Probabilistic String Similarity Joins

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Florida State University

Zhepeng Yan Ke Yi

Hong Kong University
of Science and Technology

Introduction

- Uncertainty naturally arises in string data

Yahoo!

Yahoo Labs

Yahoo

Yahoo! Labs

Introduction

- Uncertainty naturally arises in string data

??? Yahoo! ??? Yahoo Labs

Yahoo ??? Yahoo! Labs ???

Introduction

- Uncertainty naturally arises in string data

0.10	Yahoo!	0.40	Yahoo Labs
Yahoo	0.15	Yahoo! Labs	0.35

Introduction

- Uncertainty naturally arises in string data

0.10 Yahoo

Yahoo! 0.15

0.40 Yahoo! Labs

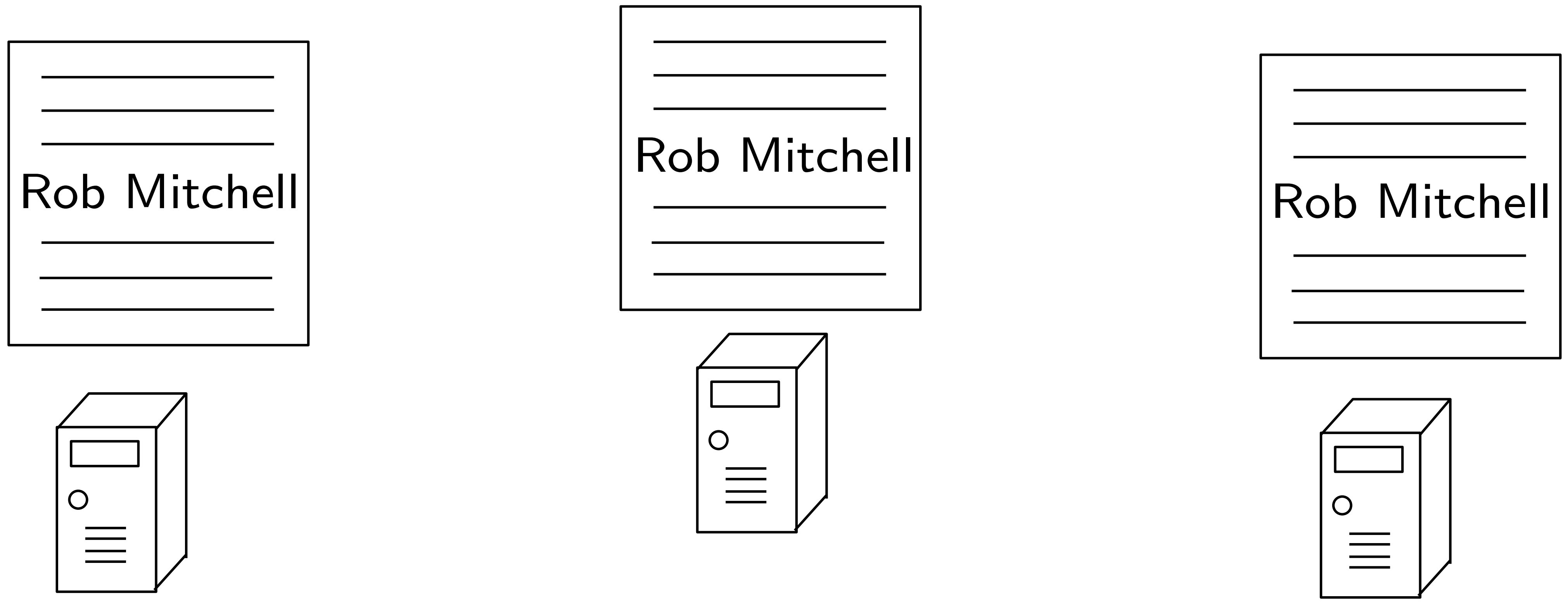
Yahoo Labs 0.35

R	
id	S
1	{ (Microsoft, 0.10), (Microsoft Research, 0.70), (Microsoft Labs, 0.20) }
2	{ (Yahoo, 0.10), (Yahoo!, 0.15), (Yahoo! Labs, 0.40), (Yahoo Labs, 0.35) }
⋮	⋮

$\bowtie_S$

T	
id	S
1	{ (Google, 1) }
2	{ (AT&T, 0.70), (ATT, 0.30) }
⋮	⋮

Probabilistic Strings: Data Integration



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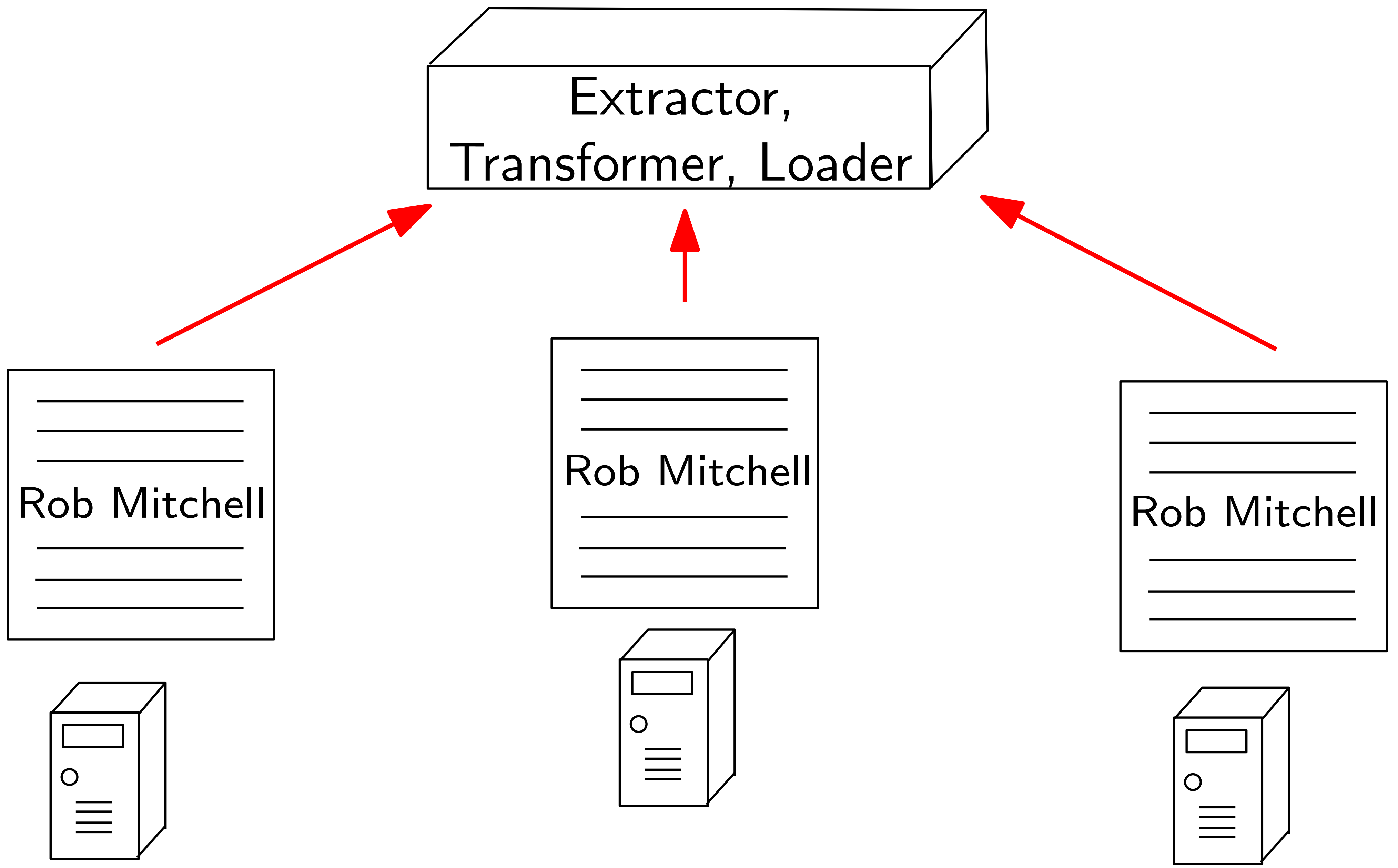

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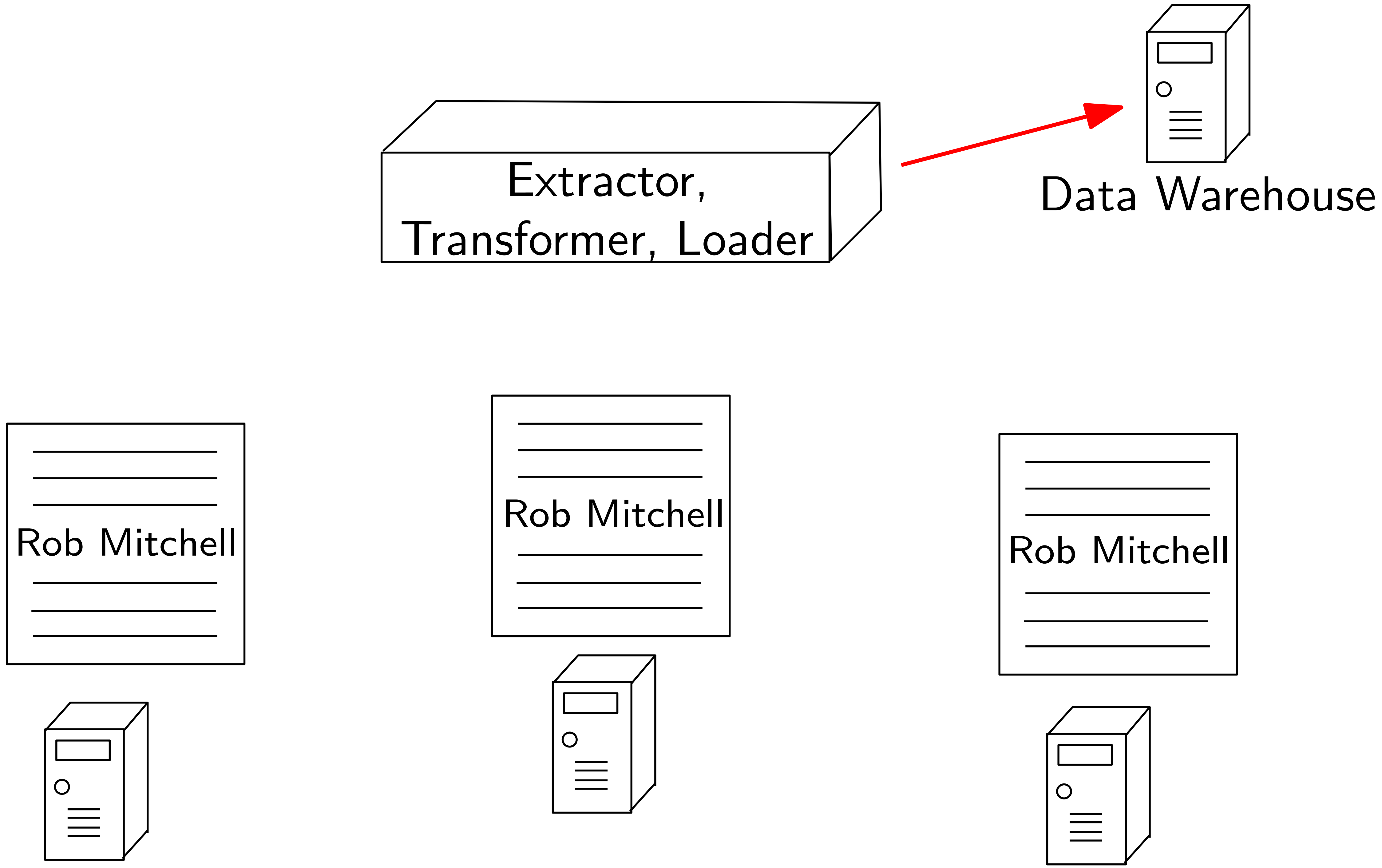

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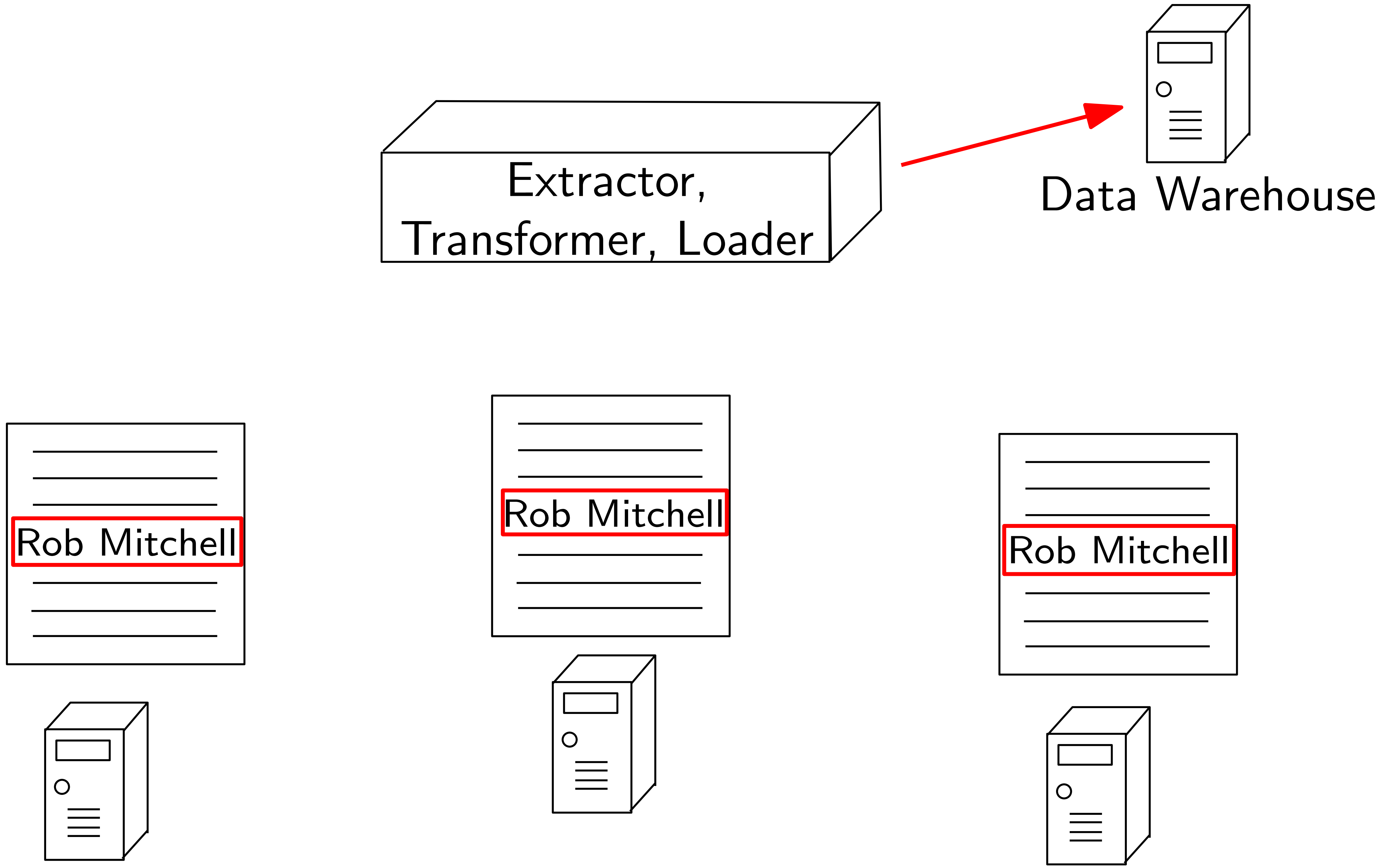

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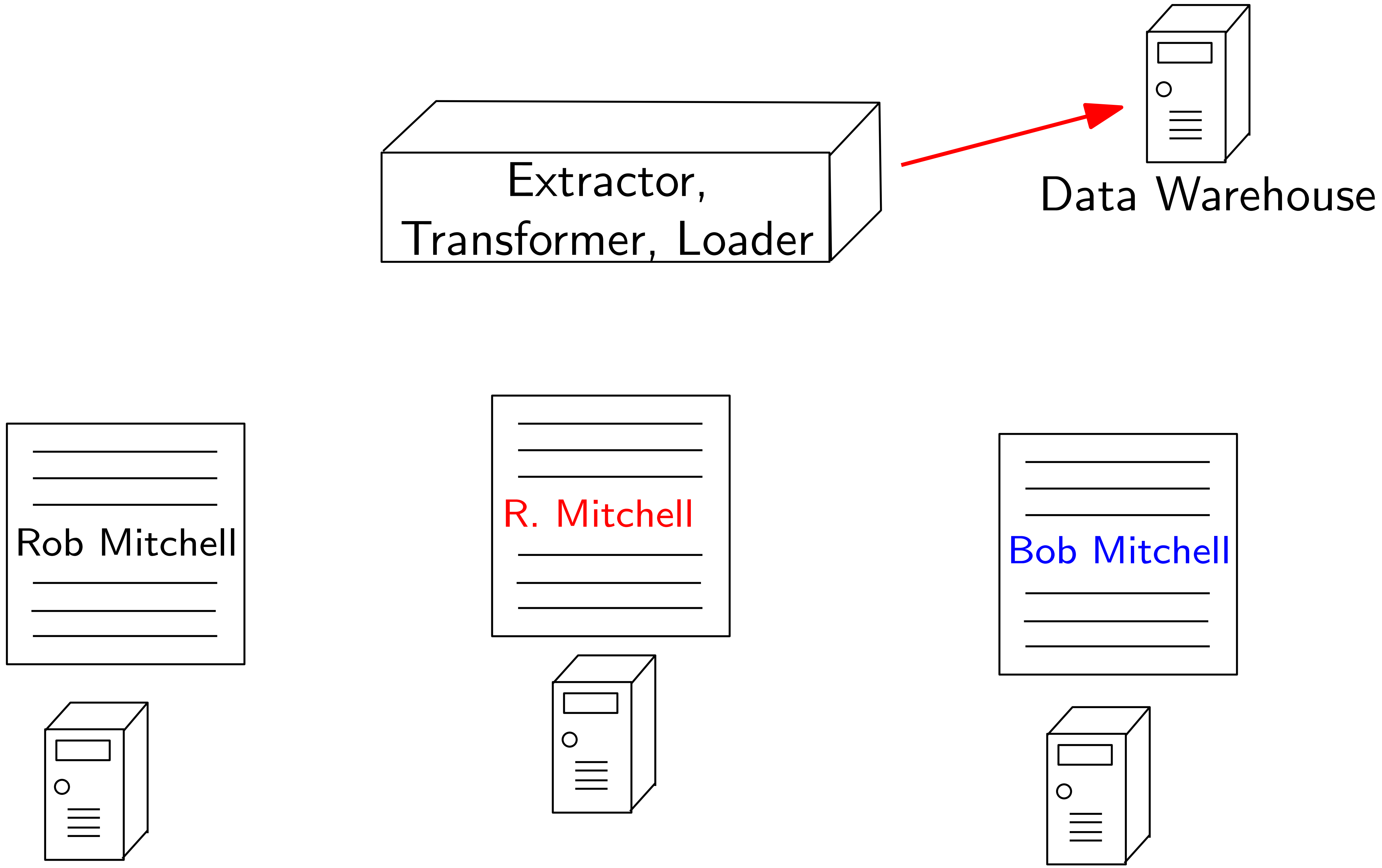

String-Level Probabilistic Model

String S (σ_i's)	Probability (p_i's)
Bob Mitchell	0.75
Rob Mitchell	0.10
Robert Mitchell	0.05
Robby Mitchell	0.10

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S(1) = (σ_1, p_1) = ("Bob Mitchell", 0.75)
S(2) = (σ_2, p_2) = ("Rob Mitchell", 0.10)
S(3) = (σ_3, p_3) = ("Robert Mitchell", 0.05)
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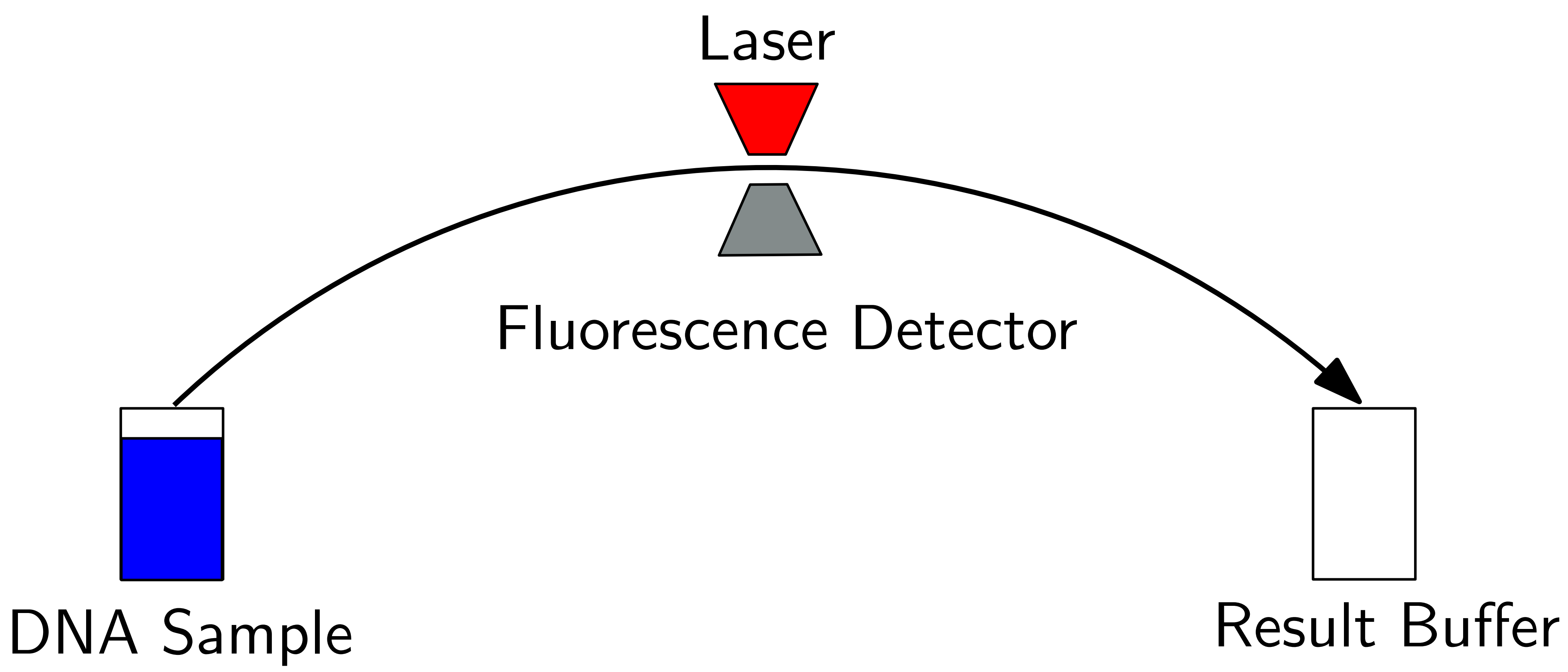
$S = \{(\sigma_1, p_1), (\sigma_2, p_2), \ldots, (\sigma_m, p_m)\}$
where $\sum_{i=1}^{m} p_i = 1$ and $\sigma_i \in \Sigma^*$

Probabilistic Strings: Scientific Applications

DNA Sequencing: Automated Dye-terminator Sequencing



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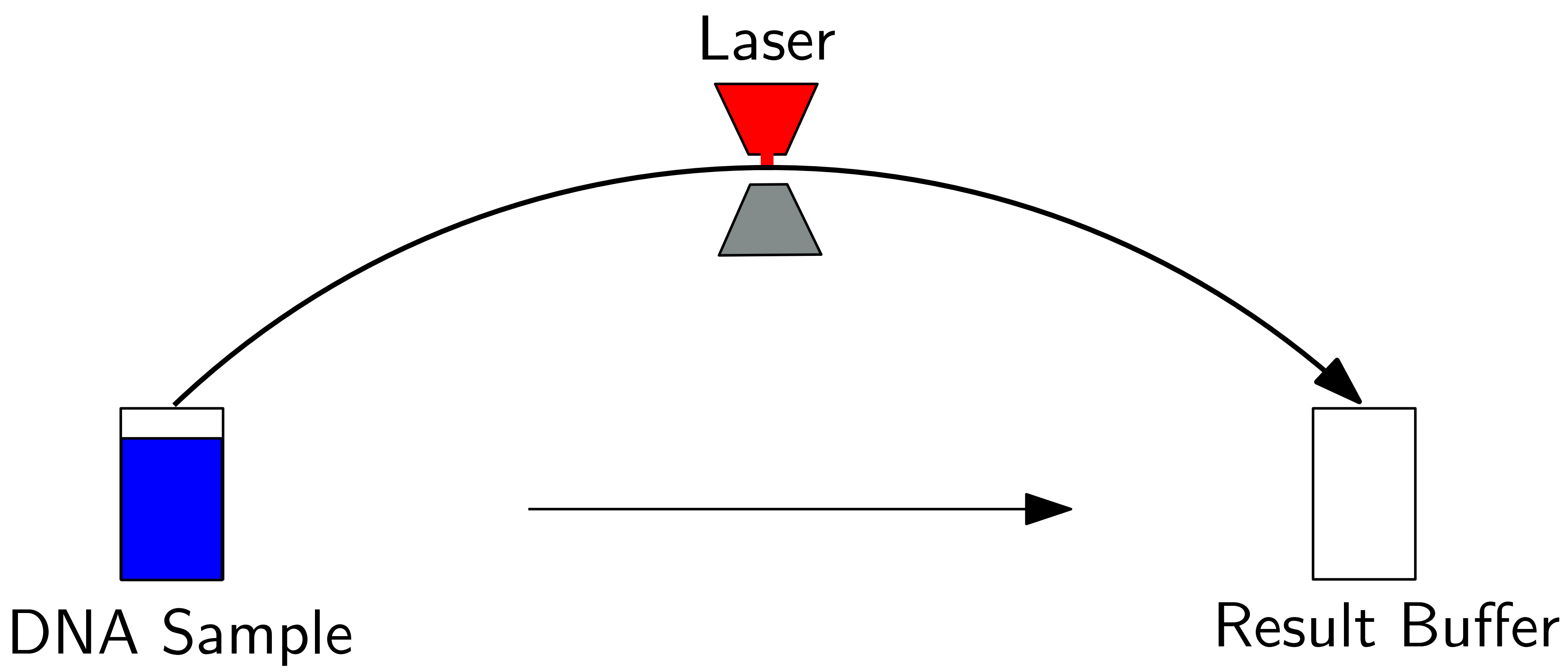

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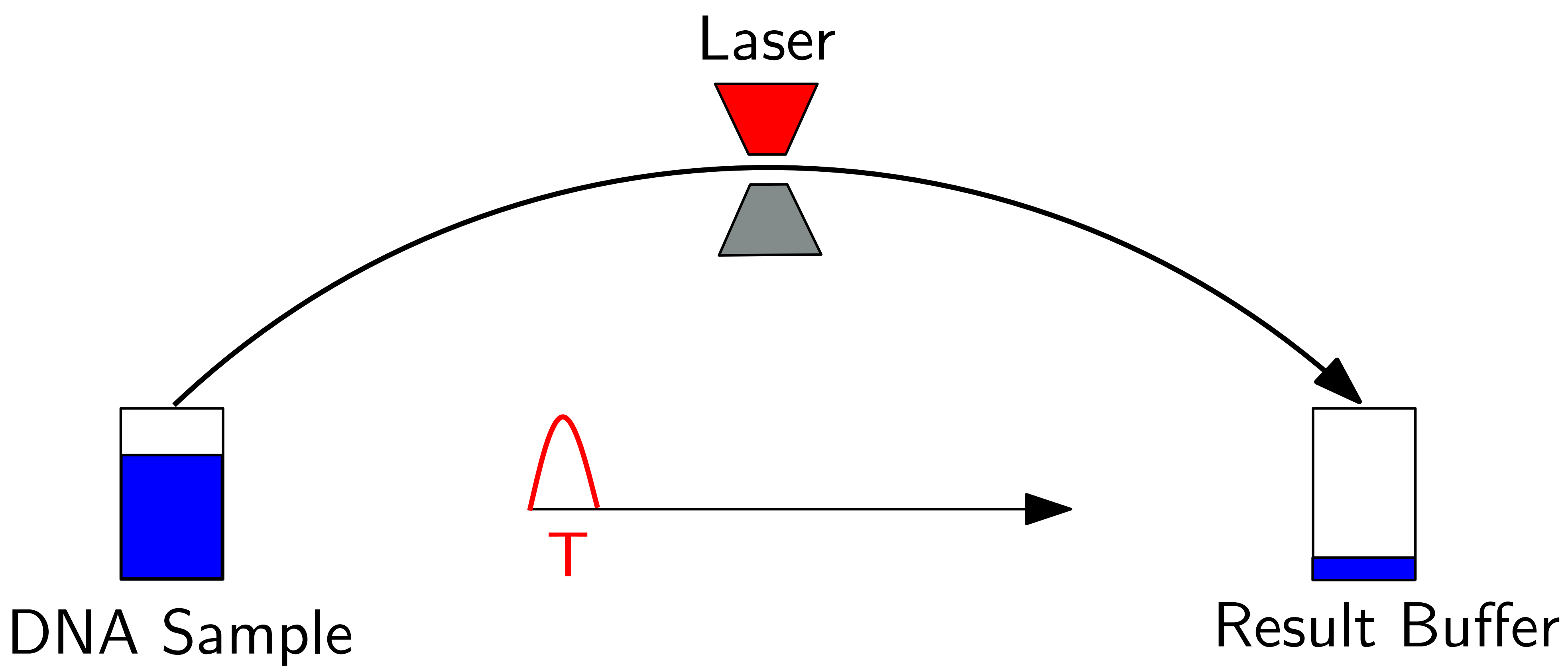

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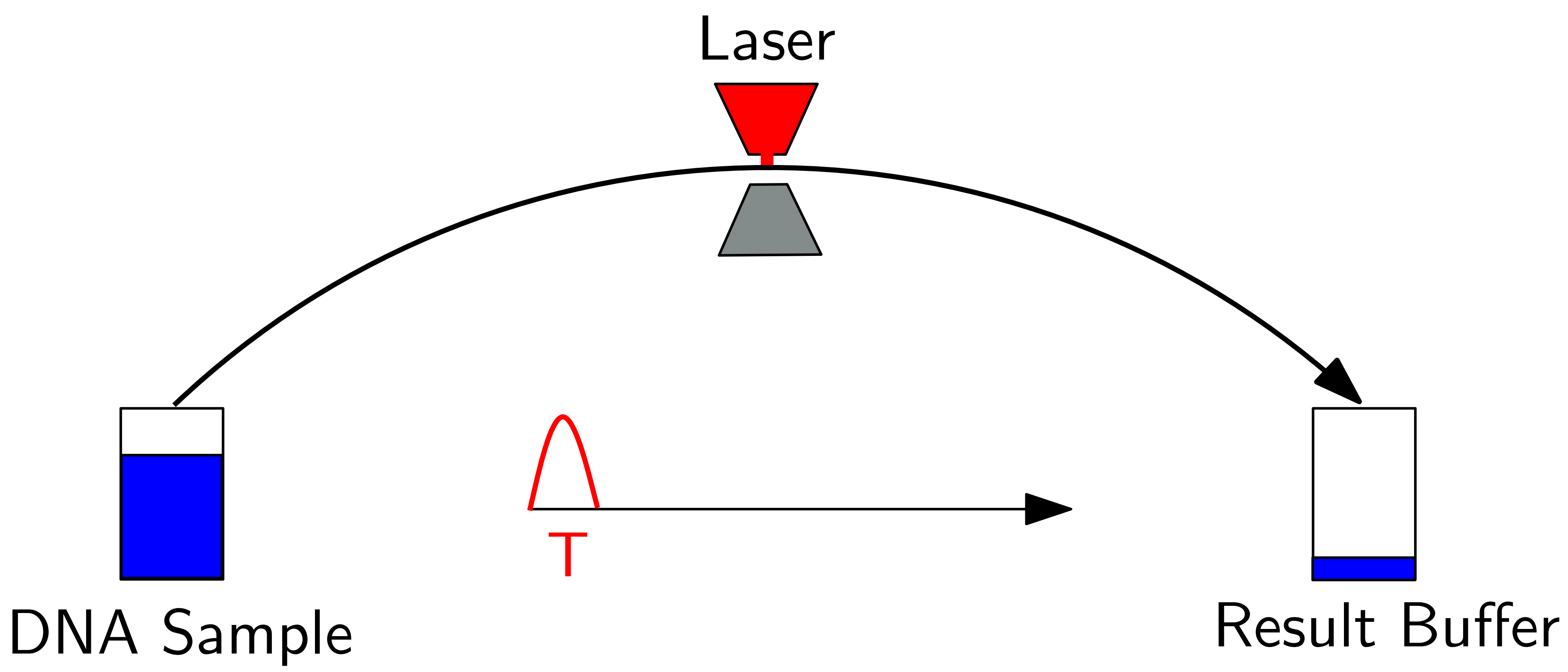

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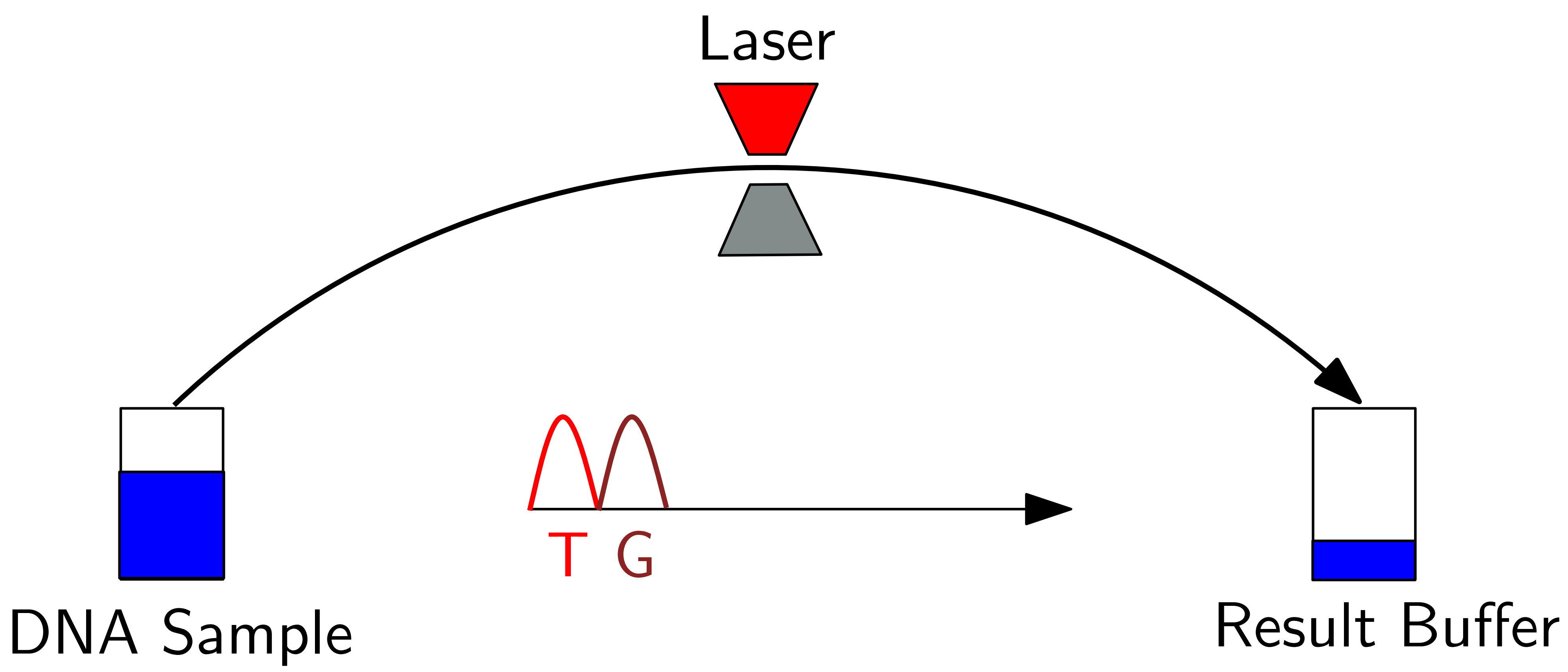

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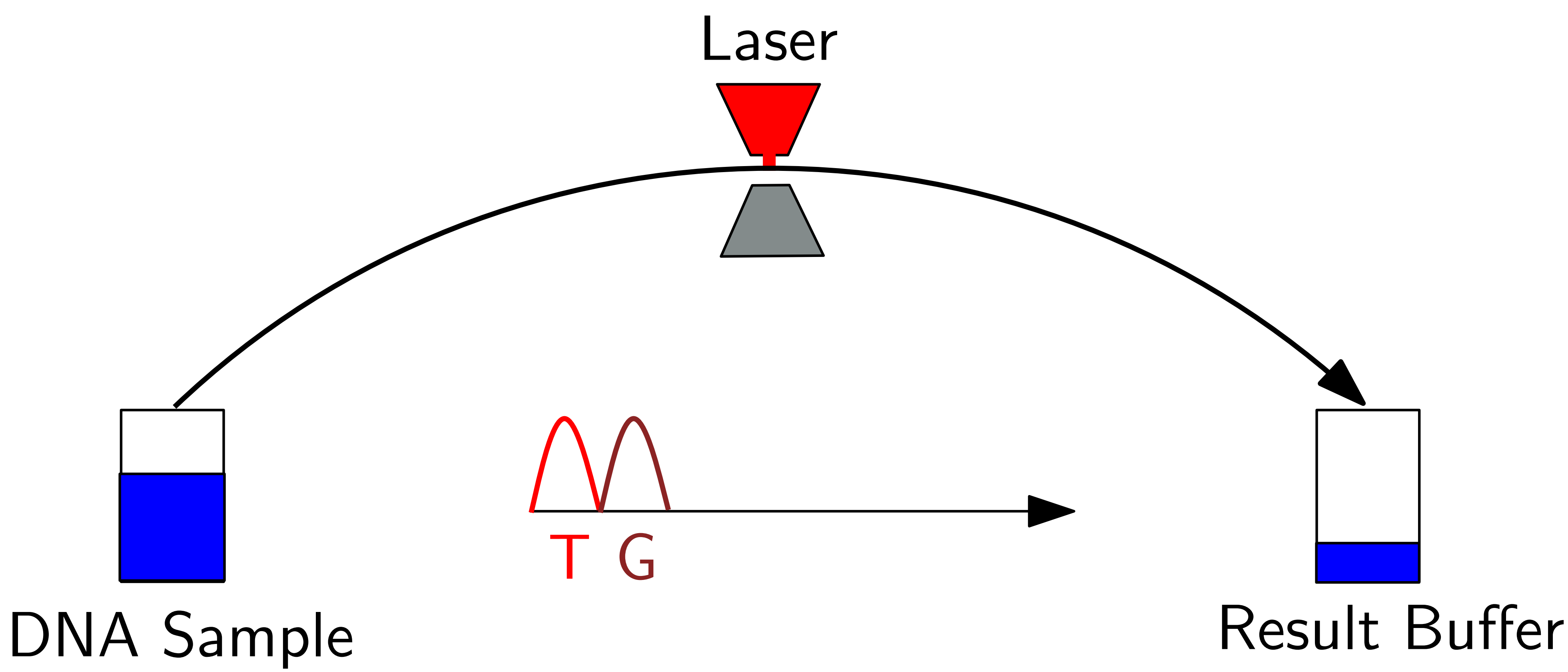

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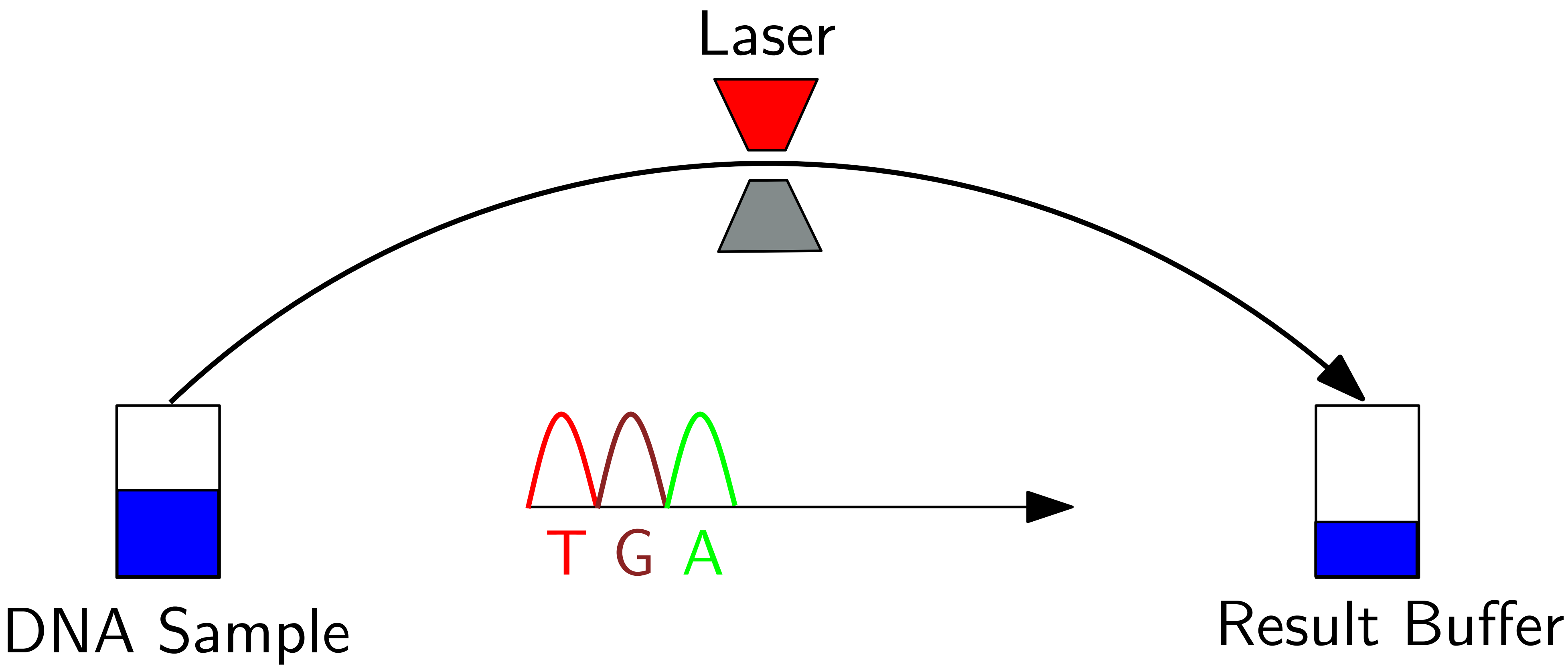

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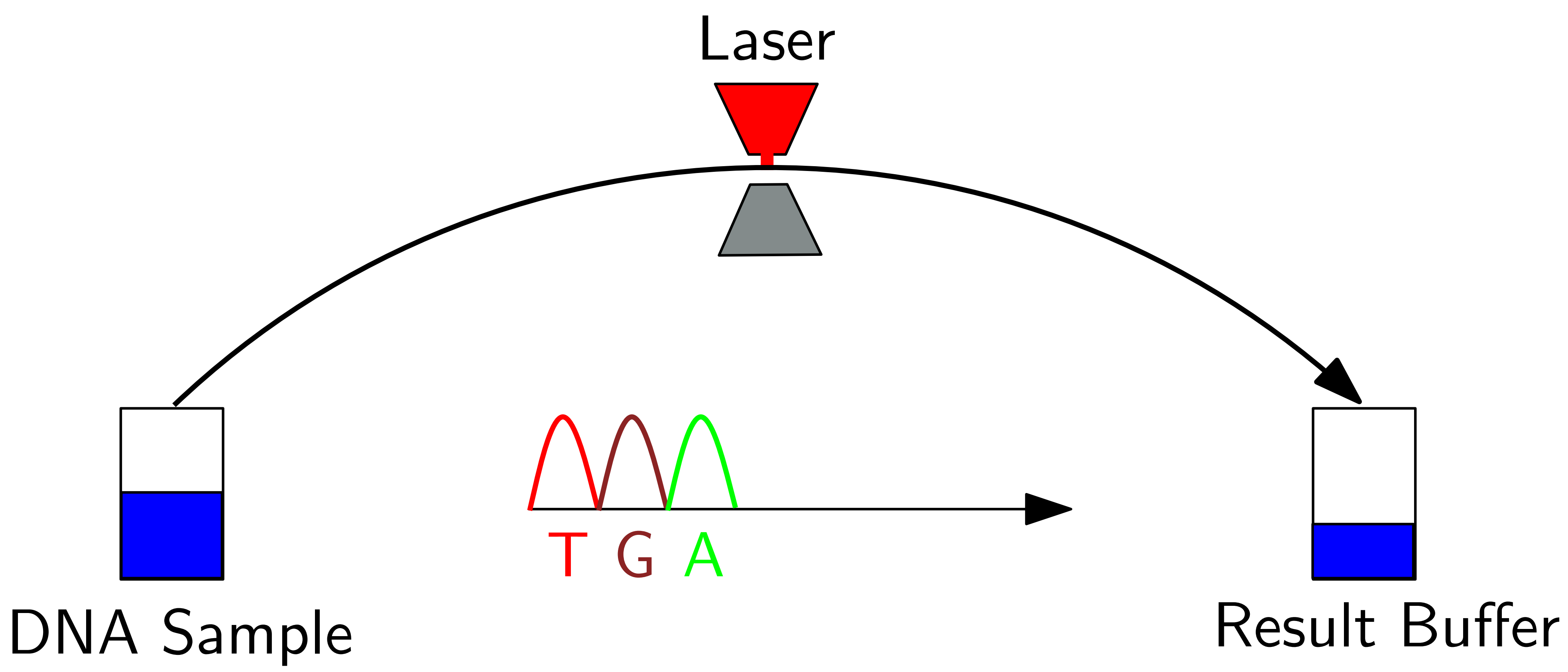

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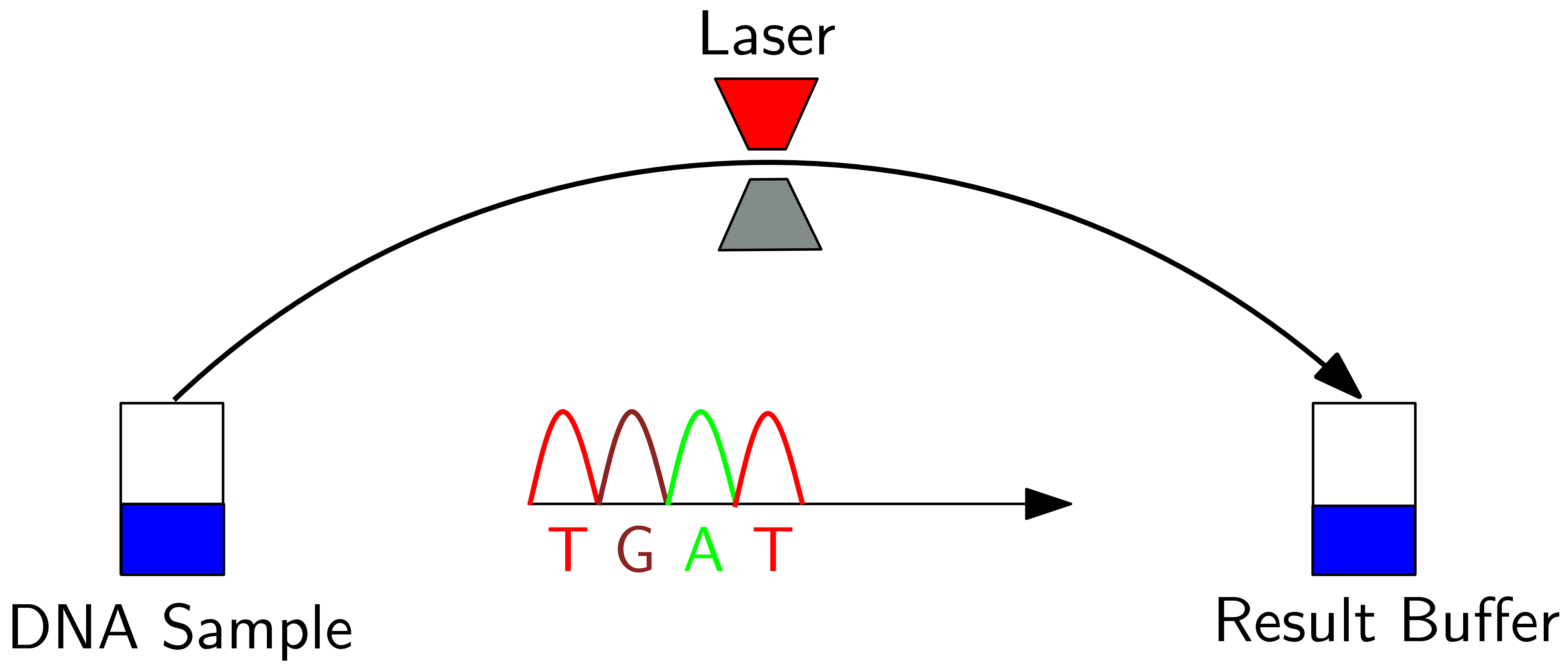

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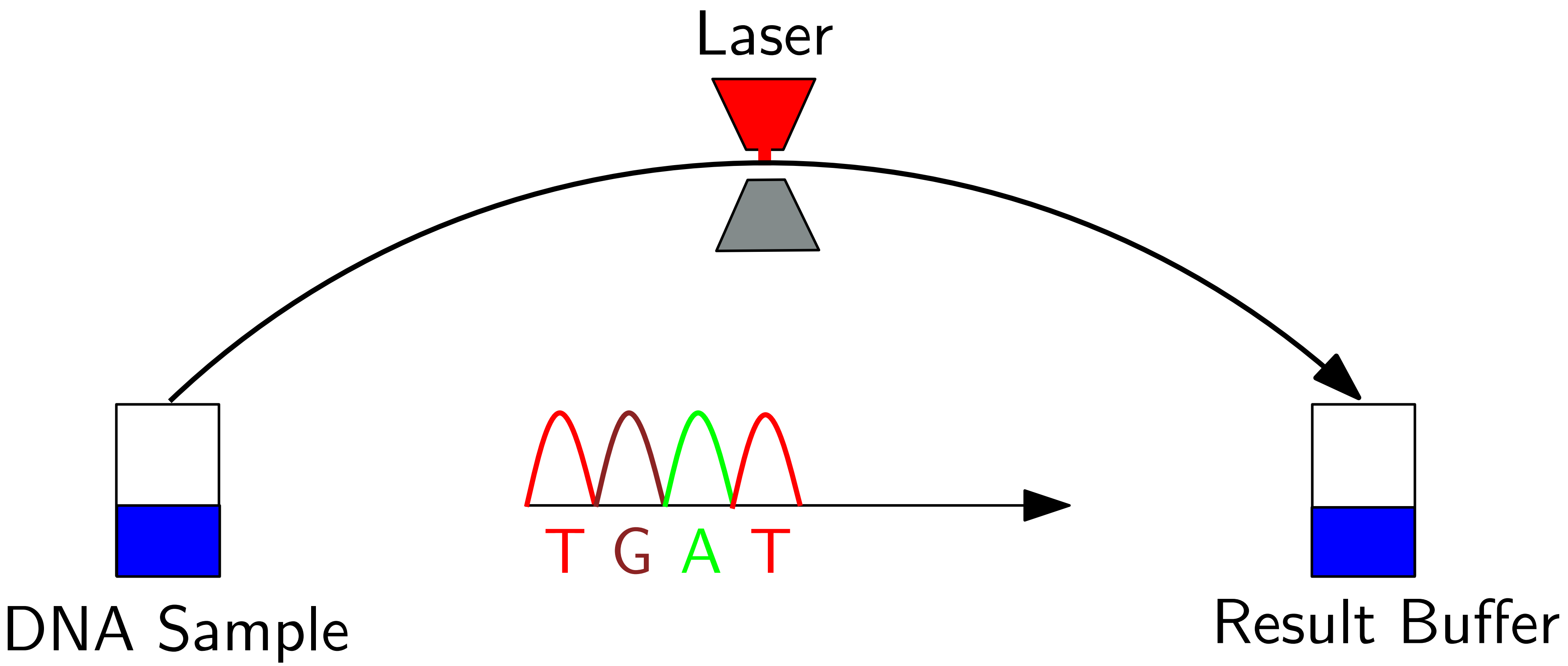

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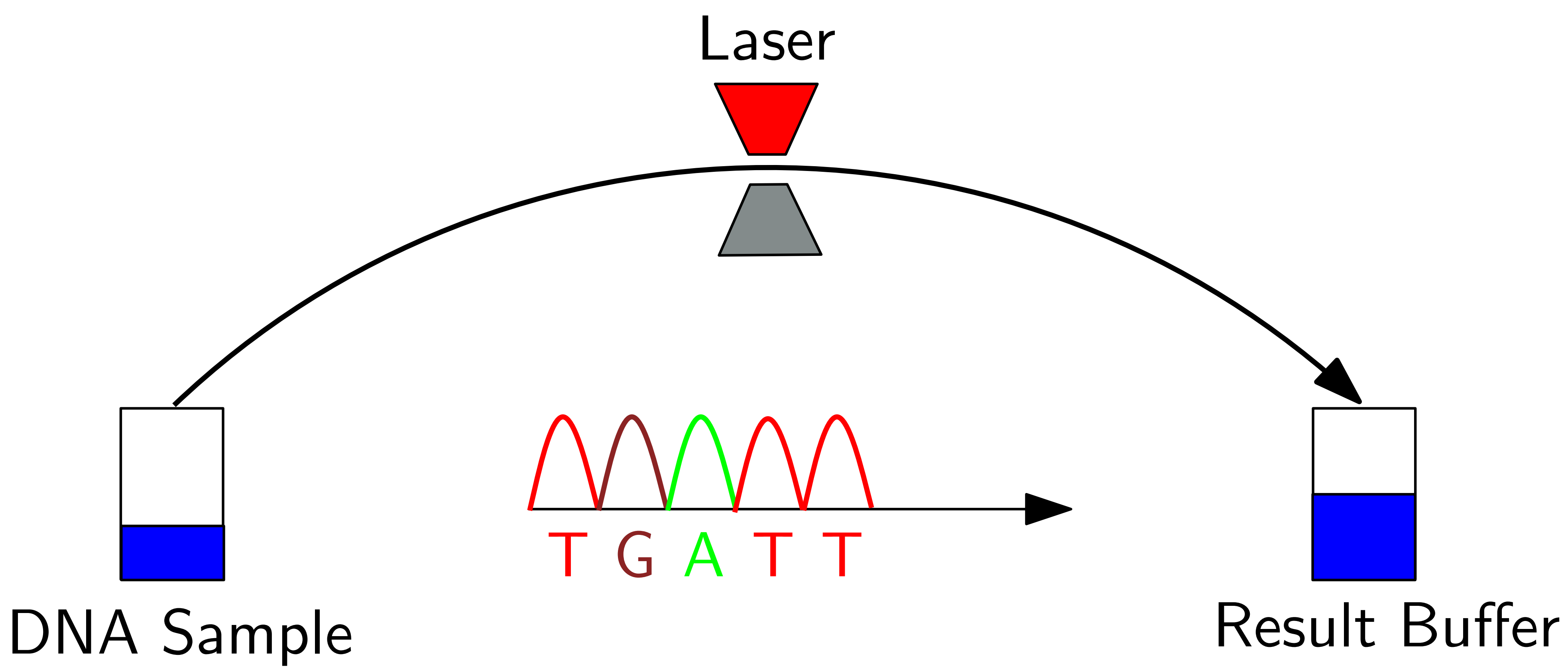

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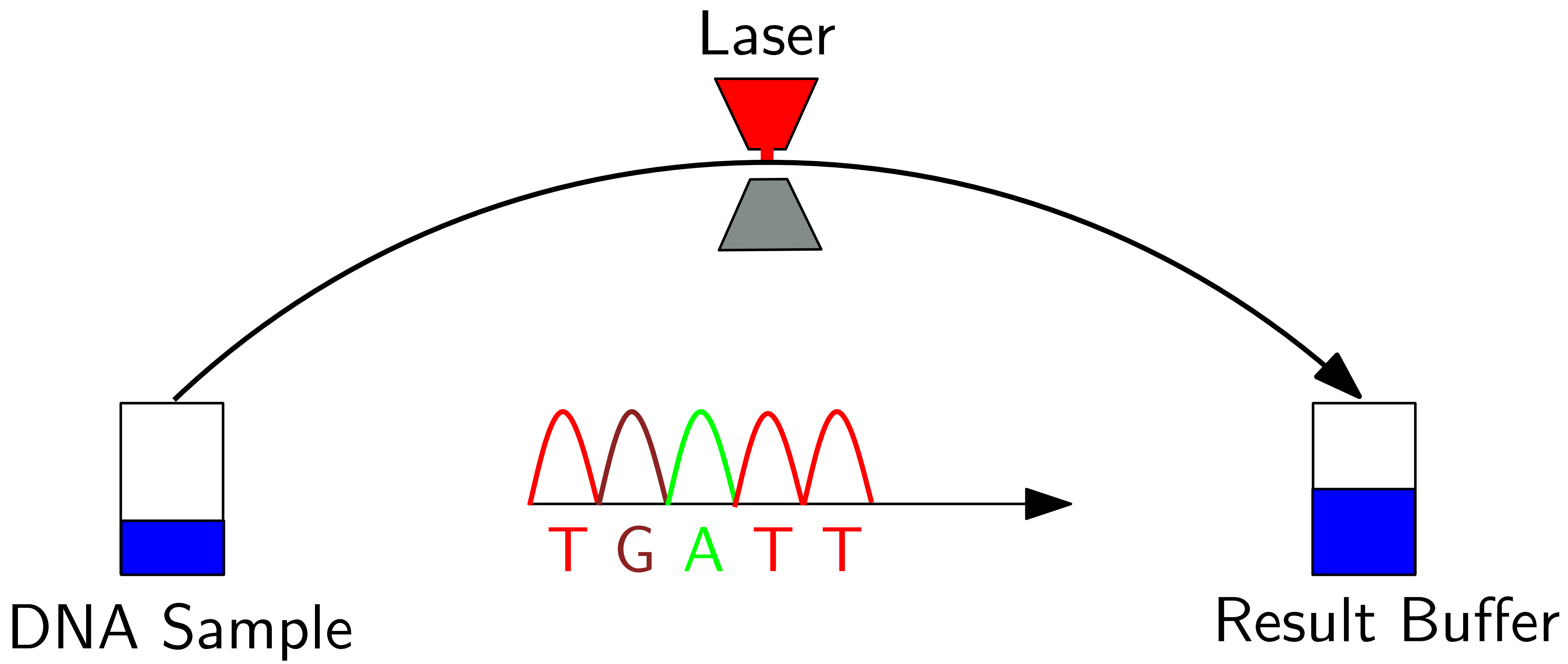

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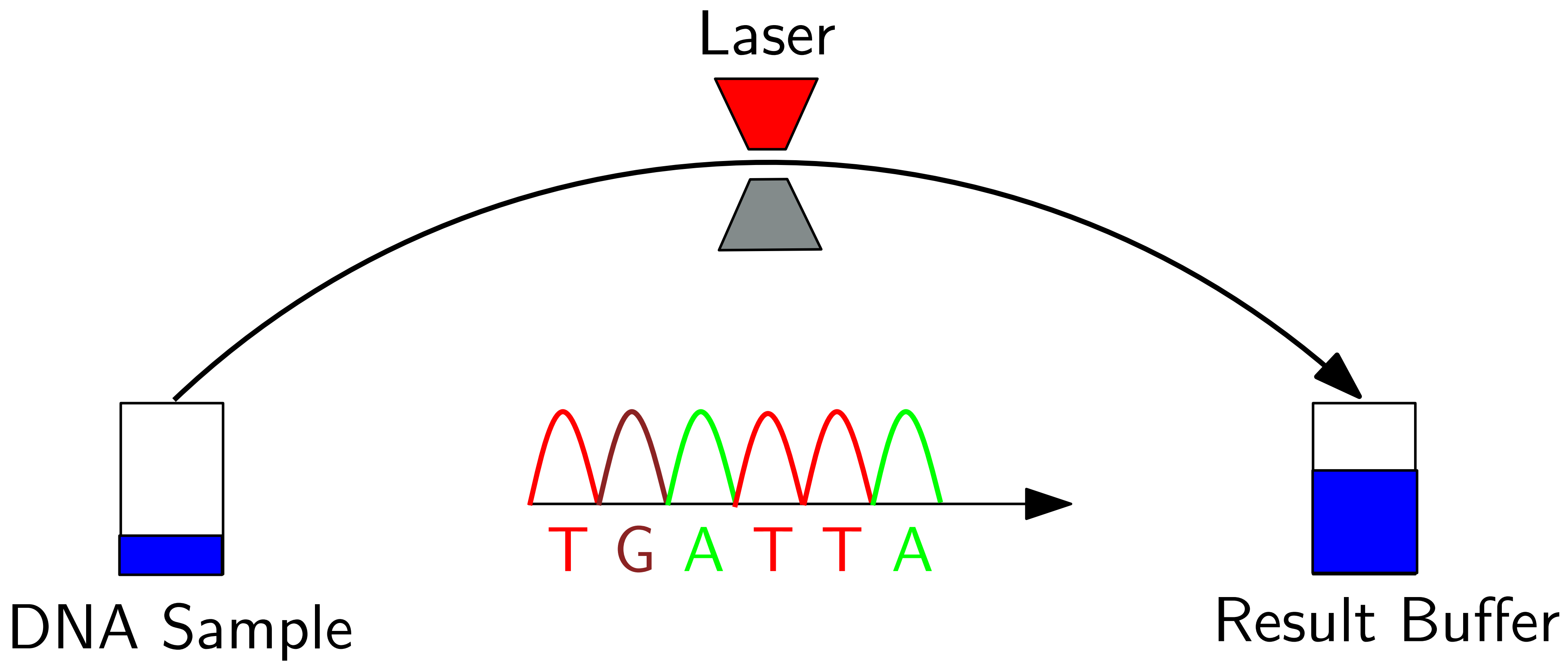

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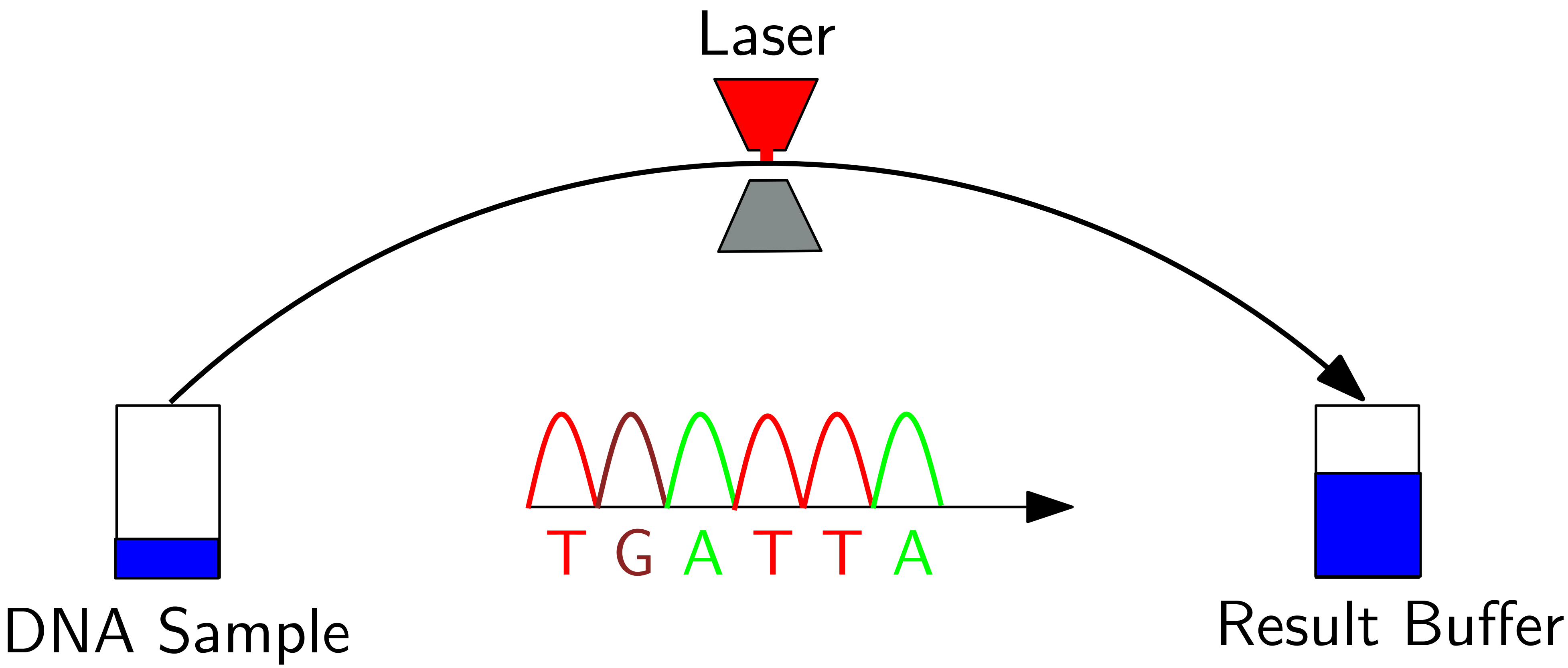

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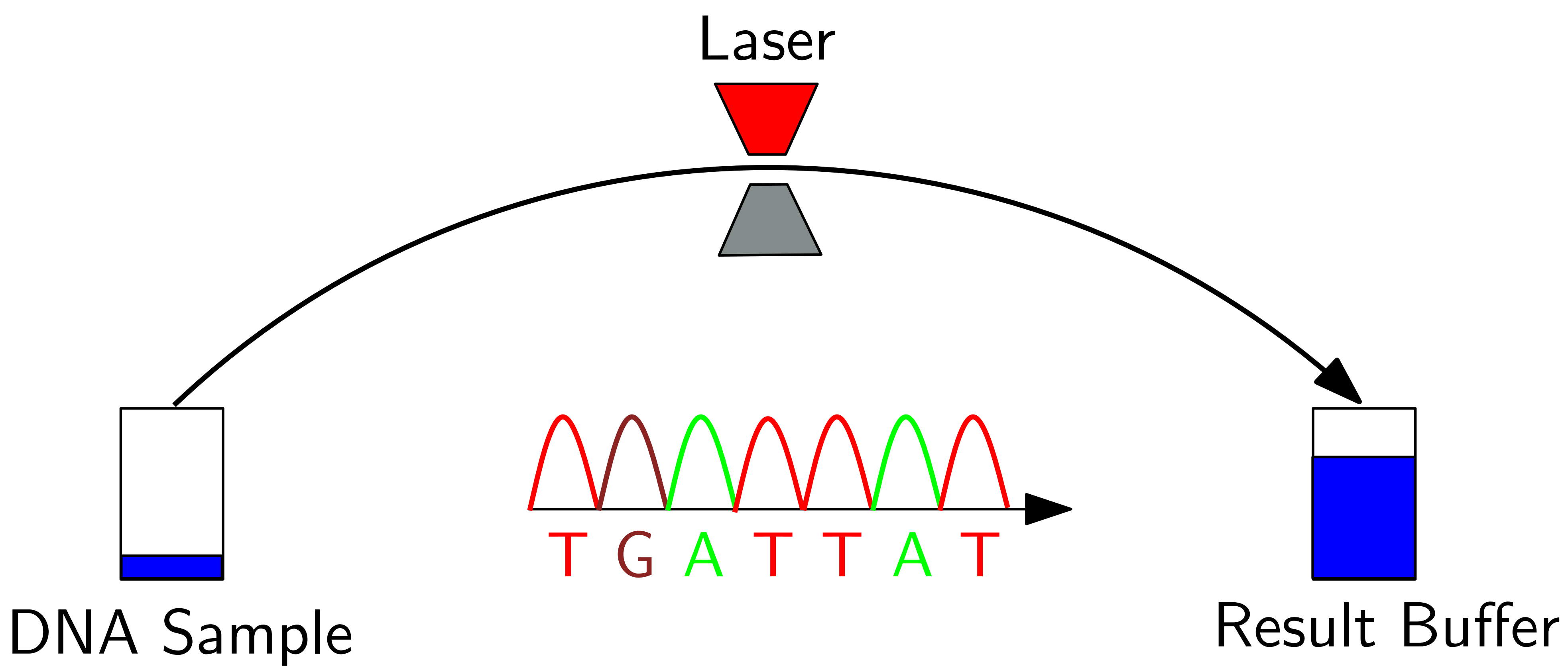

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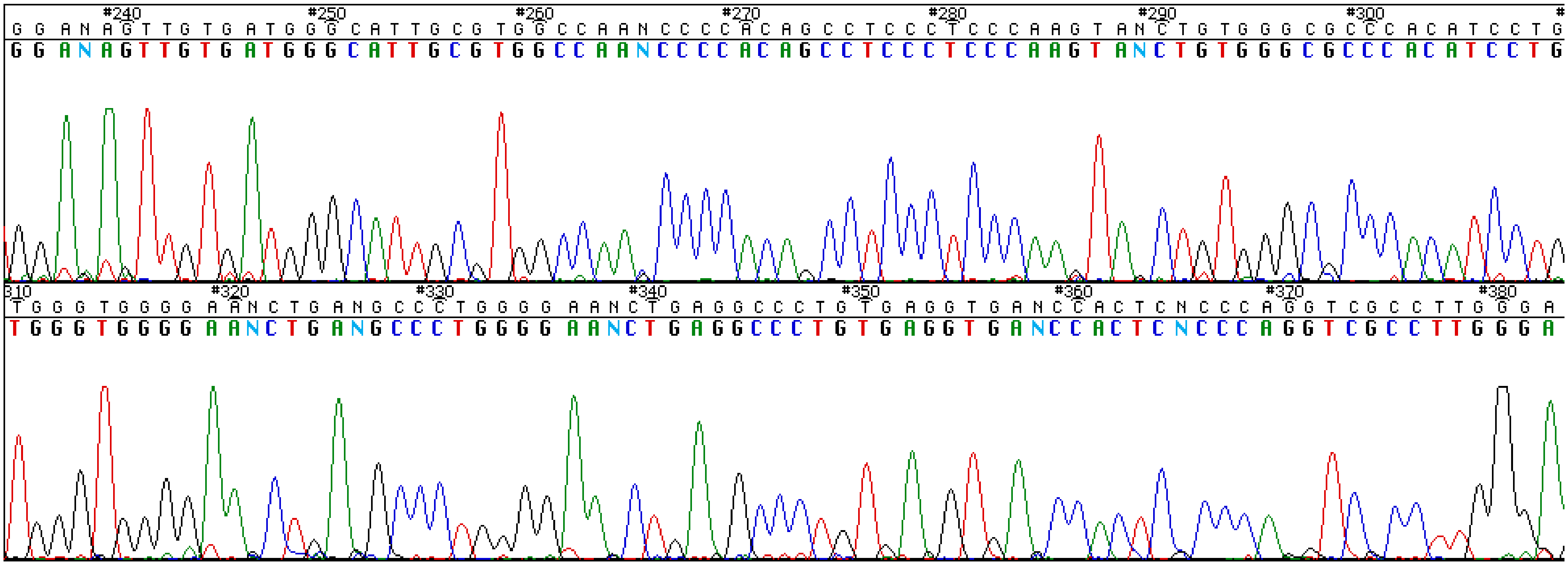

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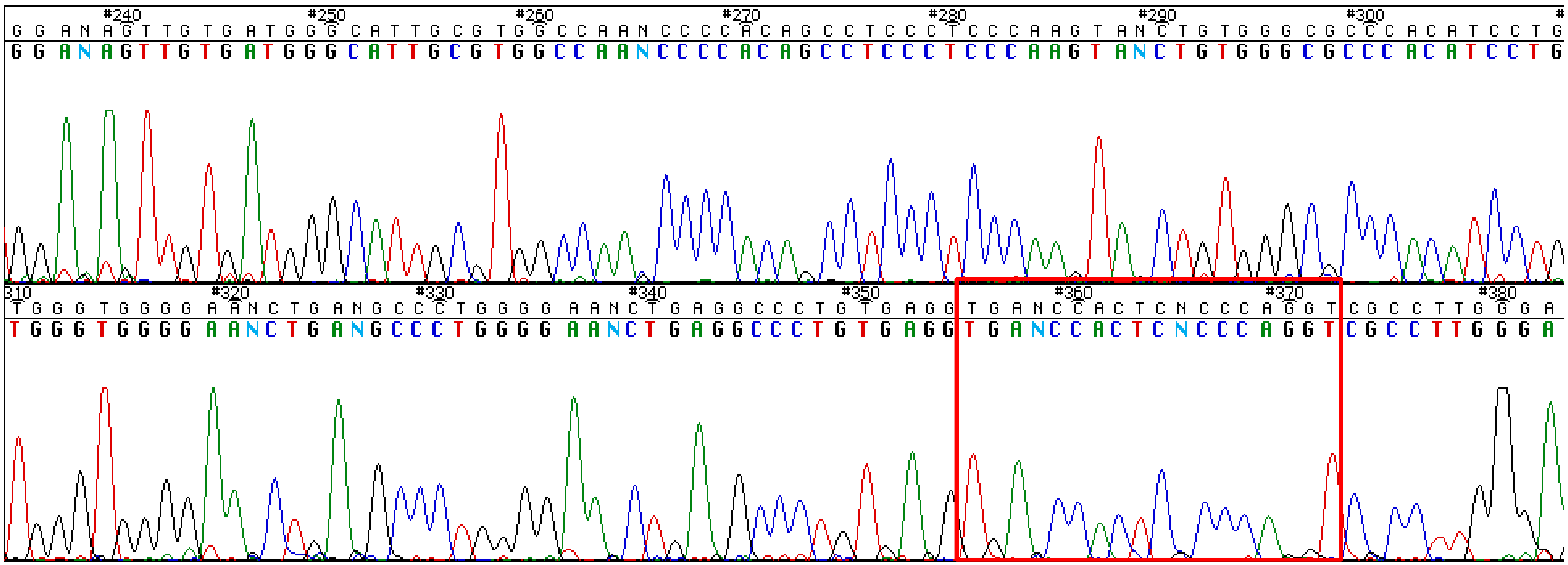

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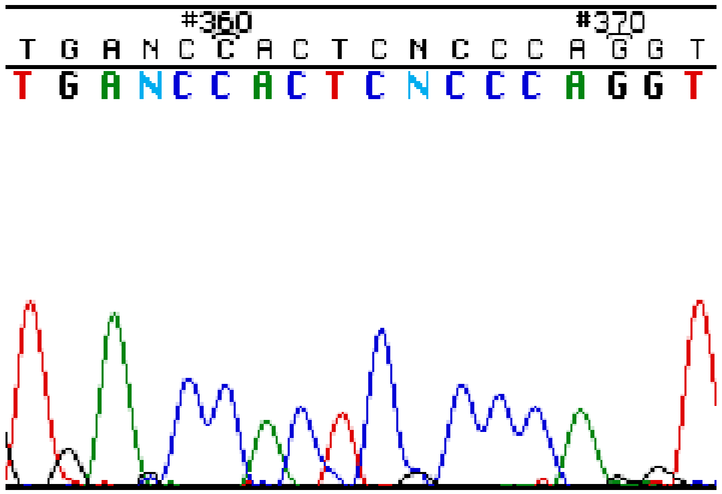

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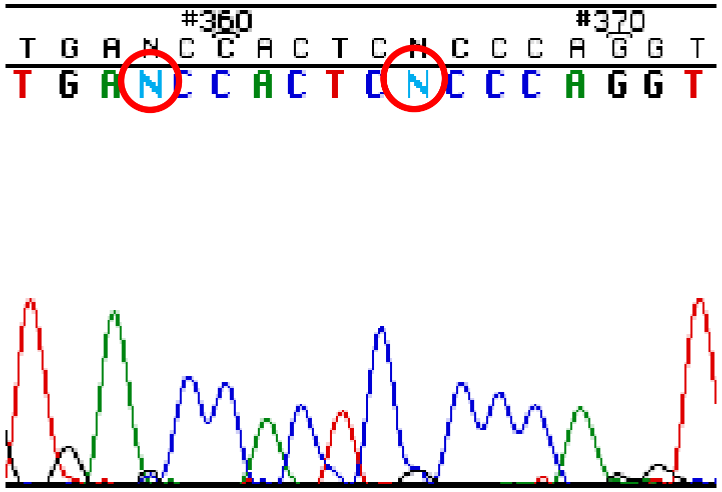

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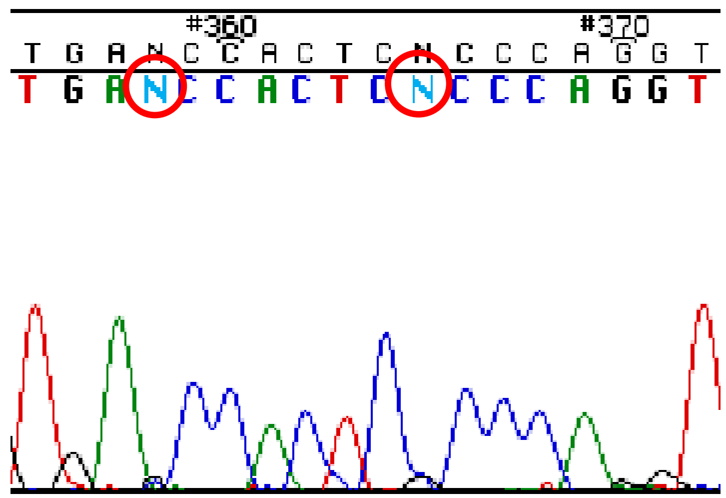

- The International Union of Pure and Applied Chemistry has issued the following convention for naming ambiguous portions of a sequence.

  - A = adenine
  - C = cytosine
  - G = guanine
  - T = thymine
  - R = G A (purine)
  - Y = T C (pyrimidine)
  - K = G T (keto)
  - M = A C (amino)
  - S = G C (strong bonds)
  - W = A T (weak bonds)
  - B = G T C (all but A)
  - D = G A T (all but C)
  - H = A C T (all but G)
  - V = G C A (all but T)
  - N = A G C T (any)

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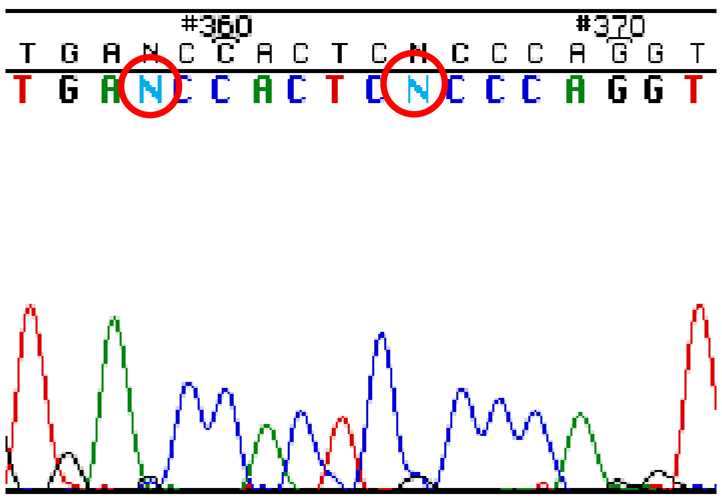

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Character-Level Probabilistic Model

DNA Sequence	Probability
TATTTCGTTGCTGCTGCGGAGC	0.75
TATATCGTTGCGGCTGCGGAGC	0.10
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$$\downarrow$$

$$\mathrm{S} = \mathrm{TAT}\ \boxed{\begin{array}{ll} \mathrm{A} & 0.20 \\ \mathrm{T} & 0.80 \end{array}}\ \mathrm{TCGTTGC}\ \boxed{\begin{array}{ll} \mathrm{G} & 0.15 \\ \mathrm{T} & 0.85 \end{array}}\ \mathrm{GCTGCGGAGC}$$

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S =	TAT	A	0.20	TCGTTGC	G	0.15	GCTGCGGAGC
		T	0.80		T	0.85	
	S[1]						S[n]

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$$S[4] = \{(A, 0.20), (T, 0.80)\}$$

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$S = S[1] \ldots S[n]$
where $S[i] = \{(c_{i,1}, p_{i,1}), \ldots, (c_{i,\eta_i}, p_{i,\eta_i})\}$
$c_{i,j} \in \Sigma$, $p_{i,j} \in (0,1]$ and $\sum_{j=1}^{\eta_i} p_{i,j} = 1$.

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“`advis_r`”

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"adviser" "advisor" "advisur"

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"`advis_r`"

"`adviser`"	"`advisor`"	"`advisur`"
✓	✓	✗

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"advis_r"

"adviser" "advisor" "advisur"

$$S = \texttt{advis} \begin{array}{cc} \texttt{e} & 0.20 \\ \texttt{o} & 0.80 \end{array} \texttt{r}$$

Edit Distance: A Classical Metric for Deterministic String Similarity Join

- The edit distance between two strings is the minimum number of *insertions* , *deletions* , and *substitutions* necessary to transform one string into the other

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How can we measure the similarity between probabilistic strings?

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How can we measure the similarity between probabilistic strings?

Key idea: Use the expected edit distance!

Possible Worlds Semantics

S_1	
$\sigma_{1,i}$	$p_{1,i}$
cat	0.50
kitty	0.50

S_2	
$\sigma_{2,i}$	$p_{2,i}$
dog	0.75
doggy	0.10
puppy	0.15

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$\sigma_{2,i}$	$p_{2,i}$
dog	0.75
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puppy	0.15

Ω					
$\sigma_{1,i}$	$p_{1,i}$	$\sigma_{2,j}$	$p_{2,j}$	w(s)	$d(\sigma_{1,i}, \sigma_{2,j})$
cat	0.50	dog	0.75	0.375	3
cat	0.50	doggy	0.10	0.05	5
cat	0.50	puppy	0.15	0.075	5
kitty	0.50	dog	0.75	0.375	5
kitty	0.50	doggy	0.10	0.05	4
kitty	0.50	puppy	0.15	0.075	4

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kitty	0.50	puppy	0.15	0.075	4

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kitty	0.50	doggy	0.10	0.05	4
kitty	0.50	puppy	0.15	0.075	4

Expected Edit Distance

$$EED = \widehat{d}(S_1, S_2) = \sum_{s \in \Omega} w(s) \cdot d(s).$$

Ω					
$\sigma_{1,i}$	$p_{1,i}$	$\sigma_{2,j}$	$p_{2,j}$	w(s)	$d(\sigma_{1,i}, \sigma_{2,j})$
cat	0.50	dog	0.75	0.375	3
cat	0.50	doggy	0.10	0.05	5
cat	0.50	puppy	0.15	0.075	5
kitty	0.50	dog	0.75	0.375	5
kitty	0.50	doggy	0.10	0.05	4
kitty	0.50	puppy	0.15	0.075	4

Expected Edit Distance

$$EED = \widehat{d}(S_1, S_2) = \sum_{s \in \Omega} w(s) \cdot d(s).$$

Ω					
$\sigma_{1,i}$	$p_{1,i}$	$\sigma_{2,j}$	$p_{2,j}$	w(s)	$d(\sigma_{1,i}, \sigma_{2,j})$
cat	0.50	dog	0.75	0.375	3
cat	0.50	doggy	0.10	0.05	5
cat	0.50	puppy	0.15	0.075	5
kitty	0.50	dog	0.75	0.375	5
kitty	0.50	doggy	0.10	0.05	4
kitty	0.50	puppy	0.15	0.075	4

$$\widehat{d}(S_1, S_2) = 0.375 \cdot 3 + 0.05 \cdot 5 + 0.075 \cdot 5 + 0.375 \cdot 5 + 0.05 \cdot 4 + 0.075 \cdot 4$$

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cat	0.50	dog	0.75	0.375	3
cat	0.50	doggy	0.10	0.05	5
cat	0.50	puppy	0.15	0.075	5
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kitty	0.50	doggy	0.10	0.05	4
kitty	0.50	puppy	0.15	0.075	4

$$\widehat{d}(S_1, S_2) = \boxed{0.375 \cdot 3} + 0.05 \cdot 5 + 0.075 \cdot 5 + 0.375 \cdot 5 + 0.05 \cdot 4 + 0.075 \cdot 4$$

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$\sigma_{1,i}$	$p_{1,i}$	$\sigma_{2,j}$	$p_{2,j}$	w(s)	$d(\sigma_{1,i}, \sigma_{2,j})$
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kitty	0.50	puppy	0.15	0.075	4

$$\widehat{d}(S_1, S_2) = 0.375 \cdot 3 + \boxed{0.05 \cdot 5} + 0.075 \cdot 5 + 0.375 \cdot 5 + 0.05 \cdot 4 + 0.075 \cdot 4$$

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$$\widehat{d}(S_1, S_2) = 4.125$$

Probabilistic String Similiarity Join

R	
id	S
1	{ (Microsoft, 0.90), (Microsoft Inc., 0.10) }
2	{ (Yahoo, 0.80), (Yahoo!, 0.20) }
$\vdots$	$\vdots$

$\bowtie_S$

T	
id	S
1	(Google, 1)
2	{ (AT&T, 0.98), (ATT, 0.02) }
$\vdots$	$\vdots$

Probabilistic String Similiarity Join

R	
id	S
1	{ (Microsoft, 0.90), (Microsoft Inc., 0.10) }
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⋮	⋮

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1	(Google, 1)
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- A join on R and T on probabilistic string attribute S returns all pairs of records (r_i, t_j) s.t. $r_i \in R$, $t_j \in T$ and $\widehat{d}(r_i.S, t_j.S) \leq \tau$

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- A join on R and T on probabilistic string attribute S returns all pairs of records (r_i, t_j) s.t. $r_i \in R$, $t_j \in T$ and $\widehat{d}(r_i.S, t_j.S) \leq \tau$
- Our goal is to find efficient means to optimize a probabilistic string join in existing relational databases

Relational Representation for String-Level Model

a **string-level probabilistic q-gram** is a quadruple (i, p, ℓ, g)

i is the choice index (cid) $\quad$ ℓ is the start position

p is the choice probability $\quad$ g is the q-gram starting at ℓ

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probabilistic strings

id	S
1	$\{$(add, 0.8), (plus, 0.2)$\}$
2	$\{$(up, 0.9), (op, 0.1)$\}$

relational representation

id	cid	A	p	len
1	1	add	0.8	3.2
1	2	plus	0.2	3.2
2	1	up	0.9	2
2	2	op	0.1	2

q-grams

id	cid	ℓ	g
1	1	1	#a
1	1	2	ad
1	1	3	dd
1	1	4	d$
1	2	1	#p
1	2	2	pl
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String-Level Probabilistic String Similiarity Join

- Solution: We derive probabilistic q-gram based lower-bound pruning techniques.

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- Our techniques are implementable in existing relational databases with pure SQL and are several of magnitude more efficient than the naive solution.

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A **character-level probabilistic** q**-gram** is a pair $(\ell, S[\ell..\ell+q-1])$
ℓ is the beginning position of the q-gram
$S[\ell..\ell+q-1]$ is the *probabilistic substring* $S[\ell]\dots S[\ell+q-1]$

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probabilistic strings

id	S
1	{(A, 0.8), (C,0.2)}, {(G, 0.7), (T, 0.3)}
2	{(A, 1)}, {(G, 0.6), (T,0.4)}
3	{(C, 1)}, {(A, 1)}, {(G, 1)}

relational representation

id	A	len
1	$\star$A0.8|C0.2$\star\star$G0.7|T0.3$\star$	2
2	A$\star$G0.6|T0.4$\star$	2
3	CAG	3

q-grams

id	p	ℓ	g
1	0.80	1	#A
1	0.20	1	#C
1	0.56	2	AG
1	0.24	2	AT
1	0.14	2	CG
1	0.06	2	CT
1	0.70	3	G$
1	0.30	3	T$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

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A **character-level probabilistic q-gram** is a pair $(\ell, S[\ell..\ell + q - 1])$
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probabilistic strings

id	S
1	{(A, 0.8), (C,0.2)}, {(G, 0.7), (T, 0.3)}
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id	A	len
1	⋆$\boxed{\text{A0.8}}$|C0.2⋆⋆G0.7|T0.3⋆	2
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1	⋆A0.8 C0.2⋆⋆G0.7 T0.3⋆	2
2	A⋆G0.6|T0.4⋆	2
3	CAG	3

q-grams

id	p	ℓ	g
1	0.80	1	#A
1	0.20	1	#C
1	0.56	2	AG
1	0.24	2	AT
1	0.14	2	CG
1	0.06	2	CT
1	0.70	3	G$
1	0.30	3	T$
⋮	⋮	⋮	⋮

Relational Representation for Character-Level Model

A **character-level probabilistic q-gram** is a pair $(\ell, S[\ell..\ell+q-1])$
ℓ is the beginning position of the q-gram
$S[\ell..\ell+q-1]$ is the *probabilistic substring* $S[\ell]\ldots S[\ell+q-1]$

probabilistic strings

id	S
1	{(A, 0.8), (C,0.2)}, {(G, 0.7), (T, 0.3)}
2	{(A, 1)}, {(G, 0.6), (T,0.4)}
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- Solution: We derive probabilistic q-gram lower and upper-bound pruning techniques.
- Our techniques are implementable in existing relational databases with pure SQL and are orders of magnitude more efficient than the naive solution.
- We also derive DP-based lower and upper-bound pruning techniques to further improve query efficiency.

Experiments: Setup

- Experimental setup
  - Microsoft SQL Server 2008 Enterprise Edition
  - Windows XP
  - Intel Xeon E5405 @ 2.00 GHz
  - 2GB memory

Experiments: Algorithms

- Due to time constraints we focus on the Character-Level model experimental results for this talk:
  - Character Basic Join (C-BJ)
  - Character q-Gram Prune Join (C-qPJ): Uses all probabilistic q-gram upper and lower bounds
  - Character Prune Join (C-PJ): Uses all probabilistic q-gram and DP upper and lower bounds

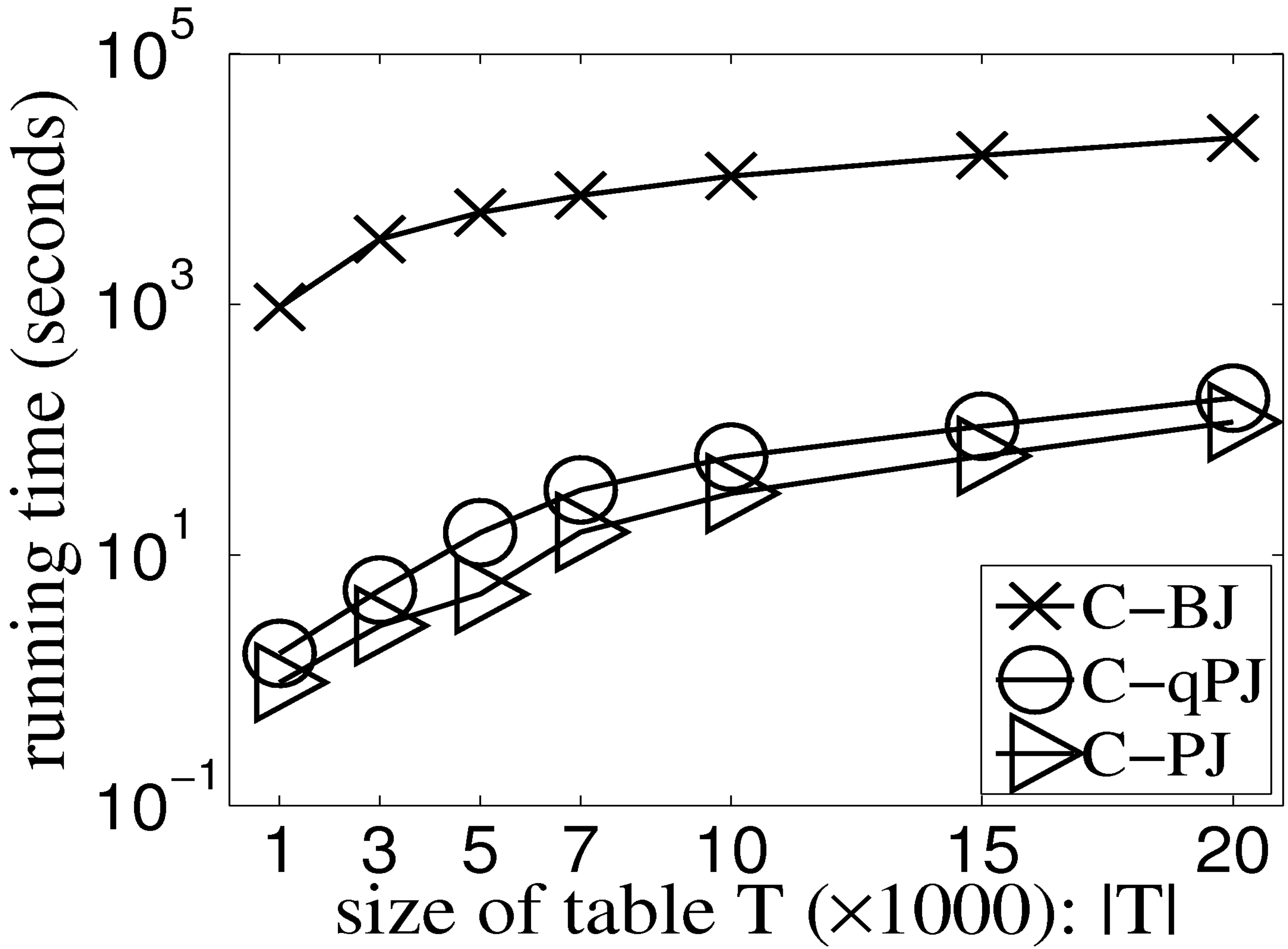
Experiments: Data Sets

- We create 2 datasets for string-level and 2 for character-level models respectively based on 3 real data sources:
  - Author Names from DBLP
  - Category-Link string field from Wikipedia category-link table
  - Genome sequence database from the Rhodococcus project.

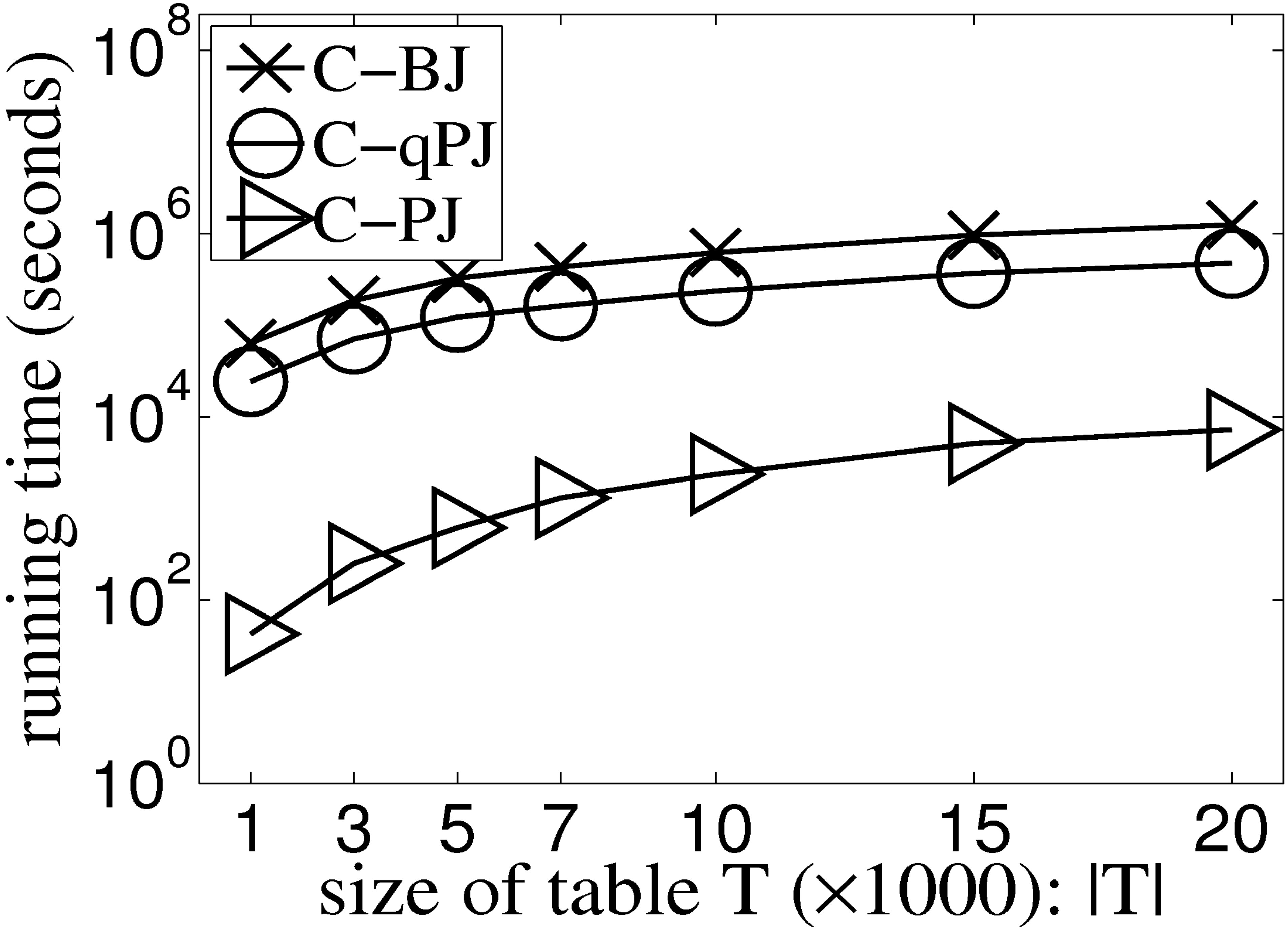
Experiments: Character-Level Default Parameters

Symbol	Definition	Default Value
χ	Max choices per position	6
θ	Max percent uncertain positions	20%
μ_C	Average string length	Author2 14.3 Genome 14.9
$\lvert R\rvert$	Size table R	1,000
$\lvert T\rvert$	Size table T	10,000
q	q-gram length	2
τ	EED threshold	2

Character-level model, *Author2* dataset



Character-level model, *Genome* dataset



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Conclusions

- We study the problem of efficient string joins in probabilistic string databases utilizing the expected edit distance
- We introduce efficient and effective techniques for both string-level and character-level models which are implementable in existing relational databases
- There are many interesting unexplored problems including methods to derive a representation in either model from a data source, indexing probabilisitc strings for similarity search, selectivity estimations, and keyword searches in probabilistic string databases
- Similarity measures other than the edit distance may be desirable, such as idf based distance, especially in the string-level model

The End

THANK YOU

Q and A

- The entire source code is available from a link at http://ww2.cs.fsu.edu/~jestes

Edit Distance: A Classical Metric for Deterministic String Similarity Join

$$d[i,j] = \min \begin{cases} d[i,j-1]+1, & \text{insertion} \\ d[i-1,j]+1, & \text{deletion} \\ d[i-1,j-1]+c(\sigma_1[i],\sigma_2[j]), & \text{substitution} \end{cases}$$

in which

$$c(\sigma_1[i],\sigma_2[j]) = \begin{cases} 1, \sigma_1[i] \neq \sigma_2[j], \\ 0, \sigma_1[i] = \sigma_2[j], \end{cases}$$

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How can we measure the similarity between probabilistic strings?

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Key idea: Use the expected edit distance!

Background: Naive Deterministic String Similarity Join

R	
id	A
1	sony
2	Microsoft
3	Toshiba
⋮	⋮

$$\bowtie_{d(A,B) \leq \tau}$$

T	
id	B
1	Micrsft
2	Sony
3	Apple
⋮	⋮

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Too expensive!!!

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- q-grams are strings of length q produced by sliding a window of length q over the characters of a string σ

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  - The RDBMS can perform pruning on tuples though the use of q-grams

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  g is the substring of length q in σ beginning from ℓ.

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Lemma 2 (positional q-gram filtering) *For strings* σ_1 *and* σ_2*,*

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Lemma 3 (length filtering) $d(\sigma_1, \sigma_2) \geq ||\sigma_1| - |\sigma_2||$

String-Level Probabilistic String Similiarity Join

- The set of probabilistic q-grams for $S(i)$ is $G_{S(i)} = \{(i, p_i, \ell_j, g_j)\}$, for all j s.t. $(\ell_j, g_j) \in G_{\sigma_i}$

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R				
id	cid	S	p	len
1	1	Microsoft	0.90	9.5
1	2	Microsoft Inc.	0.10	9.5
2	1	Yahoo	0.80	5.2
2	2	Yahoo!	0.20	5.2
⋮	⋮	⋮	⋮	⋮

T				
id	cid	S	p	len
1	1	Google	1	6
2	1	AT&T	0.98	4.1
2	2	AT&T Labs	0.02	4.1
⋮	⋮	⋮	⋮	⋮

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- S-BJ:
  `SELECT R.id,T.id FROM R,T GROUP BY R.id,T.id`
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  `HAVING SUM(R.p*T.p*d(R.A,T.A))` $\leq \tau$

  Too expensive!!!!

String-Level Probabilistic Strings: Length Filtering

- We may directly extend the **length filtering** lemma

$$d(\sigma_1, \sigma_2) \geq ||\sigma_1| - |\sigma_2||$$

to obtain,

Lemma 4 *For any string-level probabilistic strings S_1 and S_2:*

$$\widehat{d}(S_1, S_2) \geq \sum_{s \in \Omega} w(s) ||\sigma_{1,i}| - |\sigma_{2,j}||.$$

String-Level Probabilistic Strings: Length Filtering

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-
```
SELECT R.id,T.id FROM R,T
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*ABS(|R.A|-|T.A|))≤ τ
```

String-Level Probabilistic Strings: q-Gram Lower Bounds

- Inspired by deterministic **q-gram filtering**,

$$d(\sigma_1, \sigma_2) \geq \quad 1 + \frac{\max(|\sigma_1|,|\sigma_2|)}{q} - \quad \frac{|G_{\sigma_1} \cap G_{\sigma_2}|+1}{q}$$

we can derive a similar result for string-level probabilistic q-grams,

Theorem 1 *For any string-level probabilistic strings S_1 and S_2:*

$$\widehat{d}(S_1, S_2) \quad \geq \quad 1 + \frac{\max(\mathbf{E}[|S_1|], \mathbf{E}[|S_2|])}{q} - \frac{\sum_{(\rho_1,\rho_2)\in G_{S_1}\cap G_{S_2}} p(\rho_1)p(\rho_2) + 1}{q}.$$

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String-Level Probabilistic Strings: q-Gram Lower Bounds

- The next lemma will be useful in deriving $\textsc{Eed}$ lower bounds.

Lemma 5 *For two non-negative random variables X, Y:*

$$\mathbf{E}[\max(X, Y)] \geq \max(\mathbf{E}[X], \mathbf{E}[Y]).$$

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- **Proof**

$$\mathbf{E}[\max(X,Y)] \quad = \quad \mathbf{E}\left(\frac{X+Y+|X-Y|}{2}\right)$$

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- **Proof**

$$\begin{aligned} \mathbf{E}[\max(X,Y)] &= \mathbf{E}\left(\frac{X+Y+|X-Y|}{2}\right) \\ &= \frac{\mathbf{E}[X]+\mathbf{E}[Y]}{2} + \frac{\mathbf{E}[|X-Y|]}{2} \end{aligned}$$

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\mathbf{E}[\max(X,Y)] &= \mathbf{E}\left(\frac{X+Y+|X-Y|}{2}\right) \\
&= \frac{\mathbf{E}[X]+\mathbf{E}[Y]}{2} + \frac{\mathbf{E}[|X-Y|]}{2} \\
&\geq \frac{\mathbf{E}[X]+\mathbf{E}[Y]}{2} + \frac{\mathbf{E}[X-Y]}{2} = \mathbf{E}[X].
\end{aligned}$$

Similarly, $\mathbf{E}[\max(X,Y)] \geq \mathbf{E}[Y]$. $\square$

String-Level Probabilistic Strings: q-Gram Lower Bounds

- For any string-level probabilistic strings S_1 and S_2:

$$\begin{aligned}\widehat{d}(S_1, S_2) \quad &\geq \quad 1 + \frac{\max(\mathbf{E}[|S_1|], \mathbf{E}[|S_2|])}{q} \\ &\quad - \frac{\sum_{(\rho_1,\rho_2)\in G_{S_1}\cap G_{S_2}} p(\rho_1)p(\rho_2) + 1}{q}.\end{aligned}$$

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- **Proof** Let $s \in \Omega$ and $s = (S_1(i), S_2(j))$.
  By the basic **q-gram filtering** lemma we have,

$$\sum_{s\in\Omega} w(s)|G_{S_1(i)} \cap G_{S_2(j)}| \geq$$

$$\sum_{s\in\Omega} w(s)(\max(|\sigma_{1,i}|, |\sigma_{2,j}|) - 1 - q(d(s) - 1)).$$

String-Level Probabilistic Strings: q-Gram Lower Bounds

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- Since $\sum_{s \in \Omega} w(s) = 1$, the RHS equals

$$\begin{aligned} \textstyle\sum_{s \in \Omega} \quad & w(s) \max(|\sigma_{1,i}|, |\sigma_{2,j}|) - 1 - q \sum_{s \in \Omega} w(s) d(s) + q \\ = \quad & \mathbf{E}[\max(|S_1|, |S_2|)] - 1 - q \cdot \widehat{d}(S_1, S_2) + q \end{aligned}$$

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$$\mathbf{E}[\max(X, Y)] \geq \max(\mathbf{E}[X], \mathbf{E}[Y])$$

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&\geq && \boxed{\max(\mathbf{E}(|S_1|), \mathbf{E}(|S_2|))} - 1 - q(\widehat{d}(S_1, S_2) - 1),
\end{aligned}$$

$$\mathbf{E}[\max(X, Y)] \geq \max(\mathbf{E}[X], \mathbf{E}[Y])$$

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- $$\boxed{\sum_{s\in\Omega} w(s)|G_{S_1(i)} \cap G_{S_2(j)}|} \geq \sum_{s\in\Omega} w(s)(\max(|\sigma_{1,i}|, |\sigma_{2,j}|) - 1 - q(d(s) - 1)).$$

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- The LHS is

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\end{aligned}$$

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-

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GROUP BY R.id, T.id, R.len, T.len
HAVING 1+(max(R.len,T.len)-SUM(R.p*T.p)-1)/q≤ τ
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String-Level Probabilistic Strings: q-Gram Lower Bounds

- **Theorem 1** *For any string-level probabilistic strings* S_1 *and* S_2*:*

$$
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```

- We also derive SQL lower bounds which utilize positional and length information

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

- Inspired by deterministic **positional q-gram filtering**,

$$|G_{\sigma_1} \cap_{d(\sigma_1,\sigma_2)} G_{\sigma_2}| \geq \max(|\sigma_1|, |\sigma_2|) + q - 1 - qd(\sigma_1, \sigma_2)$$

we could attempt to derive a similar result for probabilistic strings

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- However, we have a problem as $d(s)$ may be different for different worlds

$$\begin{aligned} & \sum_{s \in \Omega} w(s) |G_{S_1(i)} \cap_{d(s)} G_{S_2(j)}| \\ \geq \quad & \max(\mathbf{E}(|S_1|), \mathbf{E}(|S_2|)) - 1 - q(\widehat{d}(S_1, S_2) - 1). \end{aligned}$$

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- We can enlarge the LHS to make it easier to compute

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

- Assume $\widehat{d}(S_1, S_2) \leq \tau$
  Let $\Omega' \subseteq \Omega$ be the subset of possible worlds s in which $d(s) \geq 2\tau$.
  The probability mass of Ω' is at most $1/2$. (Markov Inequality)

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- $$\sum_{s \in \Omega} w(s) |G_{S_1(i)} \cap_{d(s)} G_{S_2(j)}| \leq \sum_{s \in \Omega'} w(s) |G_{S_1(i)} \cap G_{S_2(j)}| + \sum_{s \in \Omega \setminus \Omega'} w(s) |G_{S_1(i)} \cap_{2\tau - 1} G_{S_2(j)}|$$

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  $$= \sum_{s \in \Omega} w(s) |G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}| + \sum_{s \in \Omega'} w(s) (|G_{S_1(i)} \cap G_{S_2(j)}| - |G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}|).$$

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$$= \sum_{s \in \Omega} w(s)|G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}| + \sum_{s \in \Omega'} w(s)(|G_{S_1(i)} \cap G_{S_2(j)}| - |G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}|).$$

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$$= \sum_{s\in\Omega} w(s)|G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}| + \sum_{s\in\Omega'} w(s)(|G_{S_1(i)} \cap G_{S_2(j)}| - |G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}|).$$

- Let $x(s) = |G_{S_1(i)} \cap G_{S_2(j)}| - |G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}|$

  Then we may cast it as a fractional knapsack as:
  Choose $s \in \Omega'$ s.t. $\sum_{s\in\Omega'} w(s) \le \frac{1}{2}$
  Maximize $\sum_{s\in\Omega'} w(s)x(s)$

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

- Fractional Knapsack Formulation:
  $x(s) = |G_{S_1(i)} \cap G_{S_2(j)}| - |G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}|$
  Item value : $w(s)x(s)$
  Item weight : $w(s)$

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Initialize $\Omega' = \emptyset$.
Sort $s \in \Omega$ by decreasing $\frac{w(s)x(s)}{x(s)} = x(s)$
While $\sum_{s \in \Omega'} w(s) > 1/2$
    $\Omega' \leftarrow s \in \Omega$ with next largest $x(s)$

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

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Initialize $\Omega' = \emptyset$.
Sort $s \in \Omega$ by decreasing $\frac{w(s)x(s)}{x(s)} = x(s)$
While $\sum_{s\in\Omega'} w(s) > 1/2$
  $\Omega' \leftarrow s \in \Omega$ with next largest $x(s)$

- Let s' be the last world added to Ω',

$$\begin{aligned} \mathrm{UB}_\tau &= \sum_{s\in\Omega} w(s)|G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}| \\ &\quad + \sum_{s\in\Omega'} w(s)x(s) - w(s')x(s') \frac{\sum_{s\in\Omega'} w(s) - 1/2}{w(s')} \end{aligned}$$

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

- **Theorem 2** *For any string-level probabilistic strings* S_1 *and* S_2*,*

$$\widehat{d}(S_1, S_2) \geq 1 + \frac{\max(\mathbf{E}(|S_1|), \mathbf{E}(|S_2|)) - \mathrm{UB}_\tau - 1}{q}.$$

where,

$$\begin{aligned} \mathrm{UB}_\tau \quad &= \quad \sum_{s \in \Omega} w(s) |G_{S_1(i)} \cap_{2\tau-1} G_{S_2(j)}| \\ &\quad + \sum_{s \in \Omega'} w(s)x(s) - w(s')x(s') \frac{\sum_{s \in \Omega'} w(s) - 1/2}{w(s')}. \end{aligned}$$

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- UB_τ requires a UDF to compute,
$\texttt{ub(R}_q\texttt{.cid, R.p, R}_q\texttt{.}\ell\texttt{, R}_q\texttt{.g, T}_q\texttt{.cid, T.p, T}_q\texttt{.}\ell\texttt{, T}_q\texttt{.g, }\tau\texttt{)}$

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SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q
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GROUP BY R.id, T.id, R.len, T.len
   HAVING 1 + 1/q*( max(R.len,T.len) - 1 -
   ub(R_q.cid,R.p,R_q.ℓ,R_q.g,T_q.cid,T.p,T_q.ℓ,T_q.g,τ))≤ τ
```

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Loose!!!!

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- **Observation.** For most cases if $\widehat{d}(S_1, S_2) \le \tau$,

$$\widehat{d}(S_1,S_2) \ge 1 + \frac{\max(\mathbf{E}[|S_1|], \mathbf{E}[|S_2|])}{q} - \frac{\sum_{(\rho_1,\rho_2)\in G_{S_1}\cap_{2\tau} G_{S_2}} p(\rho_1)p(\rho_2) + 1}{q}.$$

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Character-Level Probabilistic String Similarity Join

- The set of deterministic q-grams are denoted G_S^d.

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  if $(\gamma_1, \gamma_2) \in G^d_{S_1,S_2}$
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  0 otherwise

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Character-Level Probabilistic String Similarity Join

R		
id	A	len
1	AT $\star$A0.8|C0.2$\star$GAT	6
1	AT $\star$C0.2|G0.3|T0.5$\star$CG	5
$\vdots$	$\vdots$	$\vdots$

T		
id	A	len
1	T $\star$G0.95|T0.05$\star$TAC	5
1	$\star$G0.3|T0.2|A0.5$\star$CGTA	5
$\vdots$	$\vdots$	$\vdots$

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  `SELECT R.id,T.id FROM R,T WHERE ed(R.A,T.A)` $\leq \tau$

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  Too expensive!!!!

Character-Level Probabilistic Strings: Length Filtering

- We may directly extend the **length filtering** lemma

$$d(\sigma_1, \sigma_2) \geq ||\sigma_1| - |\sigma_2||$$

to obtain,

Lemma 6 *For any character-level probabilistic strings* S_1, S_2,

$$\widehat{d}(S_1, S_2) \geq ||S_1| - |S_2||.$$

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- ```
SELECT R.id AS rid,T.id AS tid FROM R,T
WHERE ABS(R.len - T.len) ≤ τ
```
```

Character-Level Probabilistic Strings: q-Gram Lower Bounds

- Inspired by deterministic **q-gram filtering**,

$$d(\sigma_1, \sigma_2) \geq 1 + \frac{\max(|\sigma_1|, |\sigma_2|)}{q} - \frac{|G_{\sigma_1} \cap G_{\sigma_2}| + 1}{q}$$

we can derive a similar result for character-level probabilistic q-grams,

Theorem 3 *For any character-level probabilistic strings* S_1, S_2*:*

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HAVING 1 + max(R.len,T.len)/q -
 SUM( T_q.p*R_q.p)/q - 1/q ≤ τ
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Character-Level Probabilistic Strings: Improving q-Gram Lower Bounds

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$$|G_{\sigma_1} \cap_{d(\sigma_1,\sigma_2)} G_{\sigma_2}| \geq \max(|\sigma_1|, |\sigma_2|) + q - 1 - qd(\sigma_1, \sigma_2)$$

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  we may attempt to derive a similar result for probabilistic strings

- However, we have a problem as $d(s)$ may be different for different worlds

$$\sum_{s \in \Omega} w(s)|G_{\sigma_1} \cap_{d(s)} G_{\sigma_2}| \geq \max(|S_1|, |S_2|) - 1 - q(\widehat{d}(S_1, S_2) - 1).$$

Character-Level Probabilistic Strings: Improving q-Gram Lower Bounds

- Inspired by deterministic **postional q-gram filtering**,

$$|G_{\sigma_1} \cap_{d(\sigma_1,\sigma_2)} G_{\sigma_2}| \geq \max(|\sigma_1|, |\sigma_2|) + q - 1 - qd(\sigma_1, \sigma_2)$$

  we may attempt to derive a similar result for probabilistic strings

- However, we have a problem as $d(s)$ may be different for different worlds

$$\sum_{s \in \Omega} w(s)|G_{\sigma_1} \cap_{d(s)} G_{\sigma_2}| \geq \max(|S_1|, |S_2|) - 1 - q(\widehat{d}(S_1, S_2) - 1).$$

- We can enlarge the LHS to make it easier to compute

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- **Lemma 7** *For any character-level probabilistic strings* S_1, S_2, $\forall \tau \geq \widehat{d}(S_1, S_2)$*:*

$$\sum_{s \in \Omega} w(s) |G_{\sigma_1} \cap_{d(s)} G_{\sigma_2}| \leq \sum_{(\gamma_1, \gamma_2) \in (G_{S_1, S_2} - G^d_{S_1, S_2})} \Pr(\gamma_1 = \gamma_2)$$
$$+ \sum_{(\gamma_1, \gamma_2) \in G^d_{S_1, S_2}, \gamma_1 = \gamma_2} \min\left(1, \frac{\tau}{\mathrm{off}(\gamma_1, \gamma_2)}\right)$$

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- **Proof:** First let $s \in \Omega$ and $s = ((\sigma_1, p_1), (\sigma_2, p_2))$
  $\gamma_1 = (i, S_1[i..i+q-1]) \in G_{S_1}$
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- We will need,

$$\Pr(d(s) \geq \mathrm{off}(\gamma_1, \gamma_2)) \leq \min\left(1, \frac{\mathbf{E}(d(s))}{\mathrm{off}(\gamma_1, \gamma_2)}\right)$$

$$= \min\left(1, \frac{\widehat{d}(S_1, S_2)}{\mathrm{off}(\gamma_1, \gamma_2)}\right) \leq \min\left(1, \frac{\tau}{\mathrm{off}(\gamma_1, \gamma_2)}\right),$$

where we define $\frac{1}{\mathrm{off}(\gamma_1,\gamma_2)} = \infty$ if $\mathrm{off}(\gamma_1, \gamma_2) = 0$

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Markov Inequality

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  $\gamma_1 = (i, S_1[i..i+q-1]) \in G_{S_1}$
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- We also define $\gamma_1 =_s \gamma_2$ as the event that $S_1[i..i+q-1] = S_2[j..j+q-1]$ in s. We have

$$\Pr(\gamma_1 =_s \gamma_2) = \begin{cases} w(s), & \text{if } \sigma_1[i..i+q-1] = \sigma_2[j..j+q-1]; \\ 0, & \text{otherwise.} \end{cases}$$

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- $$\sum_{s \in \Omega} w(s) |G_{\sigma_1} \cap_{d(s)} G_{\sigma_2}|$$
  $$= \sum_{(\gamma_1,\gamma_2) \in G_{S_1,S_2}} \sum_{s \in \Omega} \Pr(\gamma_1 =_s \gamma_2 \text{ and } \mathrm{off}(\gamma_1,\gamma_2) \leq d(s))$$

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- $$\sum_{(\gamma_1,\gamma_2)\in G^d_{S_1,S_2}} \boxed{\sum_{s\in\Omega} \Pr(\gamma_1 =_s \gamma_2 \text{ and } \mathrm{off}(\gamma_1,\gamma_2) \le d(s))}$$
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$$\le \sum_{(\gamma_1,\gamma_2)\in G^d_{S_1,S_2},\gamma_1=\gamma_2} \min\left(1, \frac{\tau}{\mathrm{off}(\gamma_1,\gamma_2)}\right)$$
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$$\Pr(d(s) \ge \mathrm{off}(\gamma_1,\gamma_2)) \le \min\left(1, \frac{\tau}{\mathrm{off}(\gamma_1,\gamma_2)}\right)$$

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$$\Pr(d(s) \ge \mathrm{off}(\gamma_1,\gamma_2)) \le \min\left(1, \frac{\tau}{\mathrm{off}(\gamma_1,\gamma_2)}\right)$$

Character-Level Probabilistic Strings: Improving q-Gram Lower Bounds

- **Theorem 4** *For any character-level probabilistic strings* S_1, S_2, $\forall \tau \geq \widehat{d}(S_1, S_2)$*:*

$$\begin{aligned}\widehat{d}(S_1, S_2) \geq & 1 + \frac{\max(|S_1|, |S_2|) - 1}{q} \\ & - \frac{\sum_{(\gamma_1,\gamma_2)\in G^d_{S_1,S_2}, \gamma_1=\gamma_2} \min\left(1, \frac{\tau}{\mathrm{off}(\gamma_1,\gamma_2)}\right)}{q} \\ & - \frac{\sum_{(\gamma_1,\gamma_2)\in G_{S_1,S_2}-G^d_{S_1,S_2}} \Pr(\gamma_1 = \gamma_2)}{q}.\end{aligned}$$

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$$

-
```
SELECT R.id AS rid,T.id AS tid FROM R,T,R_q,T_q
WHERE R_q.g=T_q.g AND R.id=R_q.id AND T.id=T_q.id
GROUP BY R.id, T.id, R.len, T.len
HAVING 1 + (max(R.len,T.len)-1)/q -
SUM( FLOOR(T_q.p*R_q.p)*
      min(1, τ/max(1,ABS(R_q.ℓ - T_q.ℓ))) )/q -
      SUM( CEILING(1-R_q.p*T_q.p)*T_q.p*R_q.p )/q ≤ τ
```

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```

Character-Level Probabilistic Strings: Dynamic Program Lower Bounds

- We can derive a DP based lower-bound on the $\textsc{Eed}$ as follows,

$$\widehat{ld}[i,j] = \min \begin{cases} \widehat{ld}[i,j-1]+1; \\ \widehat{ld}[i-1,j]+1; \\ \widehat{ld}[i-1,j-1]+lc(S_1[i],S_2[j]), \end{cases}$$

where

$$lc(S_1[i],S_2[j]) = \begin{cases} 1, \Pr(S_1[i]=S_2[j])=0; \\ 0, \Pr(S_1[i]=S_2[j])>0. \end{cases}$$

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- **Theorem 5** *For any two character-level probabilistic strings* $S_1 = S_1[1]\ldots S_1[n_1]$ *and* $S_2 = S_2[1]\ldots S_2[n_2]$,

$$\widehat{ld}[n_1,n_2] = \min_{s\in\Omega}(d(s)) \le \widehat{d}(S_1,S_2).$$

Character-Level Probabilistic Strings: q-Gram Upper Bounds

- Let $q_1 = (\ell_1, g_1)$, $q_2 = (\ell_2, g_2)$
  Where $q_1 \in G_{\sigma_1}$ and $q_2 \in G_{\sigma_2}$
  Define $q_1 \equiv q_2 \iff g_1 = g_2$ *and* $\ell_1 = \ell_2$

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- **Lemma 8** *For any two deterministic strings σ_1 and σ_2:*

$$d(\sigma_1, \sigma_2) \leq \max(|\sigma_1|, |\sigma_2|) + q - 1 - \sum_{q_1 \in G_{\sigma_1}, q_2 \in G_{\sigma_2}} (q_1 \equiv q_2)$$

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Character-Level Probabilistic Strings: q-Gram Upper Bounds

- For any character-level probabilistic strings S_1, S_2:

$$\widehat{d}(S_1, S_2) \leq \max(|S_1|, |S_2|) + q - 1 - \sum_{(\gamma_1, \gamma_2) \in G_{S_1, S_2}} Pr(\gamma_1 \equiv \gamma_2)$$

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-

```
SELECT R.id AS rid,T.id AS tid FROM R,T,R_q,T_q
WHERE R_q.g=T_q.g AND R_q.ℓ=T_q.ℓ
   AND R.id=R_q.id AND T.id=T_q.id
GROUP BY R.id, T.id, R.len, T.len
HAVING max(R.len,T.len)+q-1-SUM(T_q.p*R_q.p)≤ τ
```

Character-Level Probabilistic Strings: Dynamic Program Upper Bounds

- We can derive a DP-based upper bound on the EED as follows

$$\widehat{ud}[i,j] = \min \begin{cases} \widehat{ud}[i,j-1]+1, \\ \widehat{ud}[i-1,j]+1, \\ \widehat{ud}[i-1,j-1]+\Pr(S_1[i] \neq S_2[j]), \end{cases}$$

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- **Theorem 7** $\widehat{ud}[i,j] \geq \widehat{d}(S_1[1..i], S_2[1..j])$ *for all* i, j*.*

Character-Level Probabilistic Strings: Dynamic Program Upper Bounds

- **Theorem** $\widehat{ud}[i,j] \geq \widehat{d}(S_1[1..i], S_2[1..j])$ for all i, j

Character-Level Probabilistic Strings: Dynamic Program Upper Bounds

- **Theorem** $\widehat{ud}[i,j] \geq \widehat{d}(S_1[1..i], S_2[1..j])$ for all i, j
- **Proof:** We will need,

Lemma 10 *For two non-negative random variables X, Y,*

$$\mathbf{E}[\min(X,Y)] \leq \min(\mathbf{E}[X], \mathbf{E}[Y]).$$

and,

Corollary 2 *For three non-negative random variables X, Y, Z,*

$$\mathbf{E}[\min(X,Y,Z)] \leq \min(\mathbf{E}[X], \mathbf{E}[Y], \mathbf{E}[Z]).$$

Character-Level Probabilistic Strings: Dynamic Program Upper Bounds

- **Theorem** $\widehat{ud}[i,j] \geq \widehat{d}(S_1[1..i], S_2[1..j])$ for all i, j

- The theorem holds when $i \cdot j = 0$
  Assume it holds $\forall i' \leq i, j' \leq j$.
  Let $s = \{(\sigma_1, p_1), (\sigma_2, p_2)\}$ be a random world from Ω

$$d_s[i,j] = \min \begin{cases} d_s[i-1,j] + 1, \\ d_s[i,j-1] + 1, \\ d_s[i-1,j-1] + c(\sigma_1[i], \sigma_2[j]). \end{cases}$$

Character-Level Probabilistic Strings: Dynamic Program Upper Bounds

$$\begin{aligned}
& \widehat{d}(S_1[1..i], S_2[1..j]) \\
= \quad & \mathbf{E}\left[\min \begin{cases} d_s[i-1,j]+1 \\ d_s[i,j-1]+1 \\ d_s[i-1,j-1]+c(\sigma_1[i],\sigma_2[j]) \end{cases} \right]
\end{aligned}$$

Character-Level Probabilistic Strings: Dynamic Program Upper Bounds

$$
\begin{aligned}
& \widehat{d}(S_1[1..i], S_2[1..j]) \\
= \quad & \mathbf{E}\left[\min\begin{cases} d_s[i-1,j]+1 \\ d_s[i,j-1]+1 \\ d_s[i-1,j-1]+c(\sigma_1[i],\sigma_2[j]) \end{cases}\right] \\
\leq \quad & \min\begin{cases} \widehat{d}(S_1[1..i-1], S_2[1..j])+1 \\ \widehat{d}(S_1[1..i], S_2[1..j-1])+1 \qquad \text{(by Corollary 2)} \\ \widehat{d}(S_1[1..i-1], S_2[1..j-1])+\Pr(S_1[i]\neq S_2[j]) \end{cases}
\end{aligned}
$$

Character-Level Probabilistic Strings: Dynamic Program Upper Bounds

$$
\begin{aligned}
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\leq \quad & \min \begin{cases} \widehat{d}(S_1[1..i-1], S_2[1..j]) + 1 \\ \widehat{d}(S_1[1..i], S_2[1..j-1]) + 1 \\ \widehat{d}(S_1[1..i-1], S_2[1..j-1]) + \Pr(S_1[i] \neq S_2[j]) \end{cases} \qquad \text{(by Corollary 2)} \\
\leq \quad & \min \begin{cases} \widehat{ud}[i-1, j] + 1 \\ \widehat{ud}[i, j-1] + 1 \\ \widehat{ud}[i-1, j-1] + \Pr(S_1[i] \neq S_2[j]) \end{cases} \qquad \text{(by induction)}
\end{aligned}
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\begin{aligned}
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= \quad & \mathbf{E}\left[\min\begin{cases} d_s[i-1,j]+1 \\ d_s[i,j-1]+1 \\ d_s[i-1,j-1]+c(\sigma_1[i],\sigma_2[j]) \end{cases}\right] \\
\leq \quad & \min\begin{cases} \widehat{d}(S_1[1..i-1], S_2[1..j])+1 \\ \widehat{d}(S_1[1..i], S_2[1..j-1])+1 \\ \widehat{d}(S_1[1..i-1], S_2[1..j-1])+\Pr(S_1[i]\neq S_2[j]) \end{cases} \qquad \text{(by Corollary 2)} \\
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= \quad & \widehat{ud}[i,j]. \qquad \square
\end{aligned}
$$

Experiments: Generation of Data Sets

- For each string σ_1 in the source database D we find the similar set $A(\sigma_1) = \{\sigma_2 | \sigma_2 \in D \wedge d(\sigma_1, \sigma_2) \leq 3\}$

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  The choices are in $A(\sigma_1)$ with probability normalized to their frequencies, some strings are replaced with strings outside $A(\sigma_1)$

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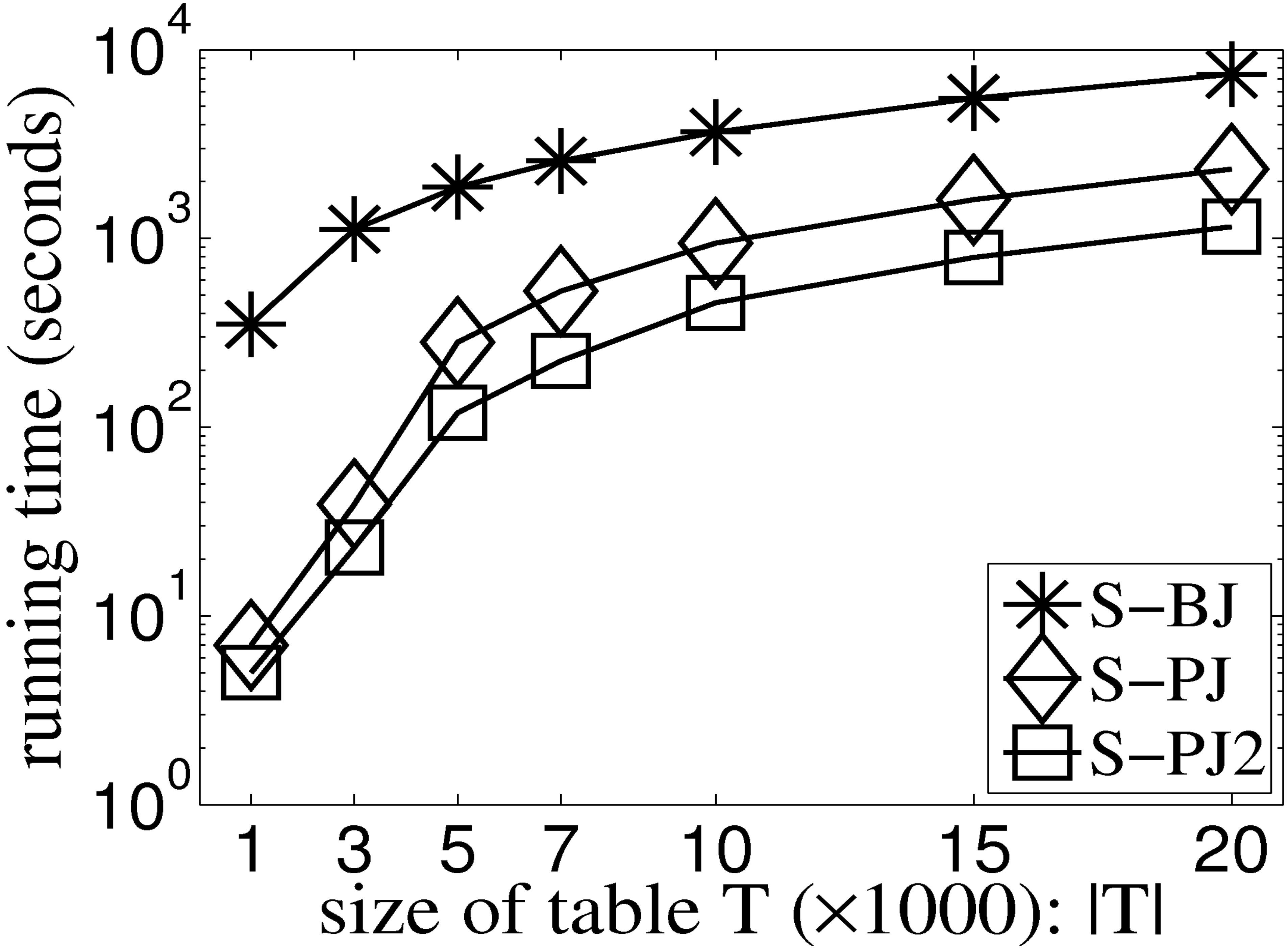
- For string-level probabilistic strings:
  The choices are in $A(\sigma_1)$ with probability normalized to their frequencies, some strings are replaced with strings outside $A(\sigma_1)$
- For a character-level probabilistic string S:
  We select a position $i \in [1, n]$ where $|\sigma_1| = n$ and generate the pdf of $S[i]$ based on the normalized frequency of characters in the ith position of strings in $A(\sigma_1)$.

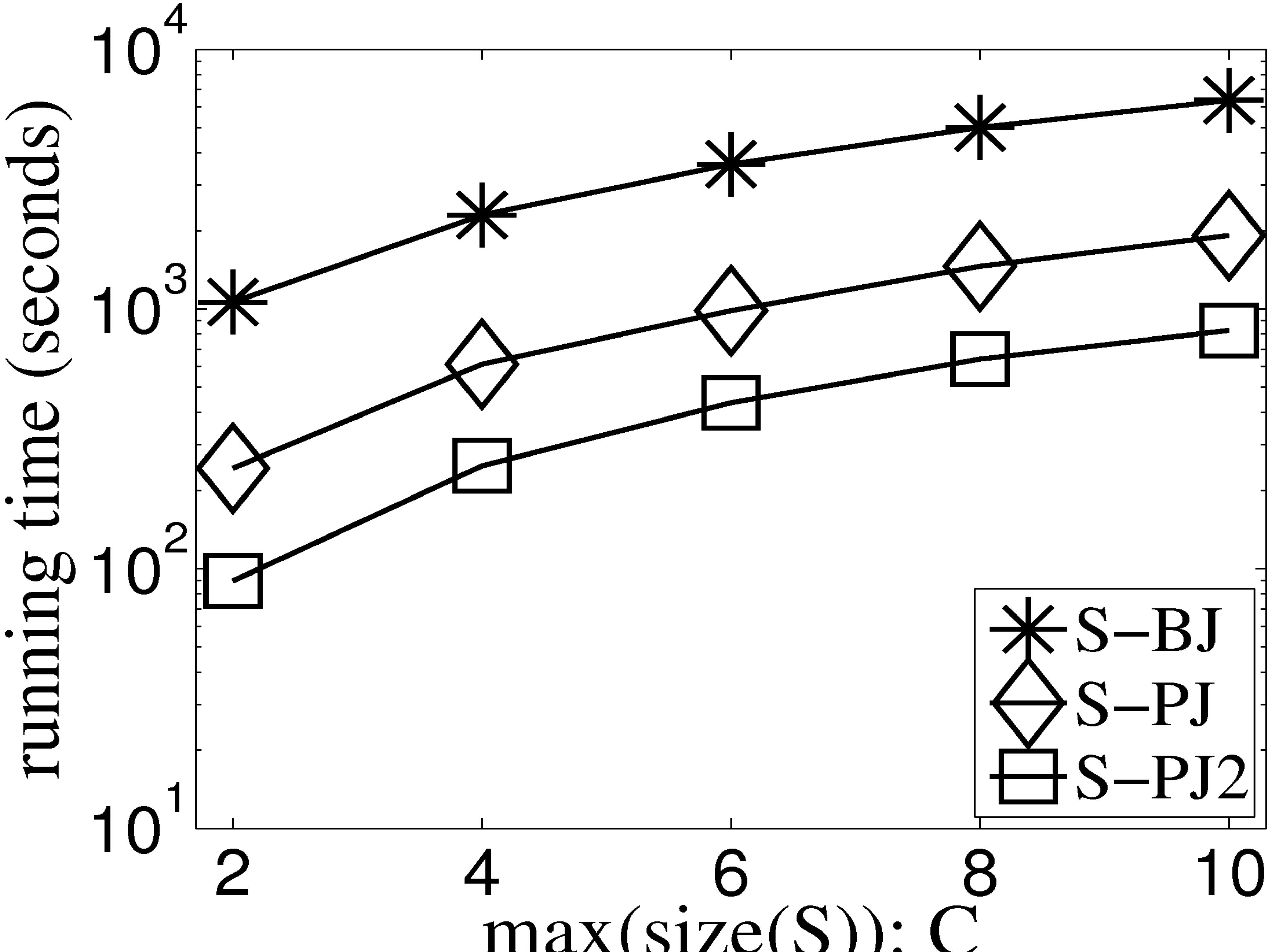
Experiments: String-Level Default Parameters

Symbol	Definition	Default Value
C	max(size(S))	6
ω	Self-concatenations	0
μ_S	Average $\lvert S(i)\rvert$	Author1 13.6 Category 19.9
$\lvert R\rvert$	Size table R	1,000
$\lvert T\rvert$	Size table T	10,000
q	q-gram	2
τ	EED threshold	2

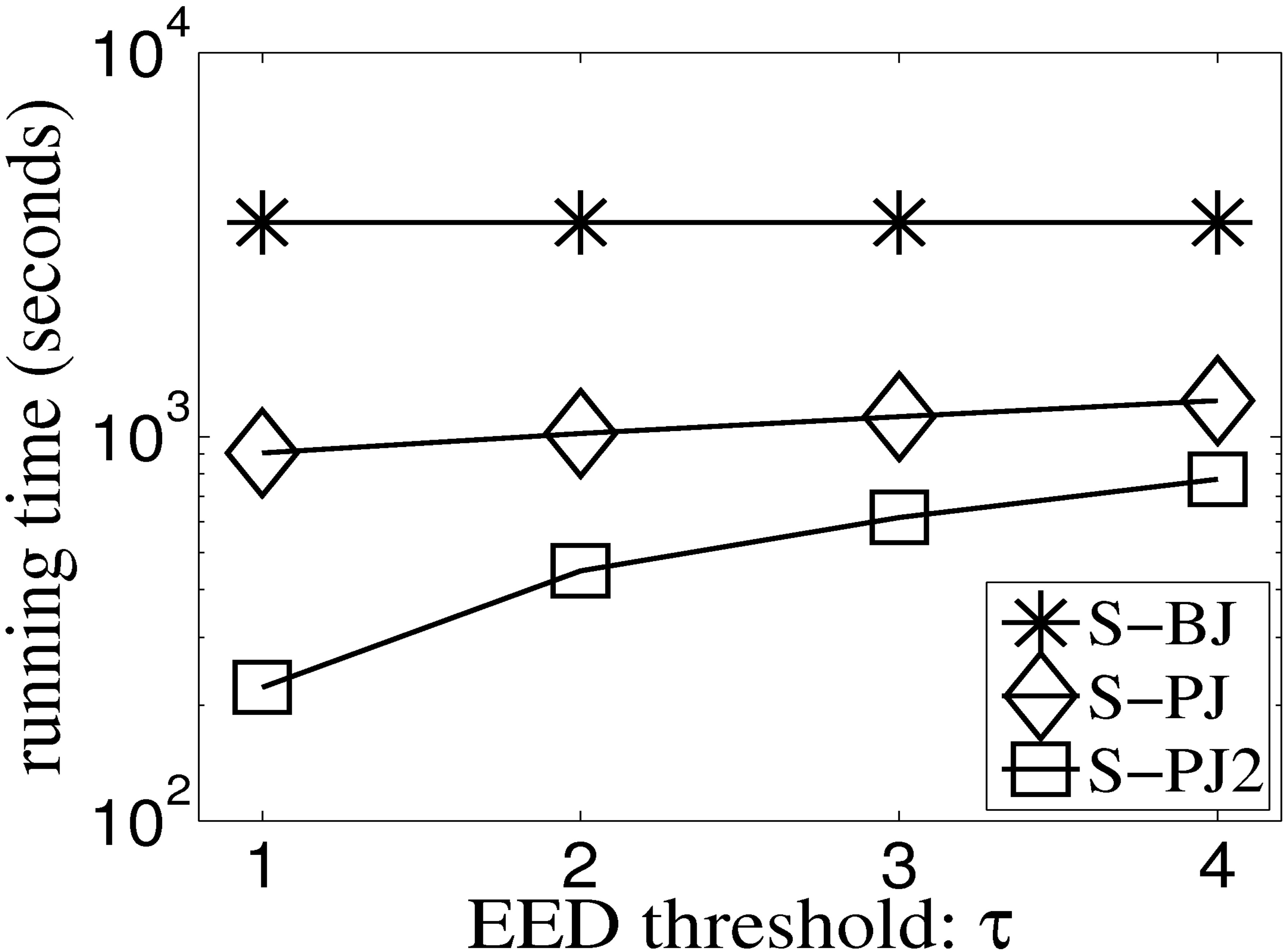
String-level model, *Author1* dataset



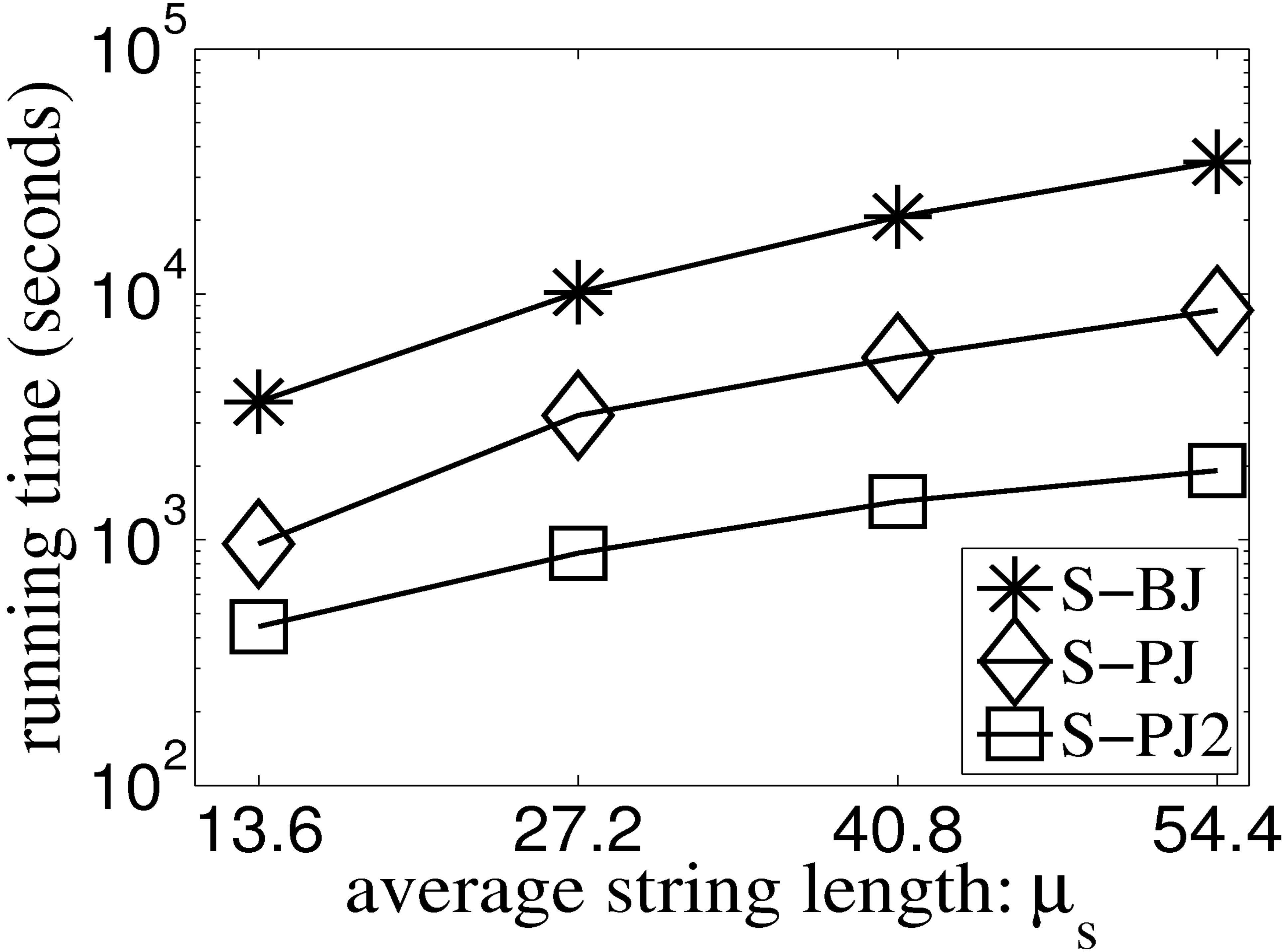
String-level model, *Author1* dataset



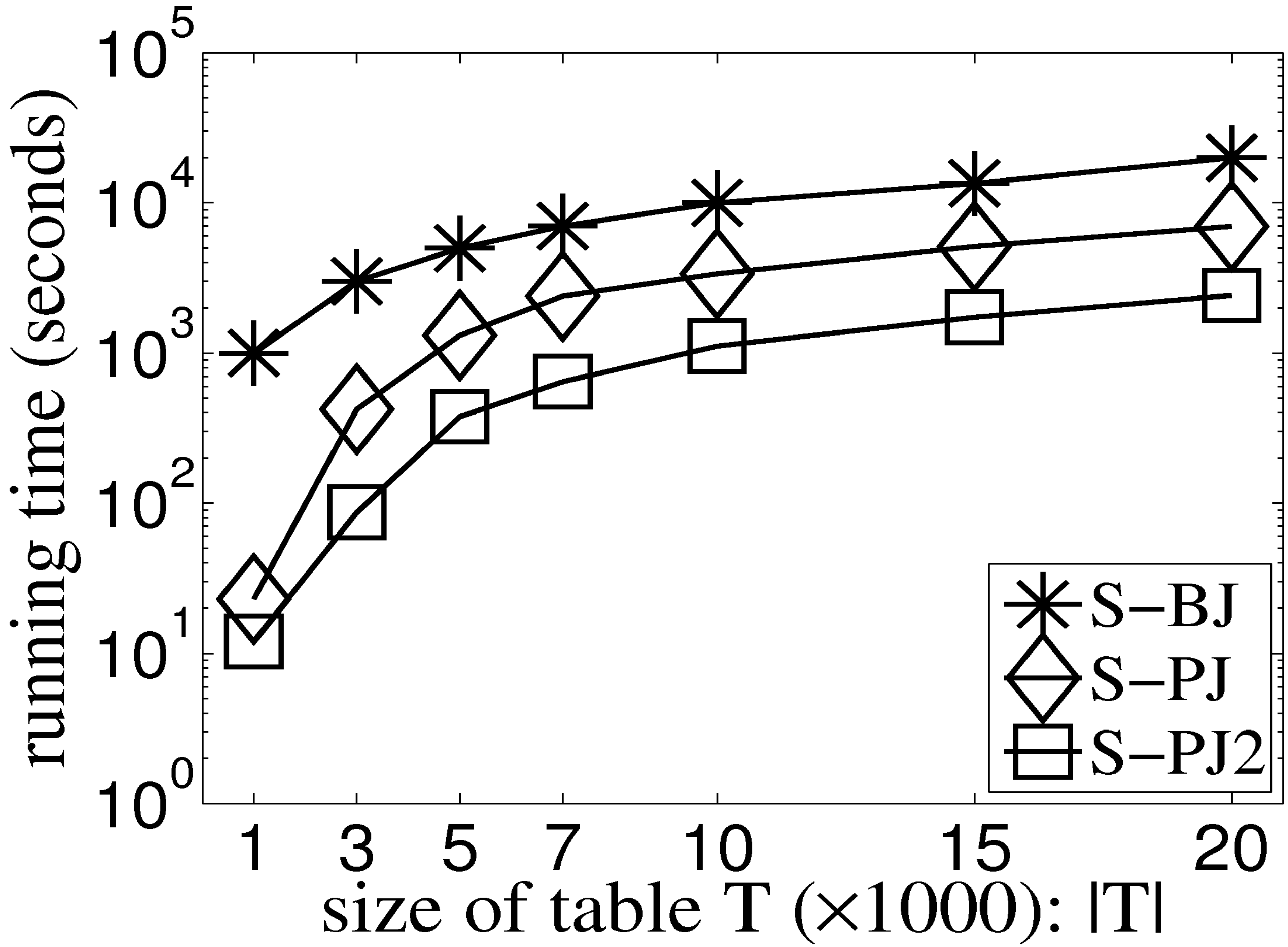
String-level model, *Author1* dataset



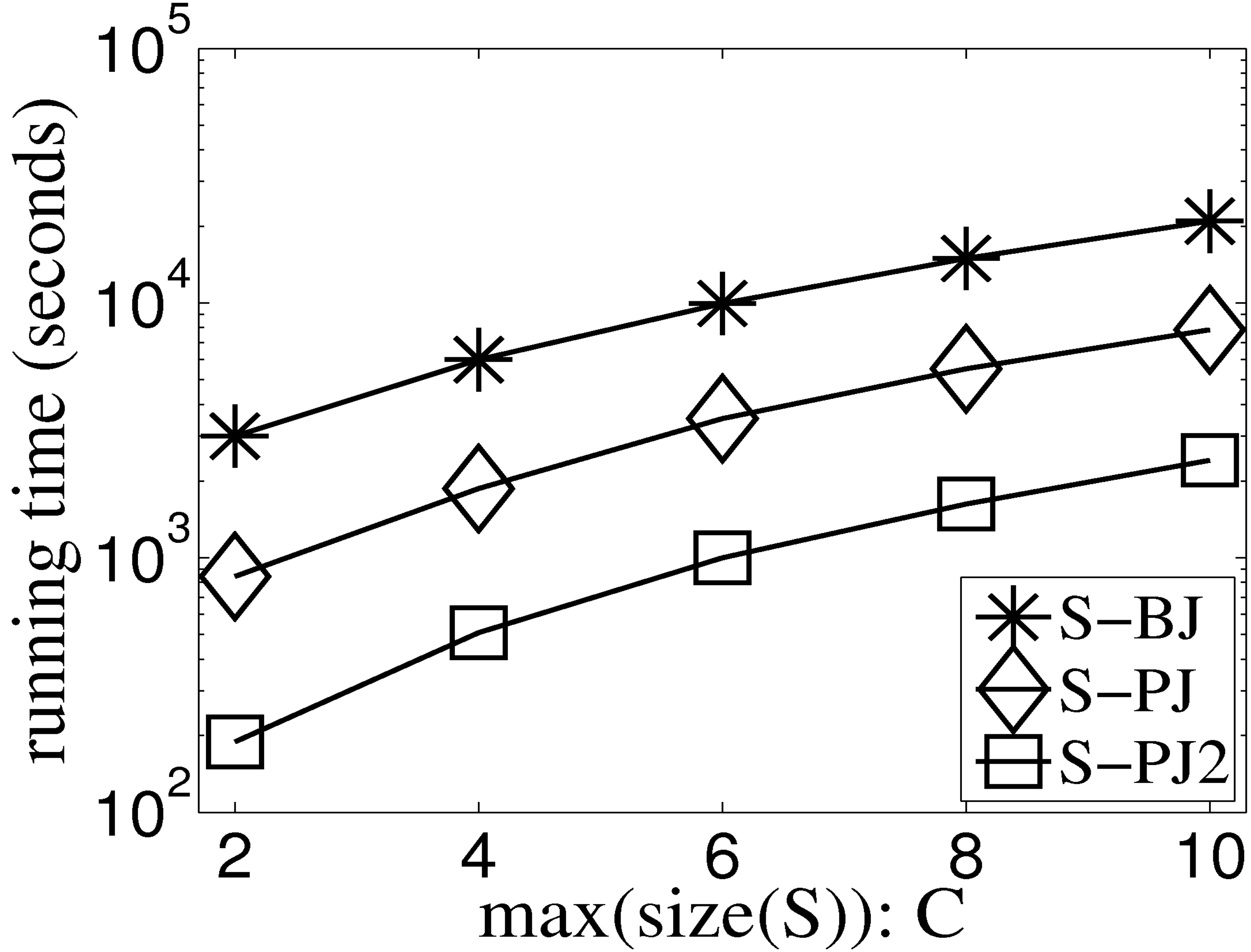
String-level model, *Author1* dataset



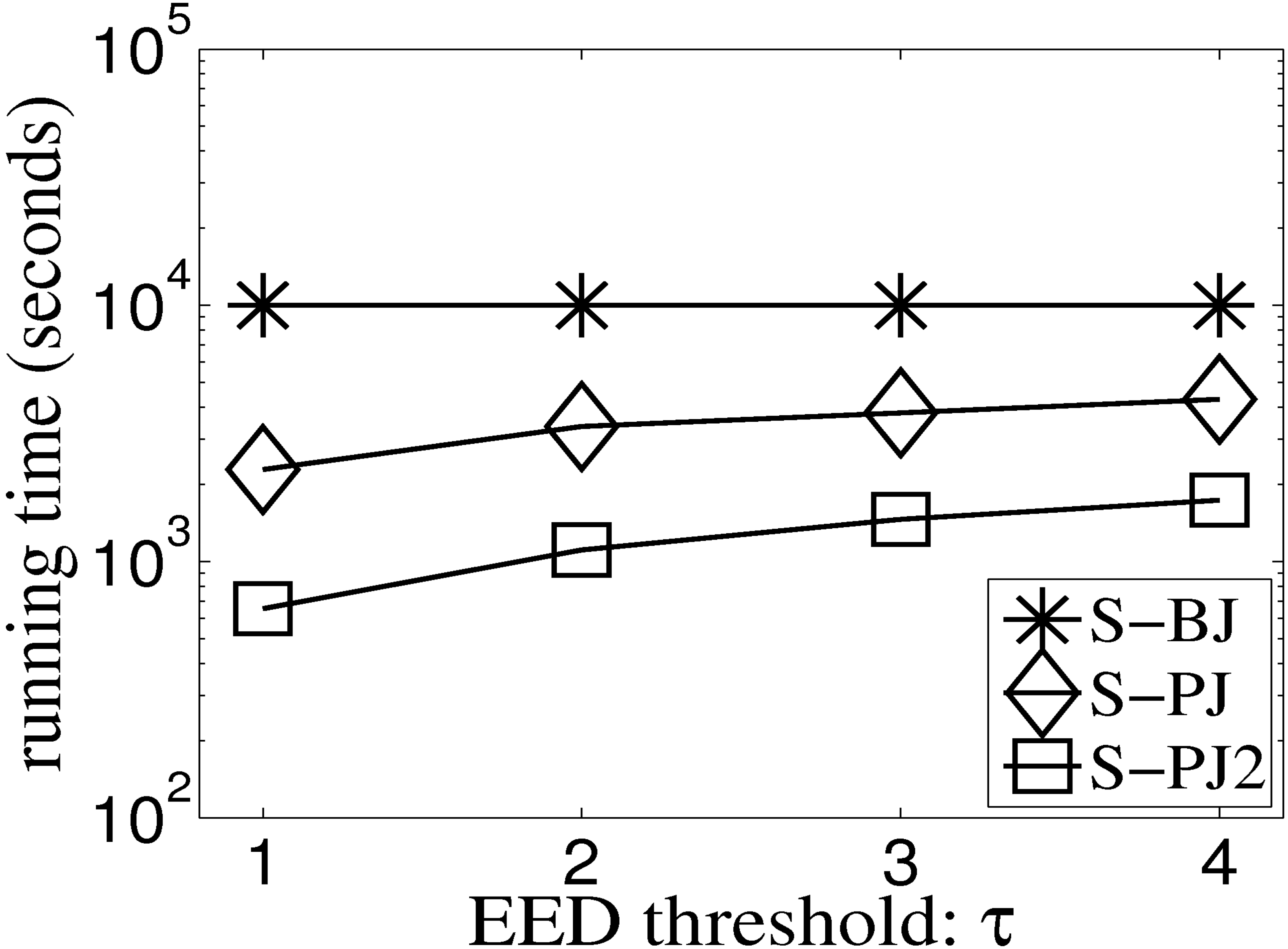
String-level model, *Category* dataset



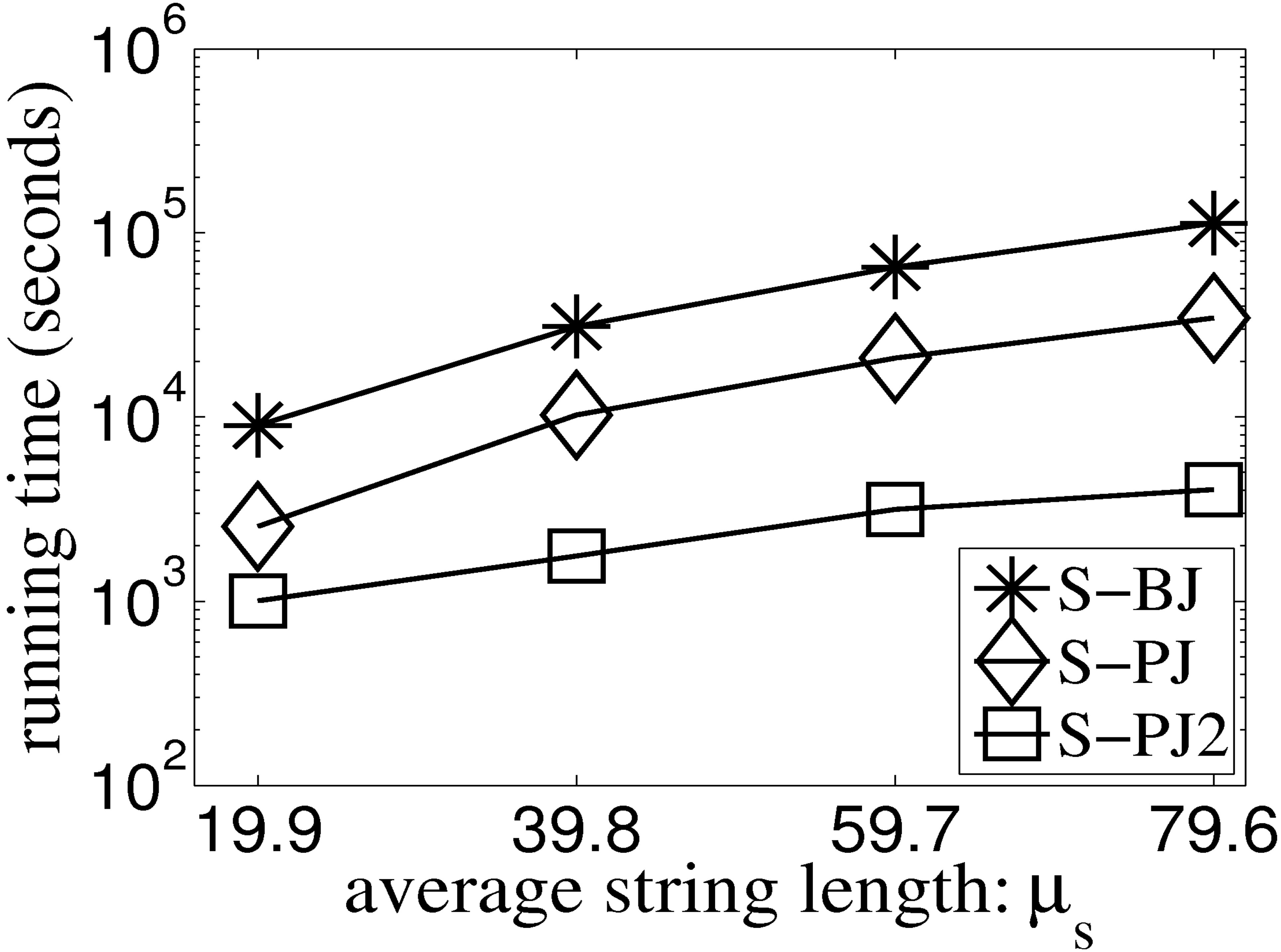
String-level model, *Category* dataset



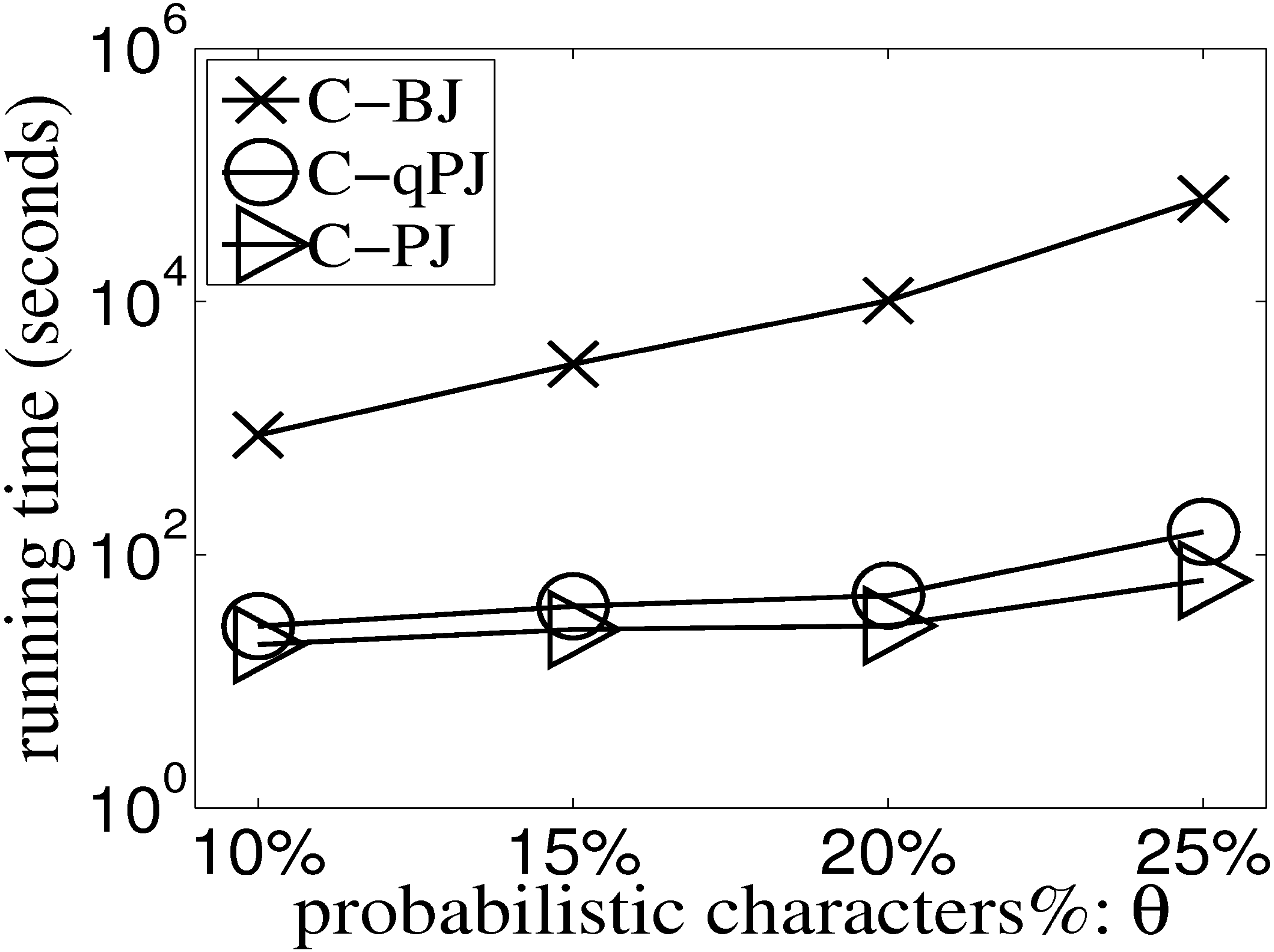
String-level model, *Category* dataset



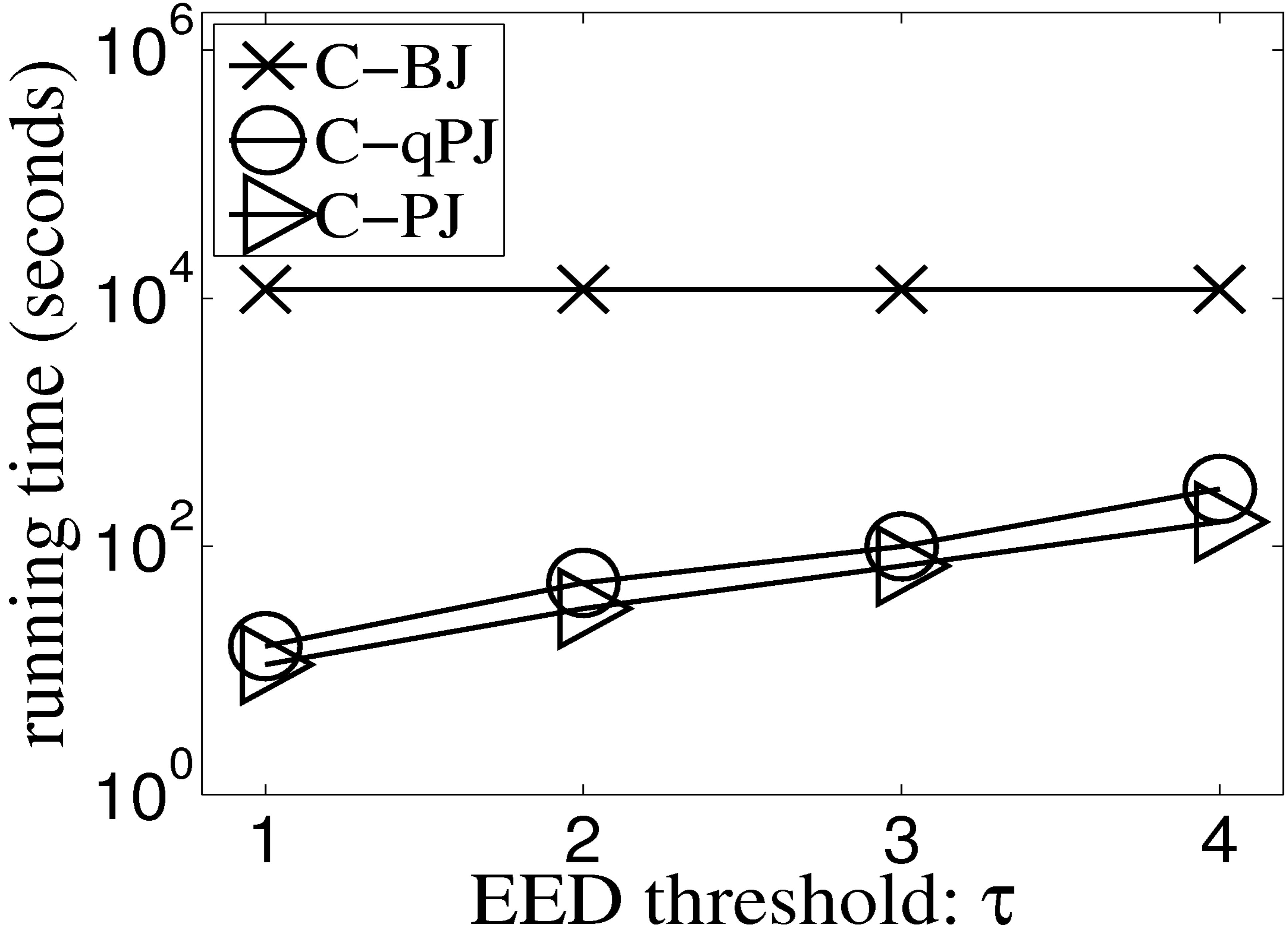
String-level model, *Category* dataset



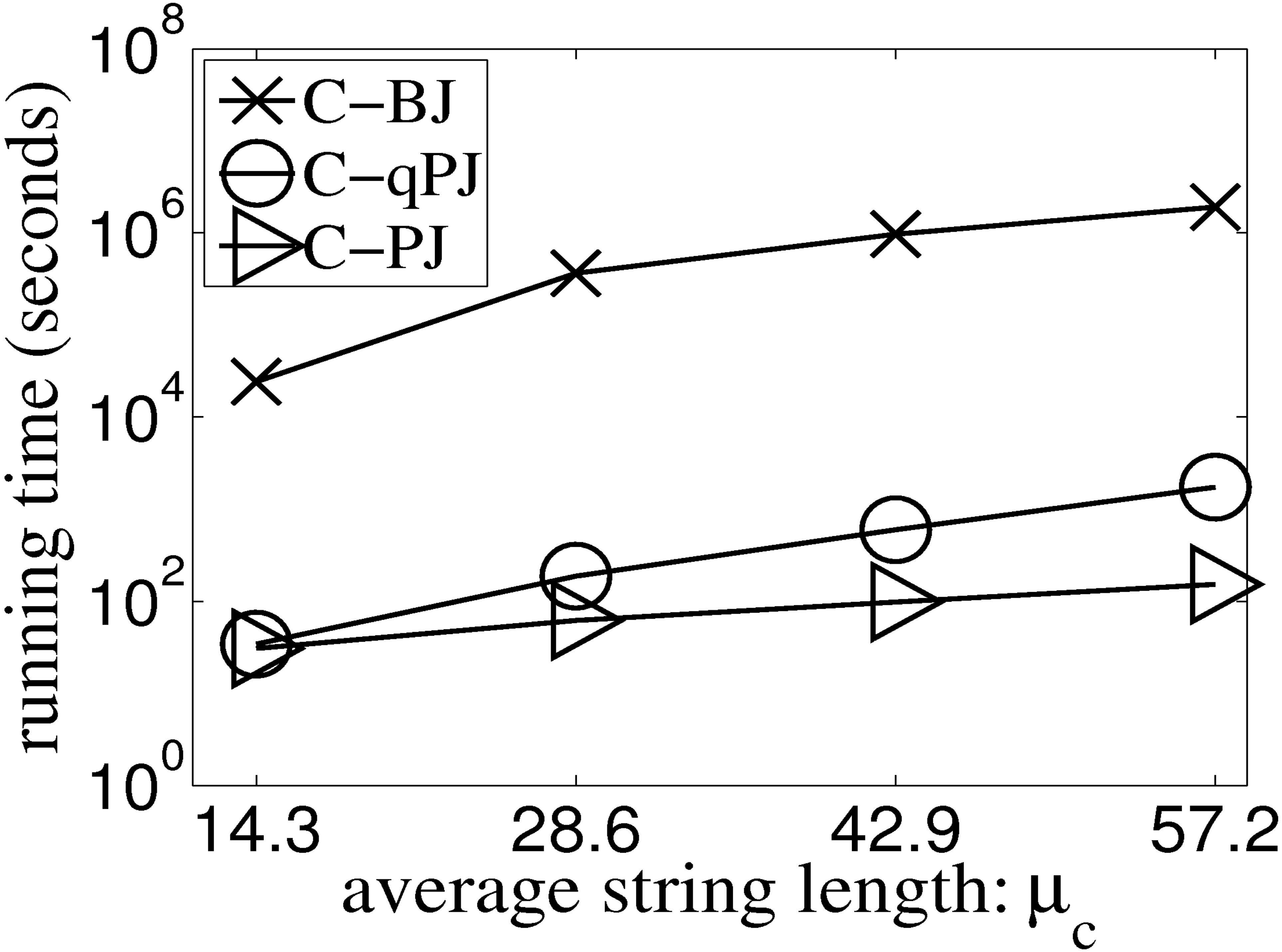
Character-level model, *Author2* dataset



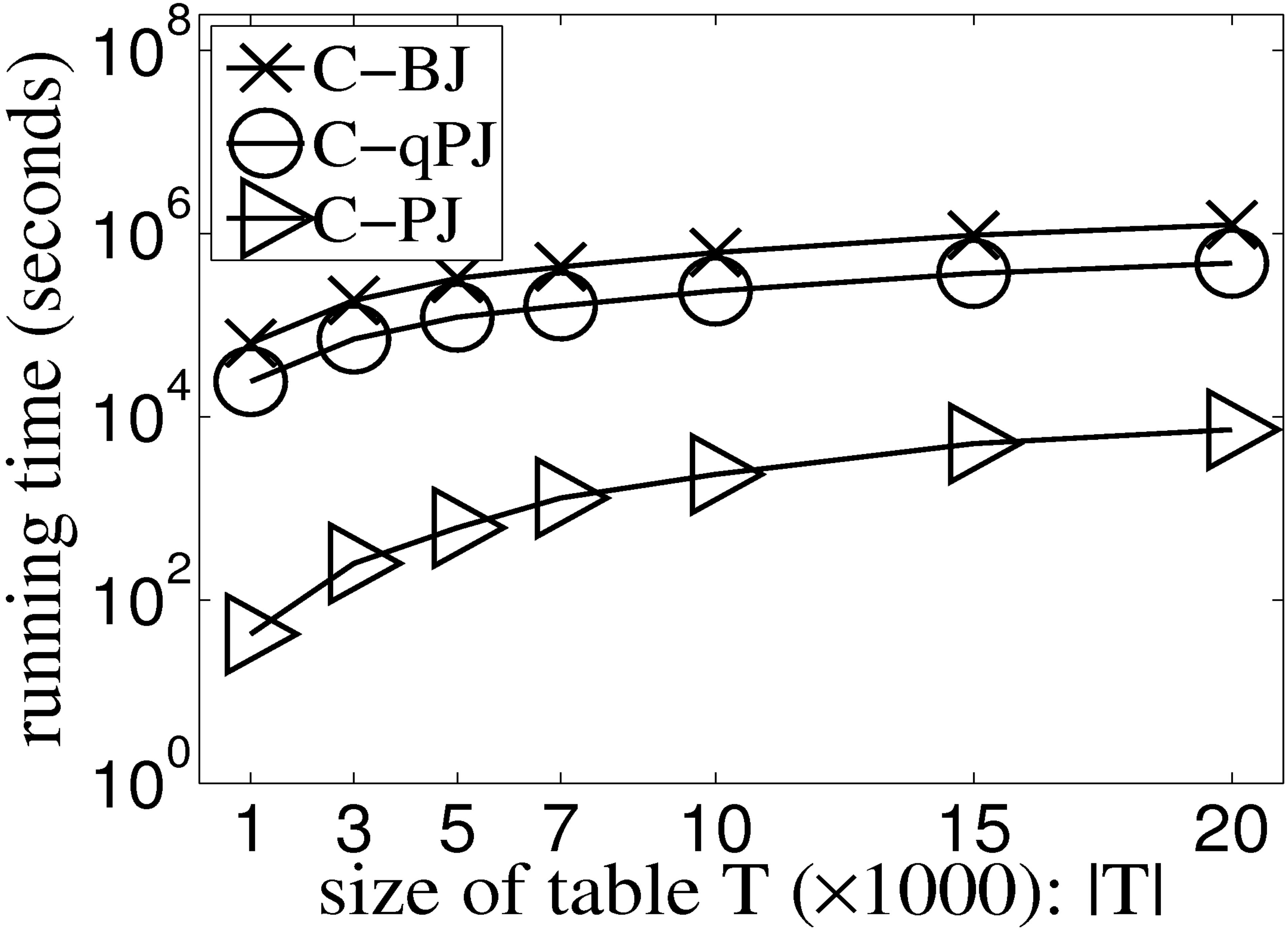
Character-level model, *Author2* dataset



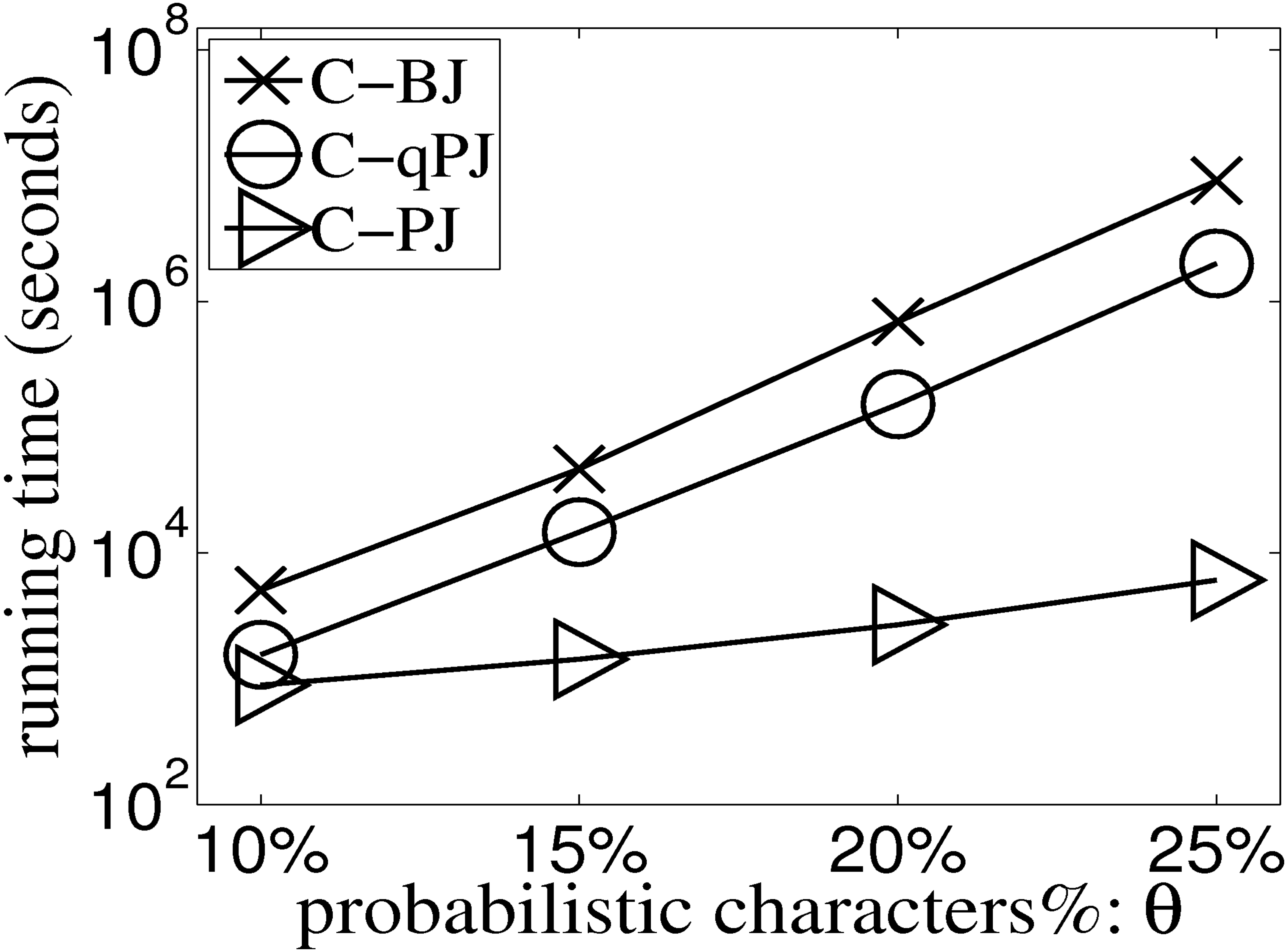
Character-level model, *Author2* dataset



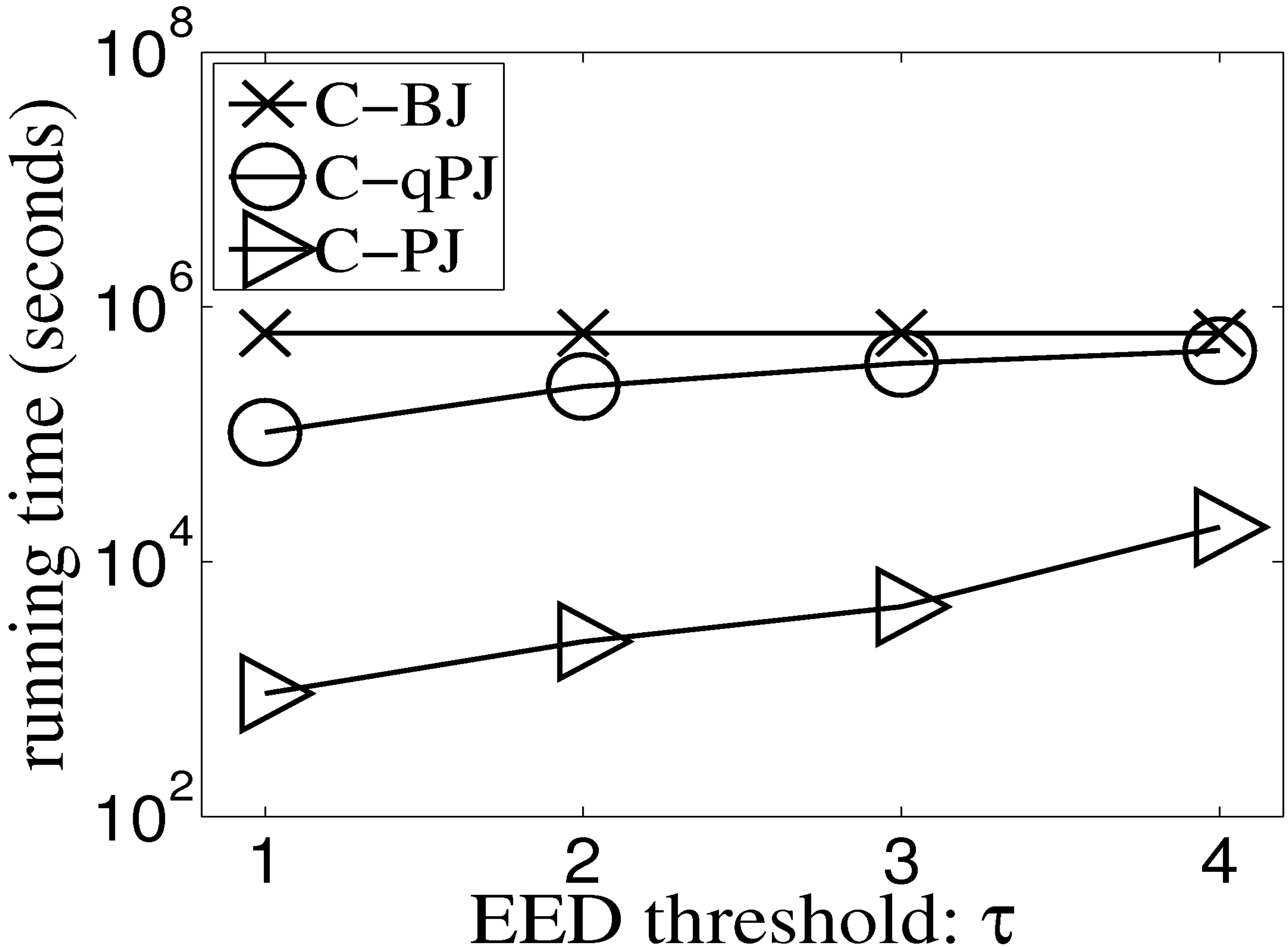
Character-level model, *Genome* dataset



Character-level model, *Genome* dataset



Character-level model, *Genome* dataset



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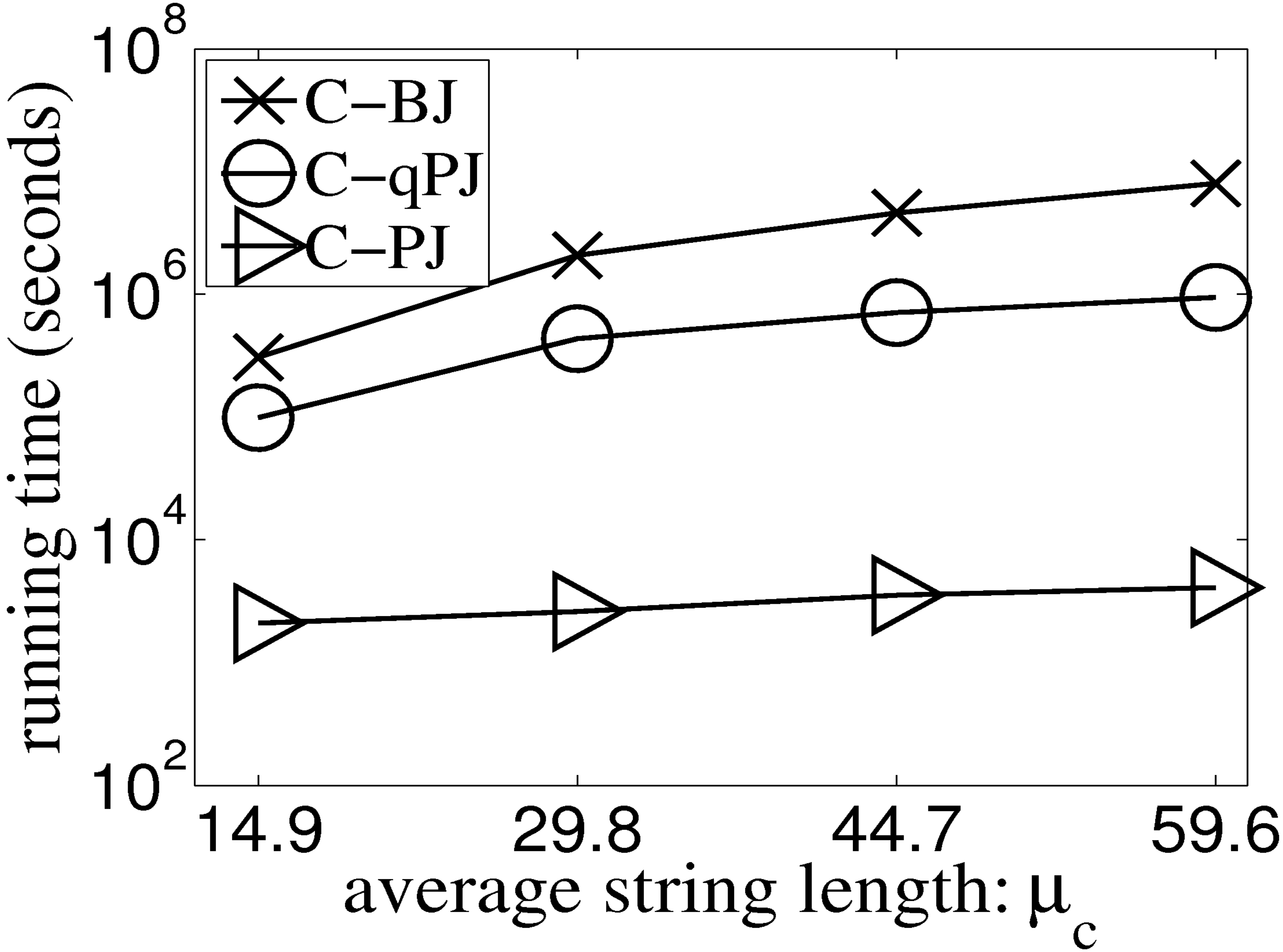
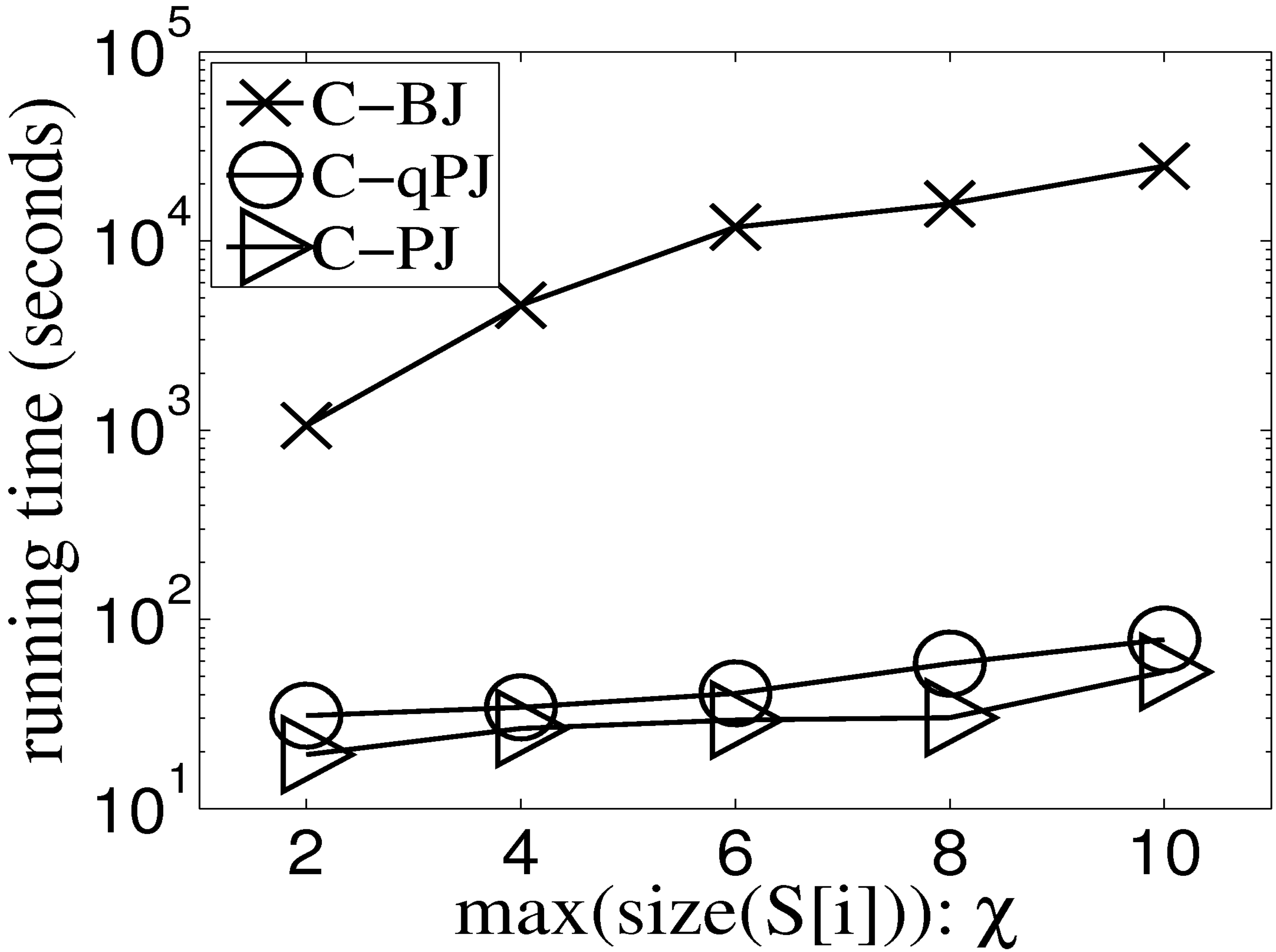


Character-level model, *Author2* dataset



The End

THANK YOU

Q and A

- The entire source code is available from a link at http:/ww2.cs.fsu.edu/~jestes

String-Level Probabilistic Strings: q-Gram Lower Bounds

- For any string-level probabilistic strings S_1 and S_2:

$$\widehat{d}(S_1, S_2) \geq 1 + \frac{\max(\mathbf{E}[|S_1|], \mathbf{E}[|S_2|])}{q} - \frac{\sum_{(\rho_1,\rho_2)\in G_{S_1}\cap G_{S_2}} p(\rho_1)p(\rho_2) + 1}{q}.$$

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-

```
SELECT R.id, T.id FROM R, T,
(SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R_q.id=R.id AND T_q.id=T.id
     AND R_q.cid=R.cid AND T_q.cid=T.cid
 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1+(max(R.len,T.len)-SUM(R.p*T.p)-1)/q≤ τ
) AS L
WHERE L.rid=R.id AND L.tid=T.id
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*d(R.A,T.A)) ≤  τ
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-

```
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     AND R_q.cid=R.cid AND T_q.cid=T.cid
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- ```
SELECT R.id, T.id FROM R, T,
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) AS L
WHERE L.rid=R.id AND L.tid=T.id
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*d(R.A,T.A)) ≤ τ
```
```

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

- For any string-level probabilistic strings S_1 and S_2,

$$\widehat{d}(S_1, S_2) \geq 1 + \frac{\max(\mathbf{E}(|S_1|), \mathbf{E}(|S_2|)) - \mathrm{UB}_\tau - 1}{q}.$$

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

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-

```
SELECT R.id, T.id FROM R, T,
(SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R_q.id=R.id AND T_q.id=T.id
     AND R_q.cid=R.cid AND T_q.cid=T.cid
 GROUP BY R.id, T.id, R.len, T.len
    HAVING 1 + (1/q)*( max(R.len,T.len) - 1 -
ub(R_q.cid,R.p,R_q.ℓ,R_q.g,T_q.cid,T.p,T_q.ℓ,T_q.g,τ))≤ τ
) AS L
WHERE L.rid=R.id AND L.tid=T.id
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*d(R.A,T.A)) ≤ τ
```

String-Level Probabilistic Strings: Improving q-Gram Lower Bounds

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ub(R_q.cid,R.p,R_q.ℓ,R_q.g,T_q.cid,T.p,T_q.ℓ,T_q.g,τ))≤ τ
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-

```
SELECT R.id, T.id FROM R, T,
(SELECT R.id AS rid, T.id AS tid FROM R,T,Rq,Tq
 WHERE Rq.g=Tq.g AND Rq.id=R.id AND Tq.id=T.id
    AND Rq.cid=R.cid AND Tq.cid=T.cid
 GROUP BY R.id, T.id, R.len, T.len
   HAVING 1 + 1/q*( max(R.len,T.len) - 1 -
ub(Rq.cid,R.p,Rq.ℓ,Rq.g,Tq.cid,T.p,Tq.ℓ,Tq.g,τ))≤ τ
) AS L
WHERE L.rid=R.id AND L.tid=T.id
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```

String-Level Probabilistic String Similiarity Join

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    AND R_q.cid=R.cid AND T_q.cid=T.cid
 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1+(max(R.len,T.len)-SUM(R.p*T.p)-1)/q≤ τ
 EXCEPT
 SELECT R.id AS rid, T.id AS tid FROM R, T
 GROUP BY R.id, T.id
 HAVING SUM(R.p*T.p*ABS(|R.A|-|T.A|)) > τ
) AS L
WHERE L.rid=R.id AND L.tid=T.id
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*d(R.A,T.A)) ≤ τ          (Q3)
```

String-Level Probabilistic String Similiarity Join

```
SELECT R.id, T.id FROM R, T,
(SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q
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     AND R_q.cid=R.cid AND T_q.cid=T.cid
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```

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```
SELECT R.id, T.id FROM R, T,
(SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R_q.id=R.id AND T_q.id=T.id
    AND R_q.cid=R.cid AND T_q.cid=T.cid
 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1+(max(R.len,T.len)-SUM(R.p*T.p)-1)/q≤ τ
 EXCEPT
 SELECT R.id AS rid, T.id AS tid FROM R, T
 GROUP BY R.id, T.id
 HAVING SUM(R.p*T.p*ABS(|R.A|-|T.A|)) > τ
 ) AS L
WHERE L.rid=R.id AND L.tid=T.id
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*d(R.A,T.A)) ≤  τ            (Q3)
```

String-Level Probabilistic String Similiarity Join

```
SELECT R.id, T.id FROM R, T,
(SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R_q.id=R.id AND T_q.id=T.id
    AND R_q.cid=R.cid AND T_q.cid=T.cid
 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1+(max(R.len,T.len)-SUM(R.p*T.p)-1)/q≤ τ
 EXCEPT
 SELECT R.id AS rid, T.id AS tid FROM R, T
 GROUP BY R.id, T.id
 HAVING SUM(R.p*T.p*ABS(|R.A|-|T.A|)) > τ
) AS L
WHERE L.rid=R.id AND L.tid=T.id
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*d(R.A,T.A)) ≤  τ          (Q3)
```

String-Level Probabilistic String Similiarity Join

- SELECT R.id, T.id FROM R, T,
  (SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q,
  $(\cdots)$ AS L *(same as lines 2-11 in Q3)*
  WHERE L.rid=R.id AND L.tid=T.id AND R_q.g=T_q.g
  AND R_q.cid=R.cid AND T_q.cid=T.cid
  AND R_q.id=R.id AND T_q.id=T.id
  GROUP BY R.id, T.id, R.len, T.len
  HAVING 1 + $\frac{1}{q}$*($\max$(R.len,T.len) - 1 -
  ub(R_q.cid,R.p,$R_q.\ell$,R_q.g,T_q.cid,T.p,$T_q.\ell$,T_q.g,τ))$\leq \tau$
) AS L2
  WHERE L2.rid=R.id AND L2.tid=T.id
  GROUP BY R.id, T.id
  HAVING SUM(R.p*T.p*d(R.A,T.A)) $\leq \tau$ (Q4)

String-Level Probabilistic String Similiarity Join

```
SELECT R.id, T.id FROM R, T,
( SELECT R.id AS rid, T.id AS tid FROM R,T,Rq,Tq,
   (···) AS L (same as lines 2-11 in Q3)
   WHERE L.rid=R.id AND L.tid=T.id AND Rq.g=Tq.g
     AND Rq.cid=R.cid AND Tq.cid=T.cid
     AND Rq.id=R.id AND Tq.id=T.id
  GROUP BY R.id, T.id, R.len, T.len
  HAVING 1 + 1/q*( max(R.len,T.len) - 1 -
 ub(Rq.cid,R.p,Rq.ℓ,Rq.g,Tq.cid,T.p,Tq.ℓ,Tq.g,τ))≤ τ
) AS L2
WHERE L2.rid=R.id AND L2.tid=T.id
GROUP BY R.id, T.id
HAVING SUM(R.p*T.p*d(R.A,T.A)) ≤  τ          (Q4)
```

String-Level Probabilistic String Similiarity Join

- SELECT R.id, T.id FROM R, T,
  (SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q,
  $(\cdots)$ AS L *(same as lines 2-11 in Q3)*
  WHERE L.rid=R.id AND L.tid=T.id AND R_q.g=T_q.g
  AND R_q.cid=R.cid AND T_q.cid=T.cid
  AND R_q.id=R.id AND T_q.id=T.id
  GROUP BY R.id, T.id, R.len, T.len
  HAVING 1 + $\frac{1}{q}$*($\max$(R.len,T.len) - 1 -
  ub(R_q.cid,R.p,R_q.ℓ,R_q.g,T_q.cid,T.p,T_q.ℓ,T_q.g,τ))$\leq \tau$
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  WHERE L2.rid=R.id AND L2.tid=T.id
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- SELECT R.id, T.id FROM R, T,
  (SELECT R.id AS rid, T.id AS tid FROM R,T,R_q,T_q,
  $(\cdots)$ AS L *(same as lines 2-11 in Q3)*
  WHERE L.rid=R.id AND L.tid=T.id AND R_q.g=T_q.g
  AND R_q.cid=R.cid AND T_q.cid=T.cid
  AND R_q.id=R.id AND T_q.id=T.id
  GROUP BY R.id, T.id, R.len, T.len
  HAVING 1 + $\frac{1}{q}$*($\max$(R.len,T.len) - 1 -
  ub(R_q.cid,R.p,$R_q.\ell$,R_q.g,T_q.cid,T.p,$T_q.\ell$,T_q.g,τ))$\leq \tau$
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  GROUP BY R.id, T.id
  HAVING SUM(R.p*T.p*d(R.A,T.A)) $\leq$ τ (Q4)

Character-Level Probabilistic Strings: q-Gram Lower Bounds

- For any character-level probabilistic strings S_1 and S_2:

$$\widehat{d}(S_1, S_2) \geq 1 + \frac{\max(|S_1|, |S_2|)}{q} - \frac{\sum_{\substack{\gamma_1 \in G_{S_1} \\ \gamma_2 \in G_{S_2}}} \Pr(\gamma_1 = \gamma_2) + 1}{q}$$

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- **Proof** Let $s \in \Omega$ and $s = ((\sigma_1, p_1), (\sigma_2, p_2))$
  By the deterministic **q-gram filtering** lemma we have,

$$\begin{aligned} & \sum_{s \in \Omega} w(s) |G_{\sigma_1} \cap G_{\sigma_2}| \\ \geq \quad & \sum_{s \in \Omega} w(s)(\max(|\sigma_1|, |\sigma_2|) - 1 - q(d(s) - 1)) \end{aligned}$$

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Character-Level Probabilistic Strings: q-Gram Lower Bounds

- Let $s \in \Omega$ and $s = ((\sigma_1, p_1), (\sigma_2, p_2))$
  $\gamma_1 = (i, S_1[i..i+q-1]) \in G_{S_1}$
  $\gamma_2 = (j, S_2[j..j+q-1]) \in G_{S_2}$

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- Define $\gamma_1 =_s \gamma_2$ as the event that $S_1[i..i+q-1] = S_2[j..j+q-1]$ in s. We have

$$\Pr(\gamma_1 =_s \gamma_2) = \begin{cases} w(s), & \text{if } \sigma_1[i..i+q-1] = \sigma_2[j..j+q-1]; \\ 0, & \text{otherwise.} \end{cases}$$

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- Recall that $|G_{\sigma_1} \cap G_{\sigma_2}|$ is the number of matching pairs of q-grams in G_{σ_1} and G_{σ_2}, so

$$\sum_{s \in \Omega} w(s) |G_{\sigma_1} \cap G_{\sigma_2}| = \sum_{s \in \Omega} \sum_{\substack{\gamma_1 \in G_{S_1} \\ \gamma_2 \in G_{S_2}}} \Pr(\gamma_1 =_s \gamma_2)$$

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$$\Pr(\gamma_1 =_s \gamma_2) = \begin{cases} w(s), & \text{if } \sigma_1[i..i+q-1] = \sigma_2[j..j+q-1]; \\ 0, & \text{otherwise.} \end{cases}$$

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$$= \sum_{\substack{\gamma_1 \in G_{S_1} \\ \gamma_2 \in G_{S_2}}} \sum_{s \in \Omega} \Pr(\gamma_1 =_s \gamma_2) = \sum_{\substack{\gamma_1 \in G_{S_1} \\ \gamma_2 \in G_{S_2}}} \Pr(\gamma_1 = \gamma_2) \qquad \square$$

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-
```
SELECT R.id,T.id FROM R,T,
(SELECT R.id AS rid,T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R.id=R_q.id AND T.id=T_q.id
 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1 + max(R.len,T.len)/q -
  SUM( T_q.p*R_q.p)/q - 1/q ≤  τ
 ) AS L
 WHERE L.rid=R.id AND L.tid=T.id
 AND ed(R.A,T.A)≤ τ
```

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-
```
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 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1 + max(R.len,T.len)/q -
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```

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-
```
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(SELECT R.id AS rid,T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R.id=R_q.id AND T.id=T_q.id
 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1 + max(R.len,T.len)/q -
  SUM( T_q.p*R_q.p)/q - 1/q ≤ τ
 ) AS L
 WHERE L.rid=R.id AND L.tid=T.id
 AND ed(R.A,T.A)< τ
```

Character-Level Probabilistic String Similarity Join

```
SELECT R.id,T.id FROM R,T,
(SELECT R.id AS rid,T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R.id=R_q.id AND T.id=T_q.id
   AND ABS(R.len - T.len) ≤ τ
 GROUP BY R.id, T.id, R.len, T.len
 HAVING 1 + (max(R.len,T.len)-1)/q -
  SUM( FLOOR(T_q.p*R_q.p)*
    min(1, τ/max(1,ABS(R_q.ℓ - T_q.ℓ))) )/q -
  SUM( CEILING(1-R_q.p*T_q.p)*T_q.p*R_q.p )/q ≤ τ
 EXCEPT
 SELECT R.id AS rid,T.id AS tid FROM R,T,R_q,T_q
 WHERE R_q.g=T_q.g AND R_q.ℓ=T_q.ℓ AND R.id=R_q.id
   AND T.id=T_q.id AND ABS(R.len - T.len) ≤ τ
 GROUP BY R.id, T.id, R.len, T.len
 HAVING max(R.len,T.len)+q-1-SUM(T_q.p*R_q.p)≤ τ
) AS L
WHERE L.rid=R.id AND L.tid=T.id AND ld(R.A,T.A)
 ≤ τ AND ud(R.A,T.A)> τ AND ed(R.A,T.A)≤ τ   (Q6)
```

Special Case: No Matching q-Grams

- Consider the string-level probabilistic strings:

$$\{(cat, 1)\} \qquad \{(dog, 1)\}$$

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$$\{(1, 1, 1, \#d), (1, 1, 2, do), (1, 1, 3, og), (1, 1, 4, g\$)\}$$

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- Our queries would miss the pair (cat,dog) when $\tau = 3$
  Solution: Add special system choices and q-grams

$$\{(\#\$, 0), (cat, 1)\} \quad \{(\#\$, 0), (dog, 1)\}$$

$$\{(0, 0, 0, \#\$), (1, 1, 1, \#c), (1, 1, 2, ca), (1, 1, 3, at), (1, 1, 4, t\$)\}$$

$$\{(0, 0, 0, \#\$), (1, 1, 1, \#d), (1, 1, 2, do), (1, 1, 3, og), (1, 1, 4, g\$)\}$$

String-level model, *Author1* dataset