## Lexical Addresses

As we saw in the last lecture, the expression

$$
\begin{aligned}
& \text { let } x=1 \quad y=2 \\
& \text { in let } \mathbf{f}=\operatorname{proc}(x)+(x, y) \\
& \quad \text { in }(\mathbf{f} \mathbf{x})
\end{aligned}
$$

might be compiled to

$$
\begin{aligned}
& \text { let }=1_{-}=2 \\
& \text { in let } \left.=\text { proc }\left(\_\right)+(<0,0\rangle,<1,1>\right) \\
& \text { in }(<0,0><1,0>)
\end{aligned}
$$

<n, $\mathbf{m}>$ means: $\mathbf{n}$ frames up in the environment, at position $\mathbf{m}$
How can we compute <n, m> for every bound variable without running the code?

## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

$$
\operatorname{proc}(\mathbf{x})+(\mathbf{x}, 7)
$$

- Count contour crossings to get $\mathbf{n}+1$
- Cross 1 contour from bound $\mathbf{x}$ to binding $\mathbf{x}$, so first part of address is 0
- Full address is $<0,0>$


## Computing Lexical Addresses

- What creates a new frame?

```
let, letrec, and (application of) proc
```

- So, to compute the $\mathbf{n}$ in $<\mathbf{n}, \mathbf{m}>$, count the number of enclosing let, letrec, and proc keywords between the bound variable and its binding
- The $\mathbf{m}$ in $<\mathbf{n}, \mathbf{m}>$ is simply the variable's position in its binding set


## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

```
proc (\mathbf{y})}\operatorname{proc}(\mathbf{x},\mathbf{z})+(\mathbf{x},-(\mathbf{y},\mathbf{z})
```

- Bound $\mathbf{x}:<0,0>$
- Bound $\mathbf{y}:<1,0\rangle$
- Bound $\mathbf{z}$ : <0, 1>


## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

```
proc (\mathbf{y})}\operatorname{proc}(\mathbf{x},\mathbf{z})+(\mathbf{x},-(\mathbf{y},\mathbf{z})
```

In general:

```
proc \(\left(\left\langle i d>_{1}, \ldots\right.\right.\), <id \(\left.>_{n}\right)<\) expr \(>\)
```


## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

$$
\begin{aligned}
& \text { let } x=5 \\
& \text { in } x
\end{aligned}
$$

## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

$$
\begin{gathered}
\text { let } x=5 \\
\text { in } x
\end{gathered}
$$

In general:

$$
\begin{aligned}
& \text { let }\langle i d\rangle_{1}=\text { <expr }>_{1} \\
& \ldots=\ldots \\
& \text { <id }>_{n}=\text { <expr }>_{n} \\
& \text { in }<\text { expr }\rangle
\end{aligned}
$$

## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

$$
\begin{aligned}
& \text { let } \begin{array}{l}
x=5 \\
y=7 \\
\text { in let } \mathbf{x}=\mathbf{x} \\
\text { in }+(\mathbf{x}, \mathbf{y})
\end{array}
\end{aligned}
$$

## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

$$
\begin{aligned}
& \text { let } x=5 \\
& y=7 \\
& \text { in let } x=x \\
& \text { in }+(\mathbf{x}, \mathbf{y})
\end{aligned}
$$

- Bound $\mathrm{x}:<0,0>$
- Bound $\mathbf{x}:<0,0>$
- Bound $\mathbf{y}$ : <1, 1>


## Computing Lexical Addresses

Visualize as countours that separate environment extension from the expressions that use it

$$
\begin{gathered}
\text { letrec } \mathbf{f}=\operatorname{proc}(\mathbf{x})+(\mathbf{x},(\mathbf{g} 7)) \\
\mathbf{g}=\operatorname{proc}(\mathbf{z})-(\mathbf{z}, 2) \\
\text { in }(\mathbf{f} 10)
\end{gathered}
$$

In general:

$$
\begin{gathered}
\text { letrec }<\text { id }>_{1}= \\
\ldots= \\
\ldots \text { expr }>_{1} \\
\text { <id }>_{n}= \\
\text { in }<\text { expr }>_{n} \\
\text { in }<\text { expr }>
\end{gathered}
$$

## Lexical Addresses are Static

- The contour approach to computing lexical addresses works because they are static
- That's why we can pre-compute them in a compiler
- Bound $\mathbf{x}:<0,0>$
- Bound $\mathbf{g}:<1,1\rangle$
- Bound $\mathbf{z}:<0,0>$
- Bound $\mathrm{f}:<0,0>$

Source Language for Compilation

```
<expr> ::= <num>
    ::= <id>
    ::= <prim> ( { <expr> }*()
    ::= let { <id> = <expr> }* in <expr>
    ::= proc ({<id> }*(,)}\mathrm{ ) <expr>
    ::= (<expr> <expr>*)
```

                concrete
    
## Source Language for Compilation

## Target Language for Compilation

```
<cexpr> ::= (lit-cexp <num>)
    ::= (var-cexp <num> <num>)
    ::= (primapp-cexp <prim> (list <cexpr>*))
    ::= (let-cexp (list <cexpr>*) <cexpr>)
    ::= (proc-cexp <cexpr>)
    ::= (app-cexp <cexpr> (list <cexpr>*))
```

                    abstract
        (no use for concrete)
    For implementation: declare a cexpression datatype with define-datatype

```
<expr> ::= (lit-exp <num>)
    ::= (var-exp <symbol>)
    ::= (primapp-exp <prim> (list <expr>*))
    ::= (let-exp (list <symbol>*) (list <expr>*) <expr>)
    ::= (proc-exp (list <symbol>*) <expr>)
    ::= (app-exp <expr> (list <expr>*))
<expr> ::= (lit-exp <num>)
```

abstract

## Compilation Function

compile-expression : expr -> cexpr

- Mostly trival: create a <cexpr> corresponding to the input <expr>
- Interesting case: var-exp
- Use an environment, almost like evaluation
- Key difference \#1: instead of apply-env, we need lexical-address-in-env
- Key difference \#2: no closures; instead, compile a proc body immediately when we encounter the proc


## Evaluation Function for the Target Language

- eval-cexpression is similar to eval-expression, except:
- The names in the environment do not matter
- Use apply-env-to-lexical-address instead of apply-env

Implementation
(implement in DrScheme)

