyclic Equations

proc(?$_1$ **x**)(**x** **x**)

T_1 T_1

no type: T_1 can't be (T_1 → ...)

- T_1 can't be `int`
- T_1 can't be `bool`
- Suppose T_1 is (T_2 → T_3)
  - T_2 must be T_1
  - So we won't get anywhere!

Type Inference: Cyclic Equations

proc(?$_1$ **x**)(**x x**)

T$_1$ **T**$_1$

no type: **T**$_1$ can’t be (**T**$_1$ $\rightarrow$ ...)

- When installing a type equivalence, make sure that the new type for **T** doesn’t already contain **T**

Implementing Type Inference

- Extend **`type`** datatype with **`tvar-type`** variant

```
(define-datatype
 type type?
 ...
 (tvar-type (serial-number integer?)
            (container vector?)))
```

- Create a new type variable record for each ?
  - Initial container value is "don’t know", **’**()
- Create a new type variable record for each application
- Change **`check-equal-type!`** to read and set type variable containers