Rounding Out Structural Rounding

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Noisy Networks

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- Exact or approximate algorithms for instances in structural classes.
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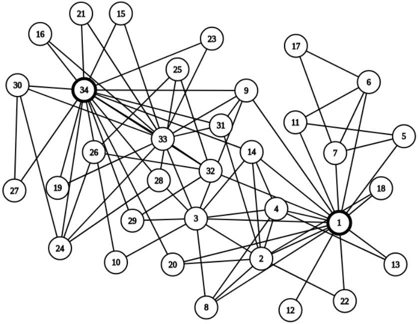
- Exact or approximate algorithms for instances in structural classes.
- But, real-world networks are noisy.
- Real-world networks tend to lie *close* to a structural class.

Noisy Networks: Social Networks close to Bounded Degree

Karate Club Network



- Max. degree = 17.

Noisy Networks: Social Networks close to Bounded Degree

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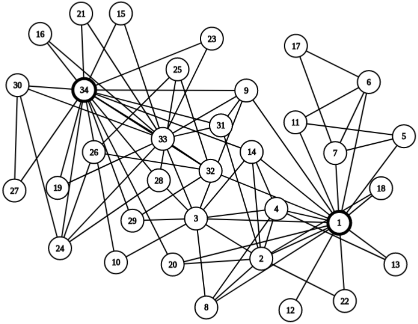

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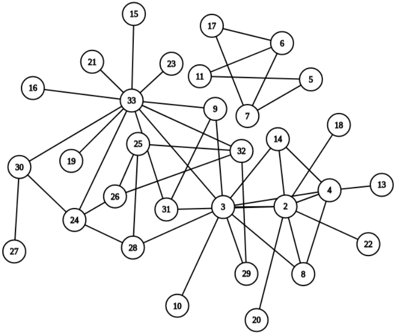
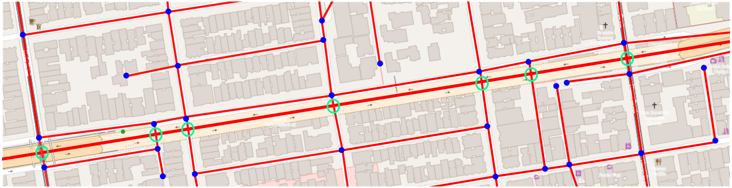
After Vertex Deletion



- Max. degree = 17.
- Max. degree = 11.

Noisy Networks: Road Networks[1] close to Planar

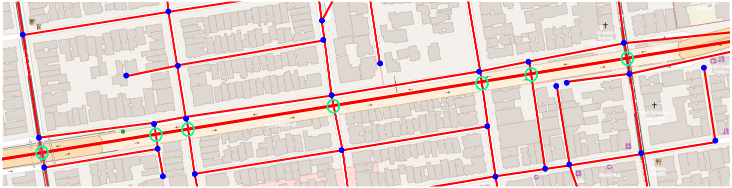
Road Network



[1] Road network image from (Eppstein & Gupta, 2017)

Noisy Networks: Road Networks[1] close to Planar

Road Network



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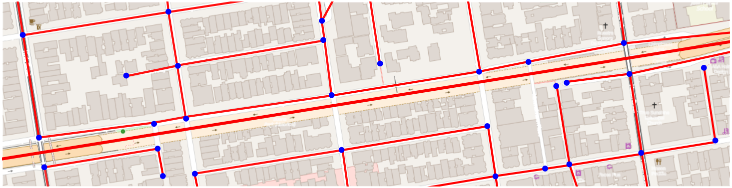
Planar After Edge Deletions



[1] Road network image from (Eppstein & Gupta, 2017)

Noisy Networks: Prior Work

- Prior work edits noisy graphs to a structural class but discard the noise[2].
- Structural Rounding gives constant factor approximations on noisy graphs *close* to a structural class.
- First looks at the structured part, then accounts for the noise.

[2](Magen & Moharrami, 2009), (Chan & Har-Peled, 2012), (Bansal & Umboh, 2017)

Structural Rounding Framework

- **Edit**[3]**:** Use edit operation φ to edit G into the class C. Let G' be the edited graph.
  - A problem is **stable** wrt φ with constant c', with d edits if $\mathrm{OPT}(G') < \mathrm{OPT}(G) + c' \cdot d$.

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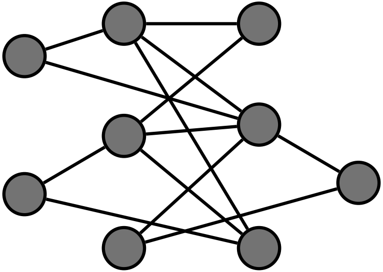
- **Solve:** on G' with algorithms for instances in class C.
- **Lift:** Extend solution on G' to G.
  - A problem can be structurally **lifted** wrt φ with constant c and a solution S' on G' can be converted in poly-time to a solution S on G with $\text{cost}(S) < \text{cost}(S') + c \cdot d$.

[3] **Edit**, **Solve** and **Lift** can be solved approximately or exactly

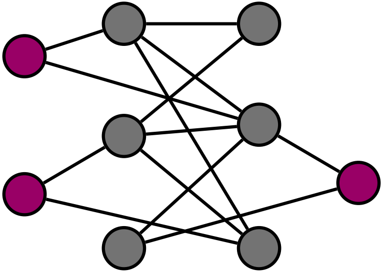
Vertex Cover in Near Bipartite Graphs



Noisy input graph G.

- The Vertex Cover problem takes as input an undirected graph $G = (V, E)$, and finds a subset $S \subseteq V$ such that every edge in E has at least one endpoint in S.

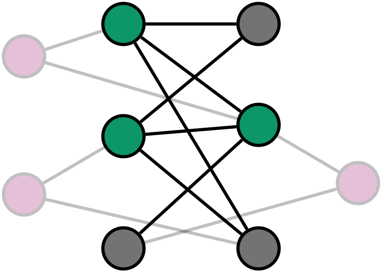
Vertex Cover in Near Bipartite Graphs: **Edit**



Edit G to a bipartite graph G'.

- Odd Cycle Transversal for editing to bipartite.
- **Stability** constant $c' = 0$ for Vertex Cover wrt vertex deletions.

VERTEX COVER in Near Bipartite Graphs: **Solve**



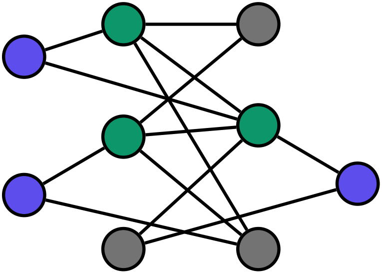
Solution S' on G'.

- **Solve** VERTEX COVER exactly in poly-time on G' using Hopcroft-Karp algorithm[4].

[4] Hopcroft and Karp, 1973

Vertex Cover in Near Bipartite Graphs: **Lift**

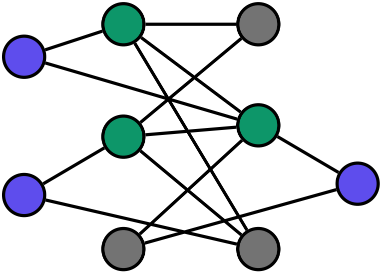
Naïve Lifting



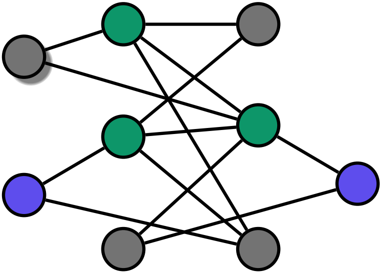
- **Lift** solution S' from edited graph G' to a solution S for the noisy graph G.
- **Lifting** constant $c = 1$ for Vertex Cover wrt vertex deletions.

Vertex Cover in Near Bipartite Graphs: **Lift**

Naïve Lifting



Greedy Lifting



- **Lift** solution S' from edited graph G' to a solution S for the noisy graph G.
- **Lifting** constant $c = 1$ for Vertex Cover wrt vertex deletions.

Structural Rounding Theorem

Theorem (Demaine et al, 2019)

Let π be a minimization problem

[5] Allows editing to family of graphs

[6] Takes $\alpha \cdot \text{OPT}$ edits to reach the class $\mathcal{C}_{\beta\lambda}$

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then there is a poly-time $((1 + c'\alpha\delta) \cdot \rho(\beta\lambda) + c\alpha\delta)$-approximation for π on graphs that are $(\delta \cdot \mathrm{OPT}_\pi(G))$-close to $\mathcal{C}_\lambda$.

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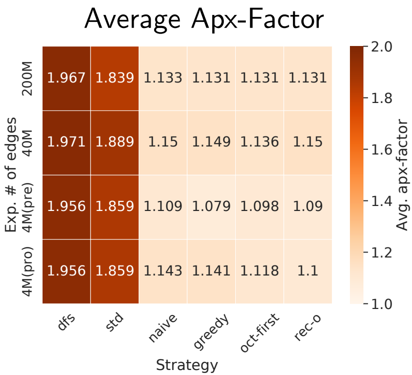
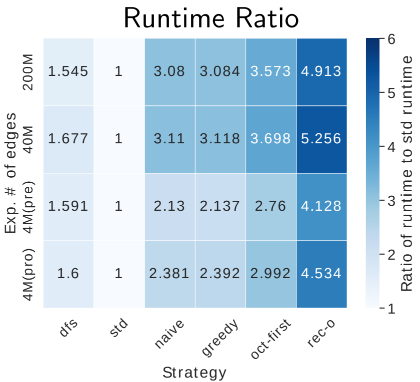
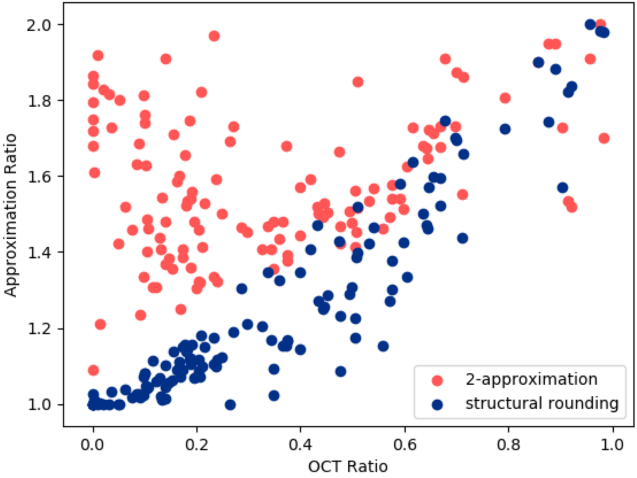
Structural Rounding for Vertex Cover: Results[7]



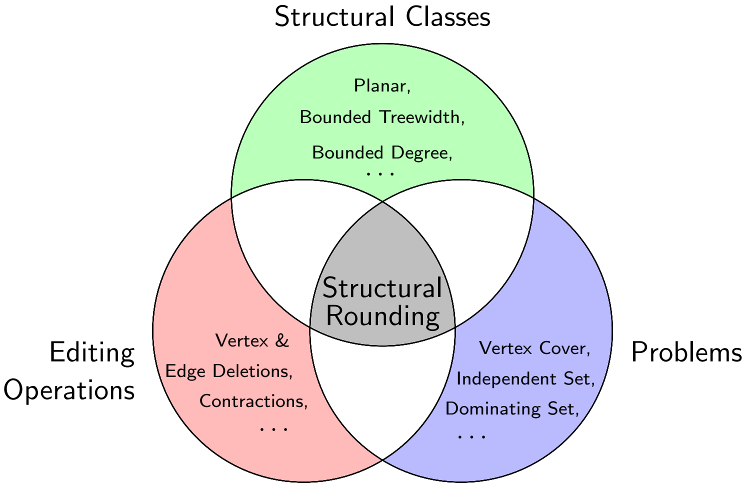


[7] B. Lavallee, H. Russell, B. D. Sullivan, and A. van der Poel. Approximating Vertex Cover using Structural Rounding. *ALENEX*, 2020

Structural Rounding for Vertex Cover: Results



Structural Rounding: Broad Applicability



Structural Rounding: Positive Editing Results

Graph Family $\mathcal{C}_\lambda$	**Bicriteria Approximations[8] into the class $\mathcal{C}_\lambda$**	
	Vertex Deletion	**Edge Deletion**
Bounded Degeneracy (r)	$(4,4)$-approx. (Density Based) $(6,6)$-approx. (LP-Based) $O(\log n)$-approx. (Greedy)	$(5,5)$-approx. $O(\log n)$-approx. (Greedy)
Bounded Treewidth (w)	$(O(\log^{1.5} n),\ O(\sqrt{\log w}))$-approx.	$(O(\log n \log\log n),\ O(\log w))$-approx.[9]
Bounded Pathwidth (w)	$(O(\log^{1.5} n),\ O(\sqrt{\log w} \cdot \log n))$-approx.	–

[8] $\alpha \cdot \mathrm{OPT}$ edits to reach the class $\mathcal{C}_{\beta\lambda}$

[9] Bansal et al, 2017

Limitations: Editing

- Editing algorithms are slow.
- Constants are too large.
- Editing is a well-studied area of research.

New Directions: Maximum Subgraphs

- Maximum subgraphs vs. approximate edit sets.
- Polylogarithmic approximation for editing to planar[10] and constant factor approximation for maximum subgraph[11].
- Accommodate the difficulty of editing to specific classes.

[10] Kawarabayashi et al., 2017

[11] Călinescu et al., 1998

New Directions: Dominating Set Problem

- Dominating Set on noisy graphs.
- Classically hard problem.
- Dominating Set and variants have applications in document summarization, ad-hoc networks, social networks, and communication networks.

Thanks!

- E. D. Demaine, T. D. Goodrich, K. Kloster, B. Lavallee, Q. C. Liu, B. D. Sullivan, A. Vakilian, and A. van der Poel, "Structural Rounding: Approximation Algorithms for Graphs Near an Algorithmically Tractable Class," *ESA*, 2019. https://arxiv.org/abs/1806.02771.
- B. Lavallee, H. Russell, B. D. Sullivan, and A. van der Poel, "Approximating Vertex Cover using Structural Rounding," *ALENEX*, 2020. https://arxiv.org/abs/1909.04611.

www.github.com/theoryinpractice