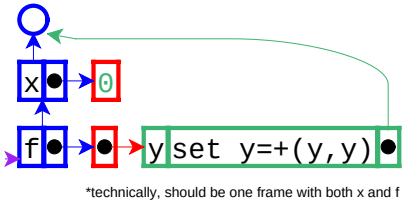
```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Starting call-by-value...


```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

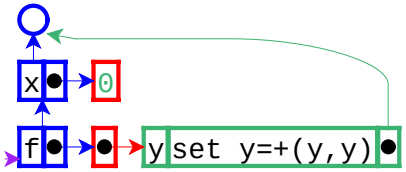
- Eval RHSs



*technically, should be one frame with both x and f

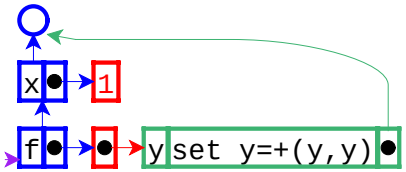
```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Bind x and f to 0 and closure, respectively



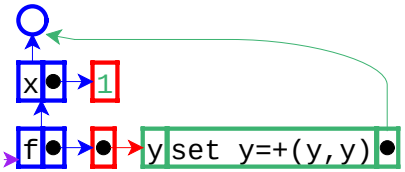

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Eval RHS for z



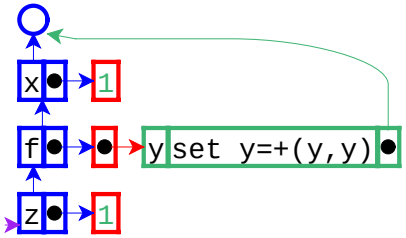

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Value for x changed to 1



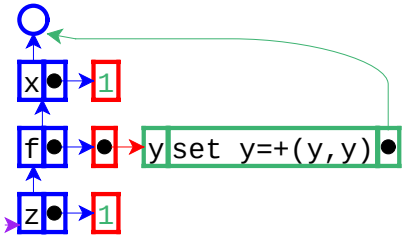

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Return x...




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

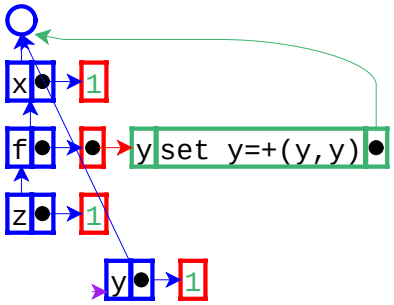
- ... and bind z to the result, 1




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call f with z

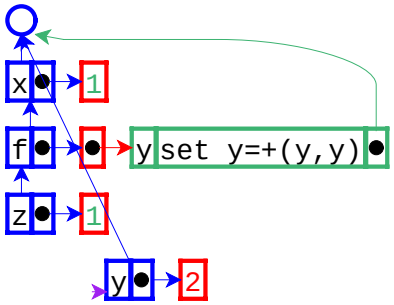
call-by-value




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call-by-value creates a new location for y

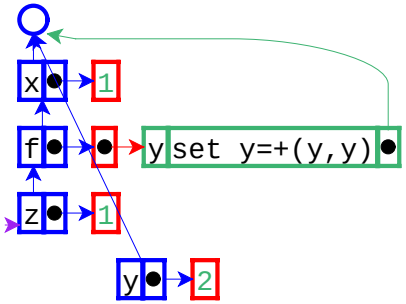
call-by-value




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Value for y changed to 2

call-by-value




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result is the current value of z: 1

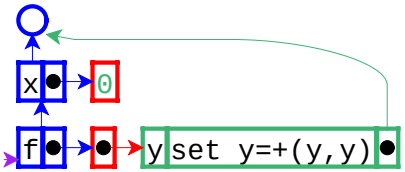
call-by-reference

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Starting call-by-reference...

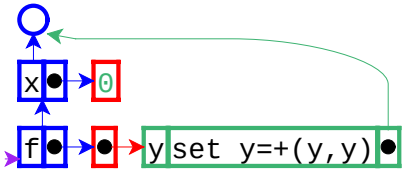
```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Eval RHSs



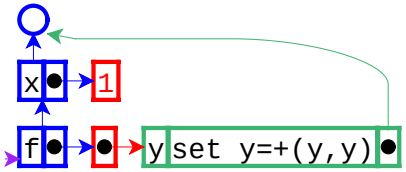

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Bind x and f to 0 and closure, respectively



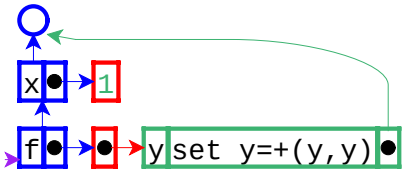

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Eval RHS for z



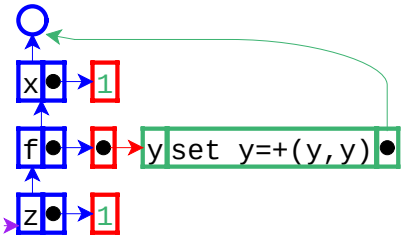

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Value for x changed to 1



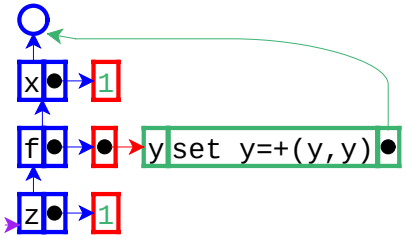

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Return x...



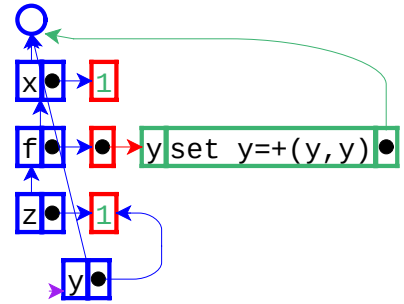

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- ... and bind z to the result, 1



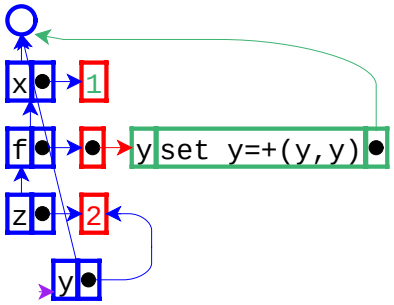

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call f with z



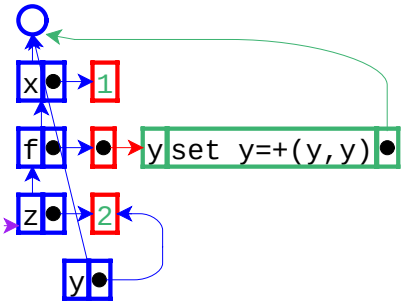

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call-by-reference shares location for z with y




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Value for y (and therefore z) changed to 2




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result is the current value of z: 2

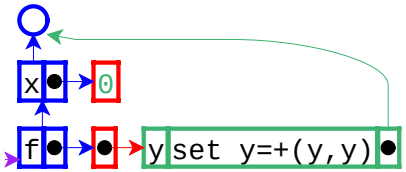

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Starting call-by-name...


```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Eval RHSs

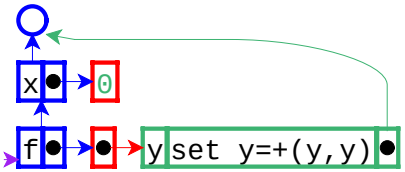
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Simple expressions: bind x and f to 0 and closure, respectively

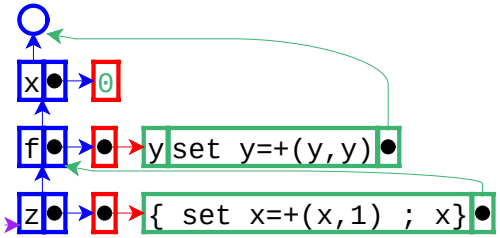
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Handle RHS of z...

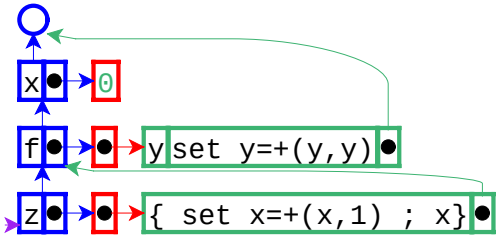
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- ... by creating a thunk for z

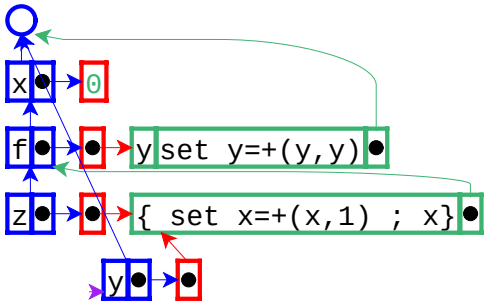
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call f with z

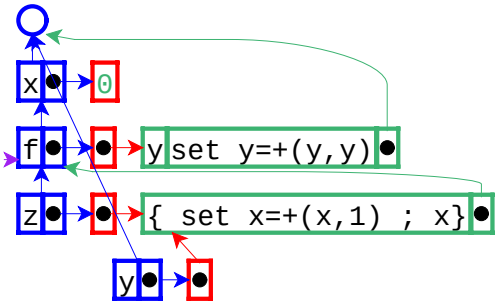
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Not by-reference; y gets a new location, containing the same thunk as z's location

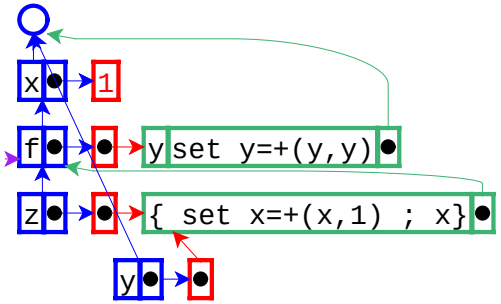
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Use of y means we eval the thunk

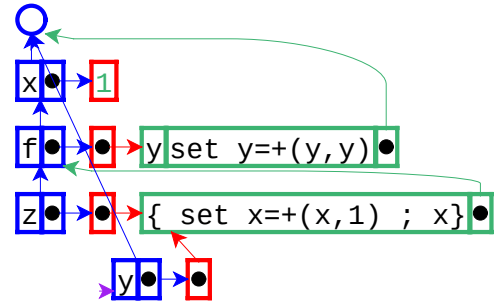
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Thunk changes value of x to 1

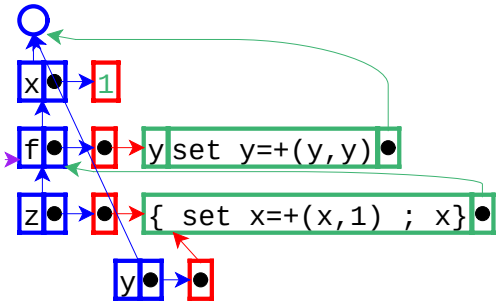
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result for first use of y is 1

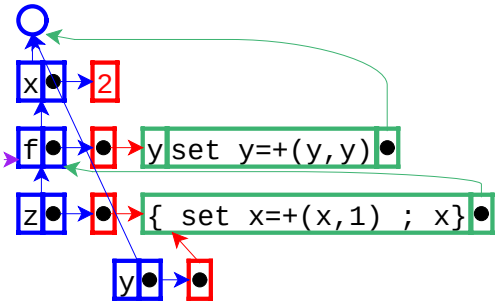
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Another use of y means we eval the thunk again

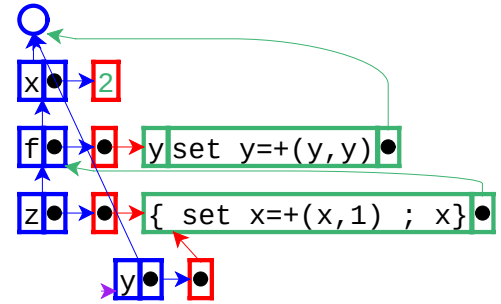
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Thunk changes value of x to 2

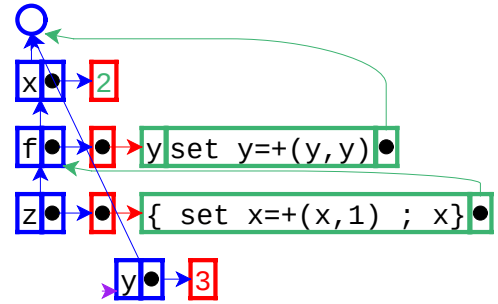
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result for second use of y is 2

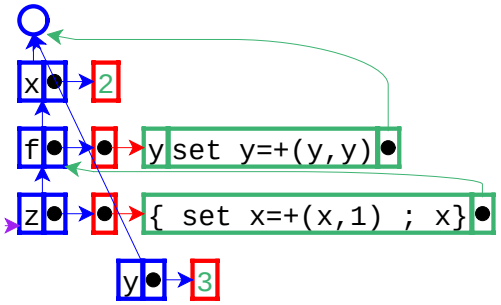
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Value for y changed to 3 (= 1 + 2)

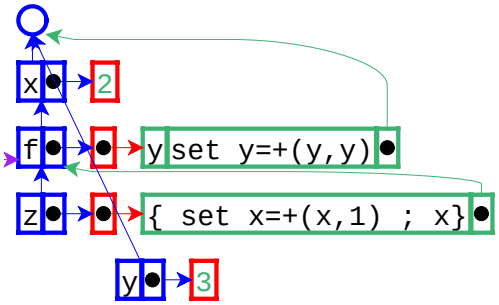
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result is the value of z...

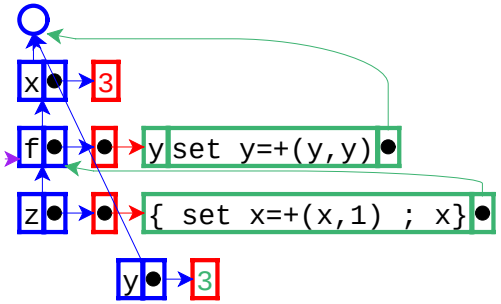
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- ... which means eval the thunk again

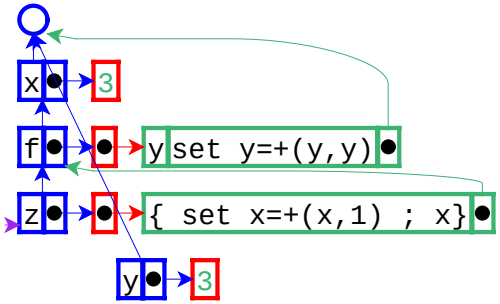
call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Thunk changes value of x to 3

call-by-name




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

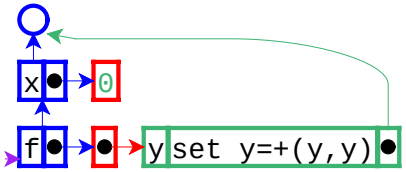
- So 3 is the final result

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Starting call-by-need...

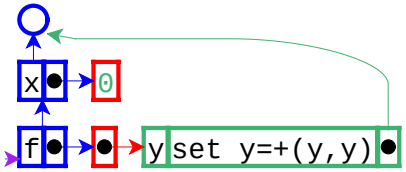

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Eval RHSs




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

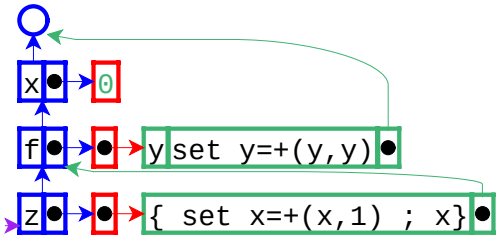
- Simple expressions: bind x and f to 0 and closure, respectively




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Handle RHS of z...

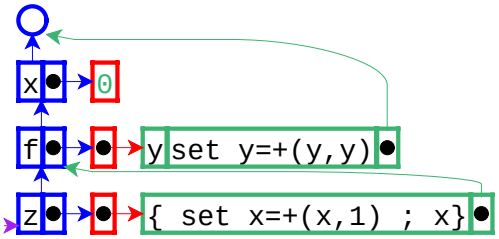
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- ... by creating a thunk for z

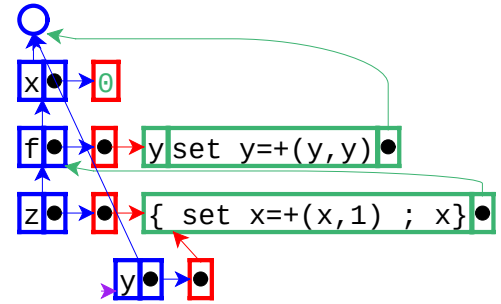
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call f with z

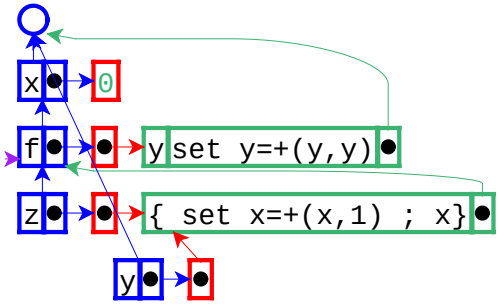
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Not by-reference; y gets a new location, containing the same thunk as z's location

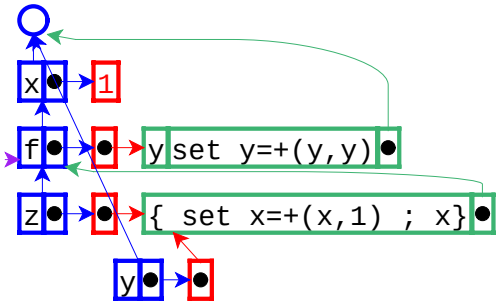
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Use of y means we eval the thunk

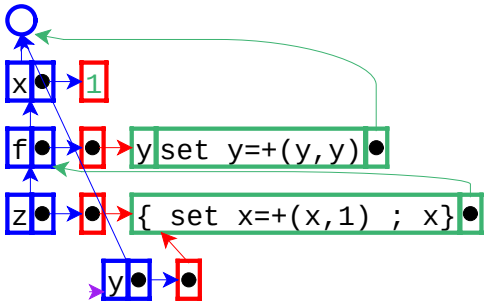
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Thunk changes value of x to 2

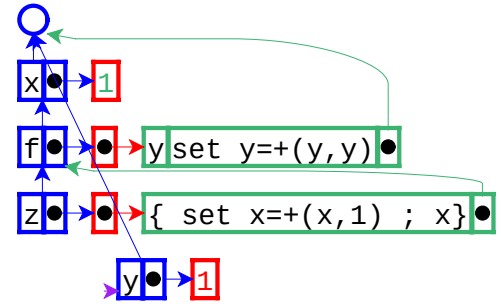
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result from first use of y was 1

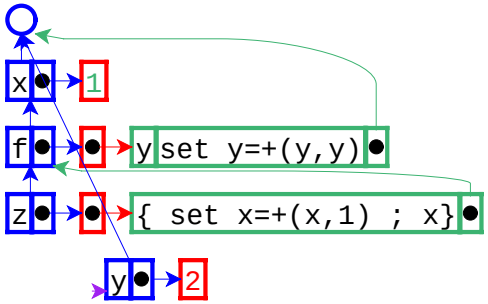
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Since this is call-by-need, install the 1 into y

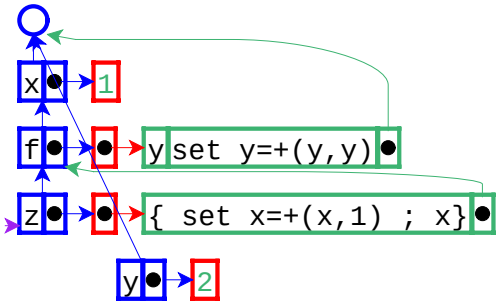
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Second use of y gets 1, set y to 2 (= 1+1)

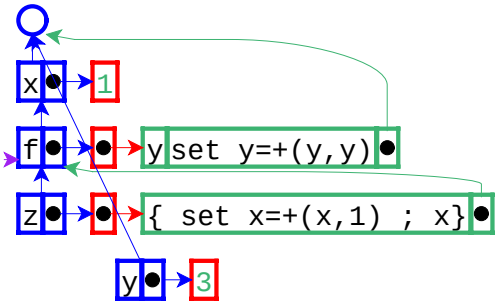
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result is value of z...

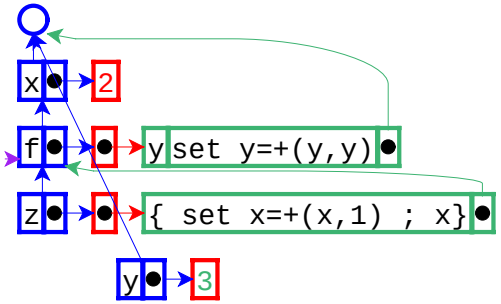
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- ... which means eval the thunk again (see note at end of this section)

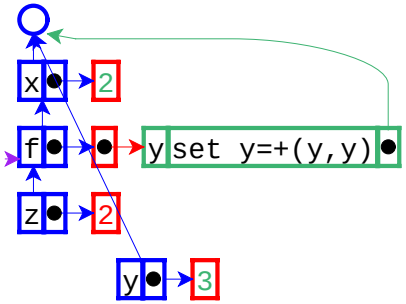
call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

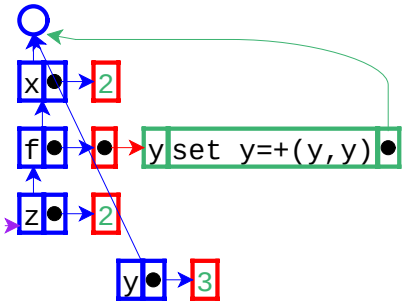
- Thunk changes value of x to 2

call-by-need




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result of thunk is 2; install result into z




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Fnal result is from z: 2

Note:

- Our interpreter implements a strange kind of call-by-need, where using a variable in a function call can cause a thunk to be evaluated multiple times.
- This strangeness is an artifact of supporting call-by-reference, where we alway treating variable arguments specially (even in the call-by-value case).

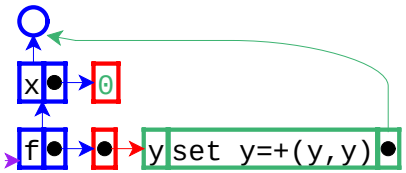

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Starting call-by-name combined with call-by-reference...

call-by-name/ref

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

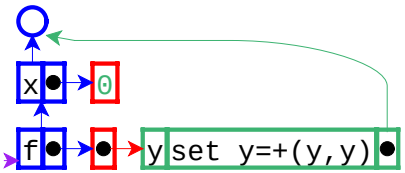
- Eval RHSs




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Simple expressions: bind x and f to 0 and closure, respectively

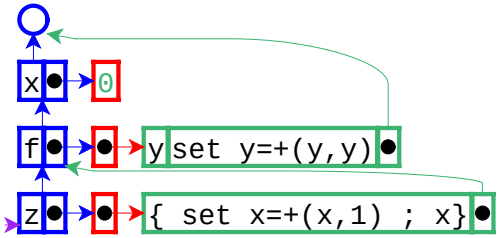
call-by-name/ref




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Handle RHS of z...

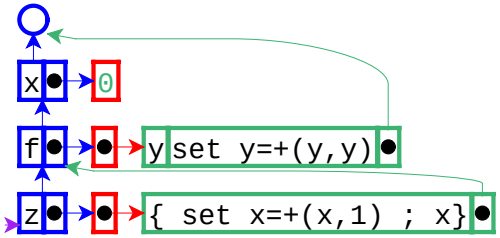
call-by-name/ref




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

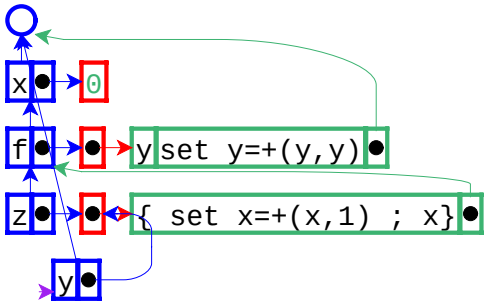
- ... by creating a thunk for z

call-by-name/ref



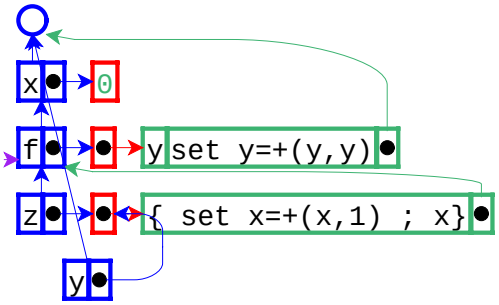

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call f with z



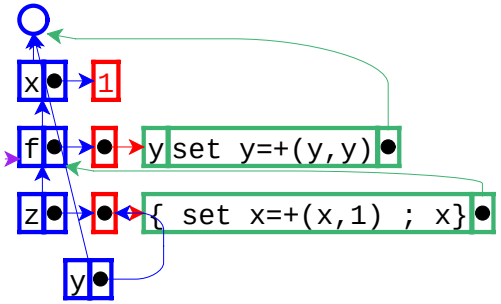

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Call-by-reference shares location for z with y



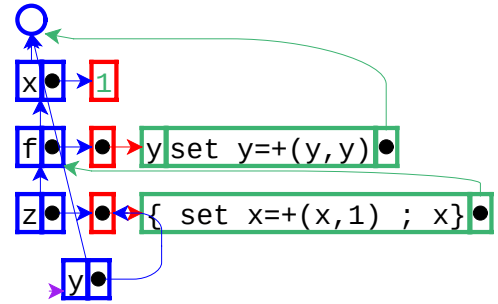

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- First use of y triggers evaluation of the thunk



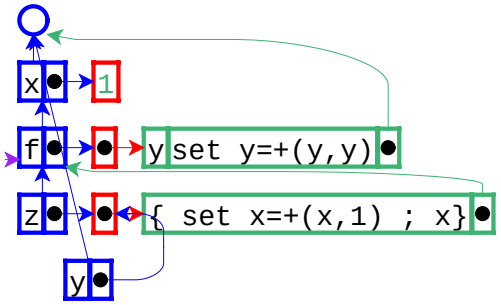

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Thunk changes value of x to 1



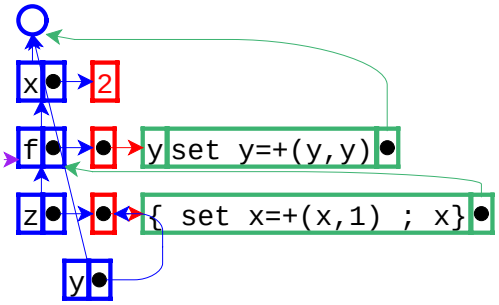

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result for first y is 1



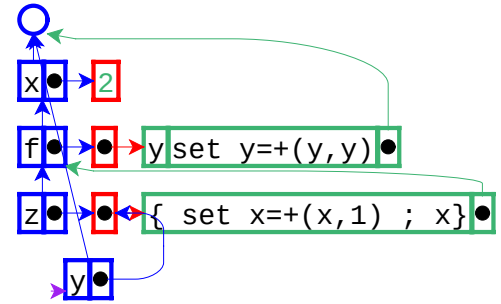

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Second use of y triggers evaluation of the thunk



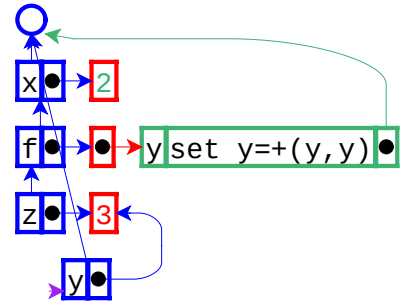

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Thunk changes value of x to 2



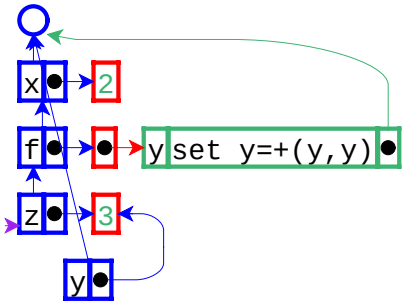

```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Result for second y is 2




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Set value of y to 3 (= 1+2)




```
let x = 0
    f = proc(y) set y=+(y,y)
 in let z = { set x=+(x,1) ; x }
     in { (f z) ;  z }
```

- Final result is the value of z: 3