Building Wavelet Histograms on Large Data in MapReduce

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Introduction

Record ID	User ID	Object ID	...
1	1	12872	...
2	8	19832	...
3	4	231	...
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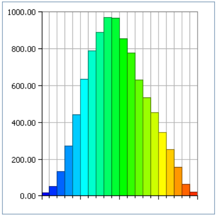

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Outline

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3	5	10	8	2	2	10	14	$\ell = 3$
$\mathbf{v}(1)$	$\mathbf{v}(2)$	$\mathbf{v}(3)$	$\mathbf{v}(4)$	$\mathbf{v}(5)$	$\mathbf{v}(6)$	$\mathbf{v}(7)$	$\mathbf{v}(8)$	

Original data signal at level $\ell = \log_2 u$.

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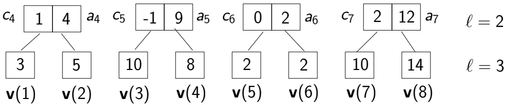

Compute *detail coefficients* c_i and *average coefficients* a_i for level $\ell = 2$.

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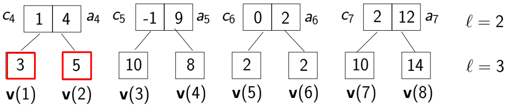

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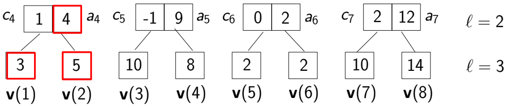
$a_4 = (\mathbf{v}(2) + \mathbf{v}(1))/2$



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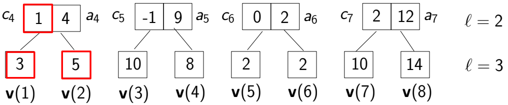

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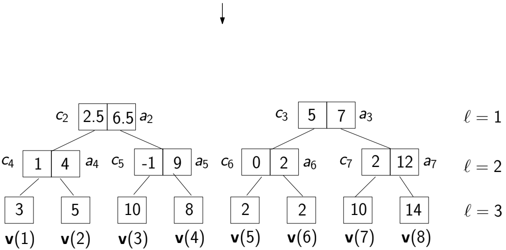

Compute *detail coefficients* c_i and *average coefficients* a_i for level $\ell = 1$.

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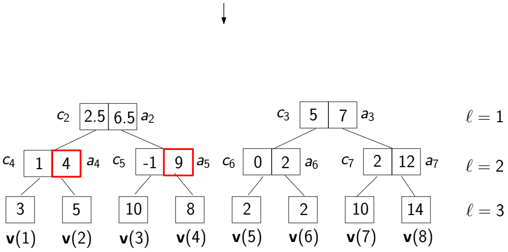

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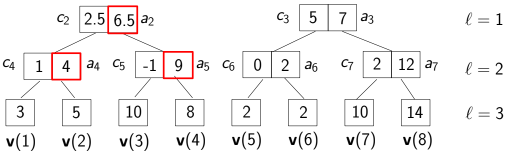

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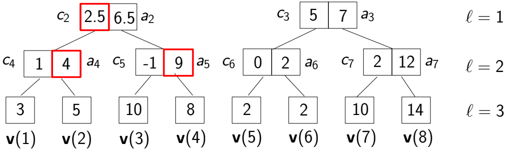
$c_2 = (a_5 - a_4)/2$



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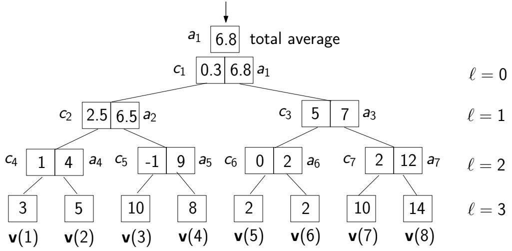

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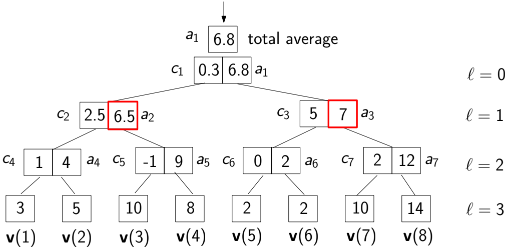

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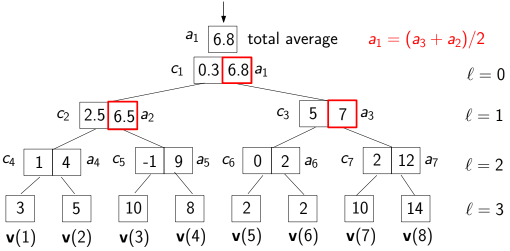

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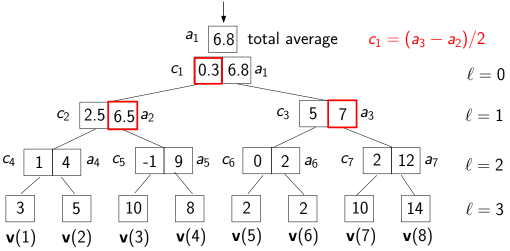

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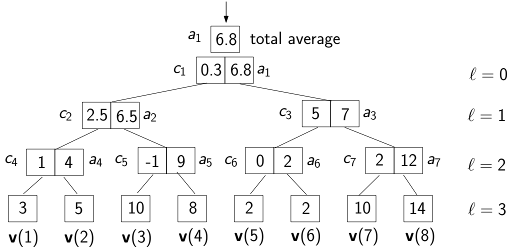

The *wavelet coefficients* w_i
are $[\ a_1, c_1, \ldots, c_{u-1}\ ]$.

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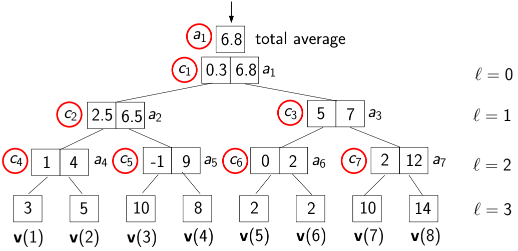

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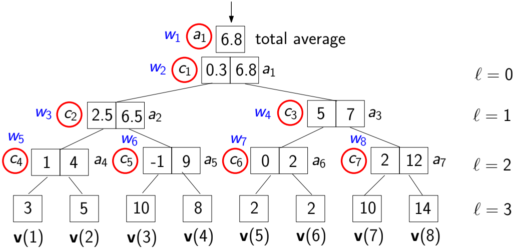

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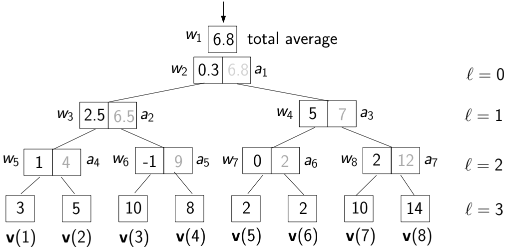

One scales w_i by $\sqrt{u/2^\ell}$ to preserve energy, i.e.
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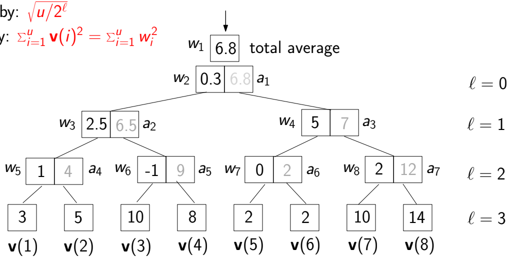

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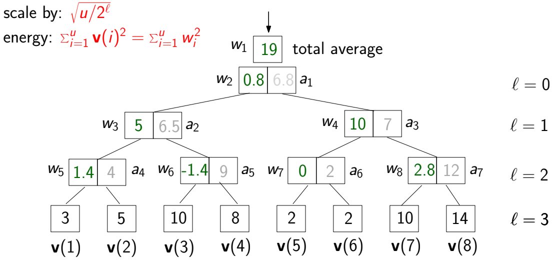

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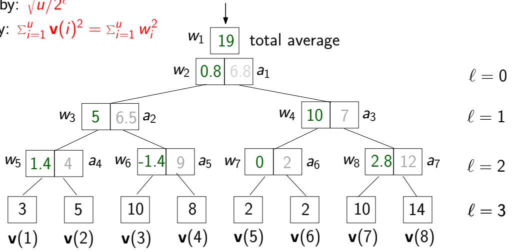

Select top-k w_i in the *absolute value* to obtain best k-term representation minimizing error in energy, i.e. minimize $\Sigma_{i=1}^{u} \mathbf{v}(i)^2 - \Sigma_{i=1}^{u} w_i^2$

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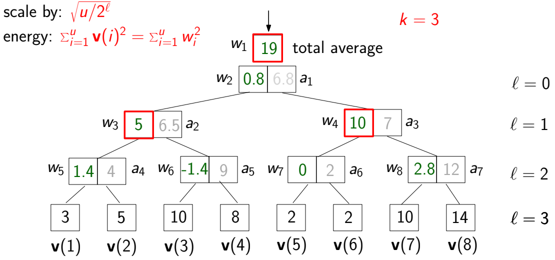

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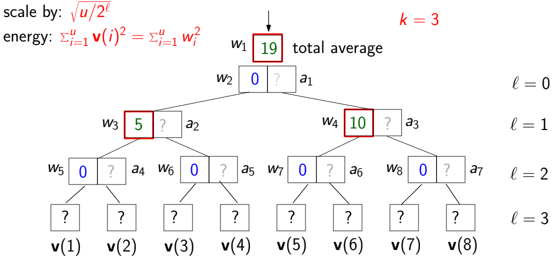

We maintain the best k-term w_i.
Other w_i are treated as 0.

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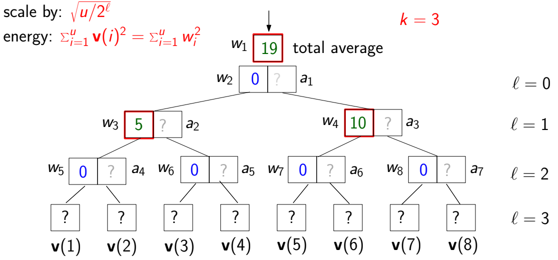

The error in energy is
$\Sigma_{i=1}^{u} \mathbf{v}(i)^2 - \Sigma_{i=1}^{u} w_i^2 = 502 - 489.5 = 12.5$.

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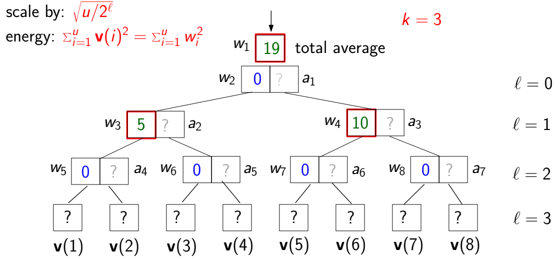

To reconstruct the original signal we compute the *average* and *difference coefficients* in reverse, i.e. top to bottom.

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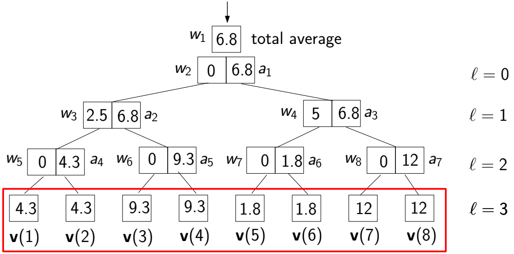

The reconstructed signal is a reasonably close approximation.

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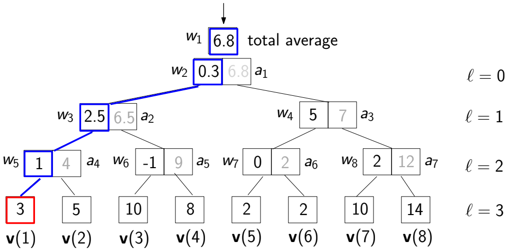

All w_i can be calculated in $O(u \log u)$ time:

1. We maintain $O(\log u)$ partial w_is at a time.
2. Compute affected w_i and contribution from each $\mathbf{v}(x)$ in $O(\log u)$ time.
2. Process $\mathbf{v}(x)$s in sorted order. [GKMS01]

[GKMS01] A.C. Gilbert, et al. Surfing wavelets on streams: One-pass summaries for approximate aggregate queries. In *VLDB*, 2001.

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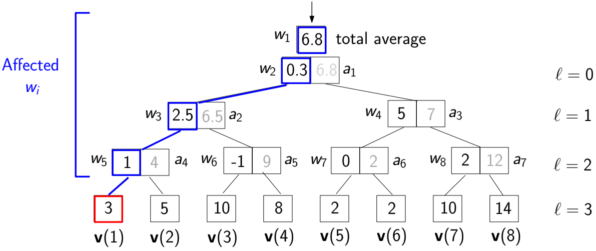

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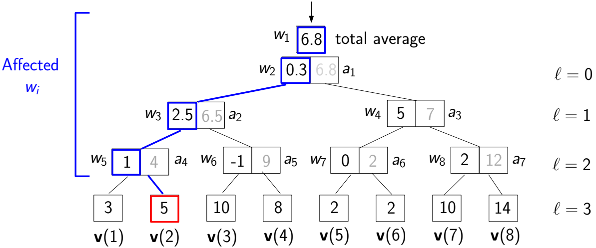

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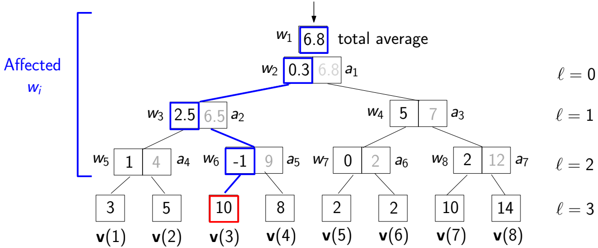
- We obtain the Haar wavelet coefficients w_i recursively as follows:

x	1	2	3	4	5	6	7	8
$\mathbf{v}(x)$	3	5	10	8	2	2	10	14



All w_i can be calculated in $O(u \log u)$ time:

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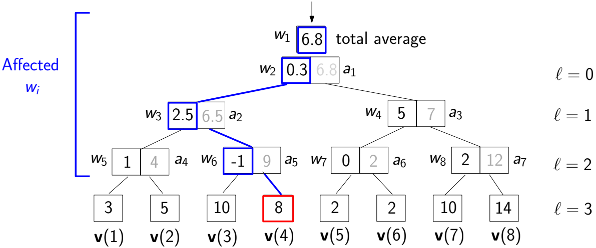

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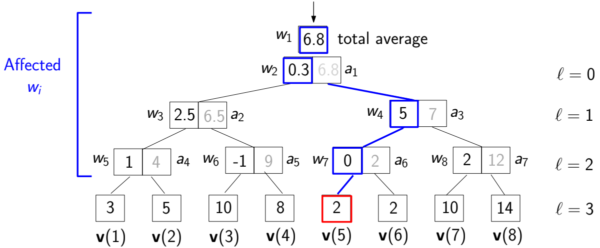

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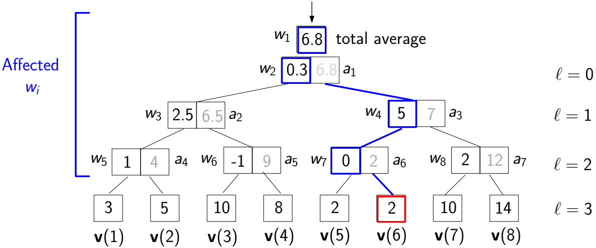

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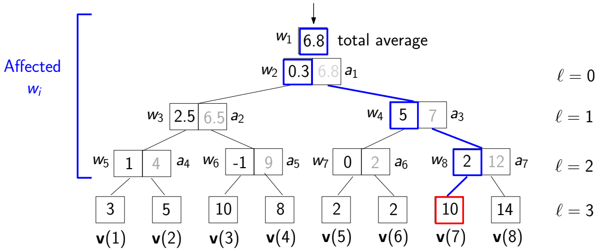

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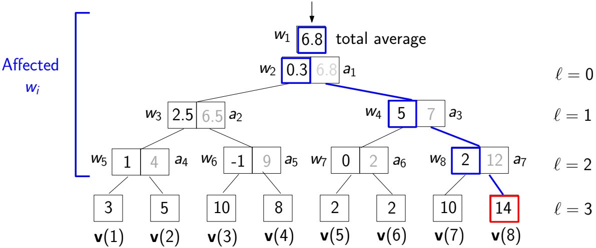

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Introduction: Histograms

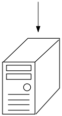
- We may also compute w_i with the wavelet basis vectors ψ_i.
  - $w_i = \mathbf{v} \cdot \psi_i$ for $i = 1, \ldots, u$

Outline

Introduction: MapReduce and Hadoop

R

Record ID	User ID	Object ID	...
1	1	12872	...
2	8	19832	...
3	4	231	...
⋮	⋮	⋮	⋮



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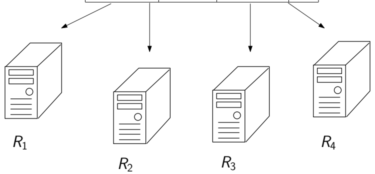

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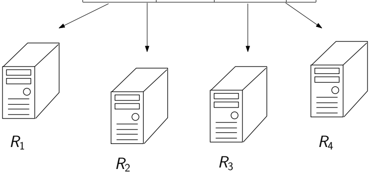

$$R = R_1 \cup R_2 \cup R_3 \cup R_4$$

- Traditionally data is stored in a centralized setting.
- Now stored data has sky rocketed, and is increasingly distributed.
- We study computing the top-k coefficients efficiently on distributed data in MapReduce using Hadoop to illustrate our ideas.

Background: Hadoop Distributed File System (HDFS)

- Hadoop requires a Distributed File System (DFS), we utilize the Hadoop Distributed File System (HDFS).

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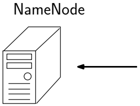

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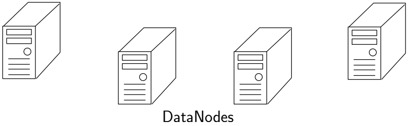

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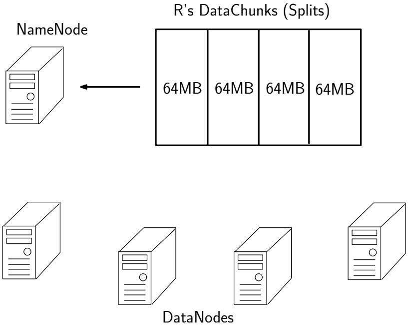

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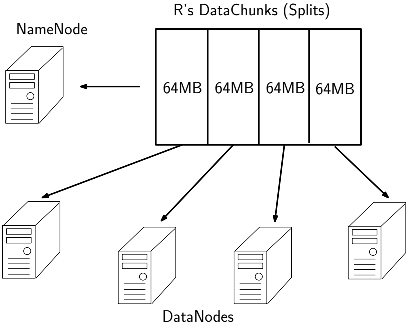

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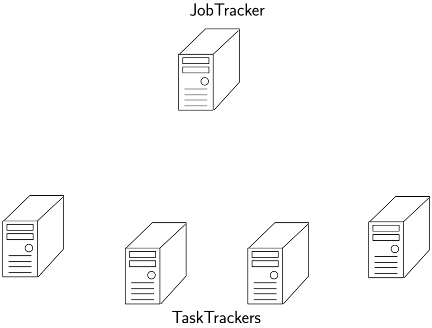

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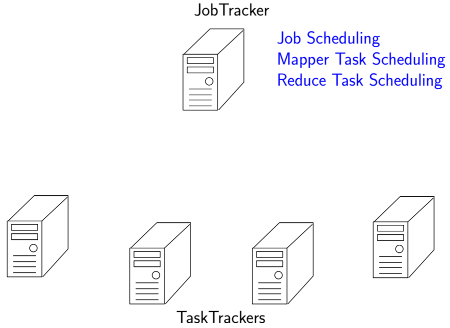

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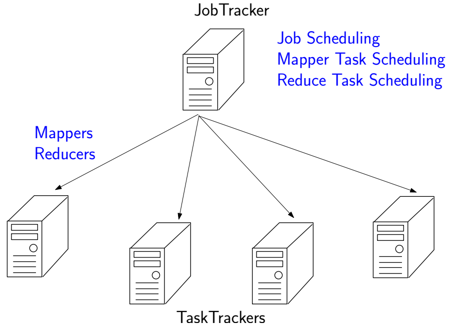

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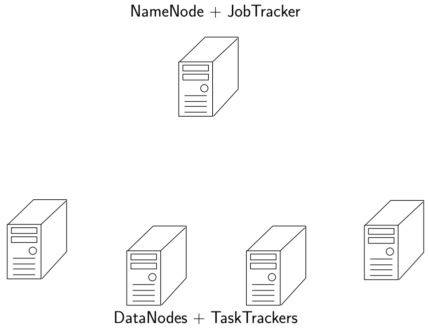

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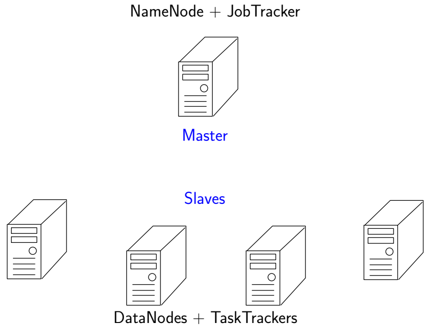
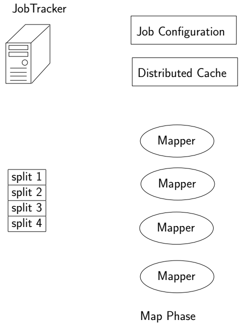
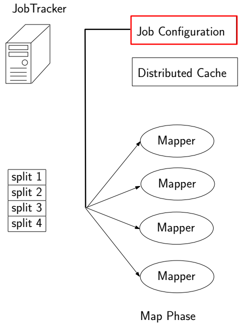


Background: MapReduce Job Overview



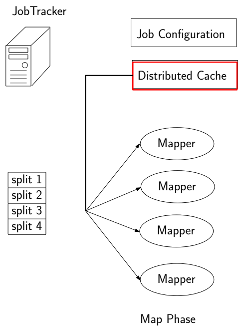
- Next we look at an overview of a typical MapReduce Job.

Background: MapReduce Job Overview



- Job specific variables are first placed in the *Job Configuration* which is sent to each *Mapper Task* by the *JobTracker*.

Background: MapReduce Job Overview



- Large data such as files or libraries are then put in the *Distributed Cache* which is copied to each *TaskTracker* by the *JobTracker*.

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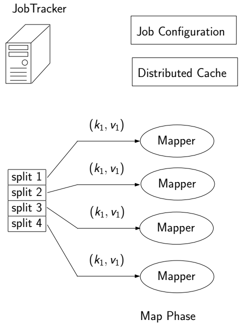

- The JobTracker next assigns each *InputSplit* to a *Mapper* task on a TaskTracker, we assume m Mappers and m InputSplits.

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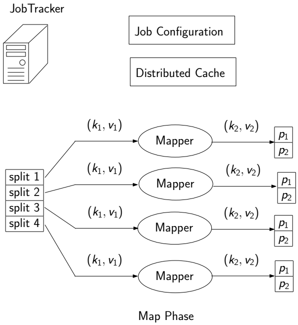

- Each Mapper maps a (k_1, v_1) pair to an intermediate (k_2, v_2) pair and partitions by k_2, i.e. $hash(k_2) = p_i$ for $i \in [1, r]$, $r = |reducers|$.

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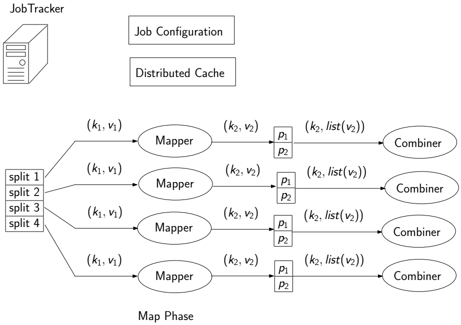

- An optional *Combiner* is executed over $(k_2, list(v_2))$.

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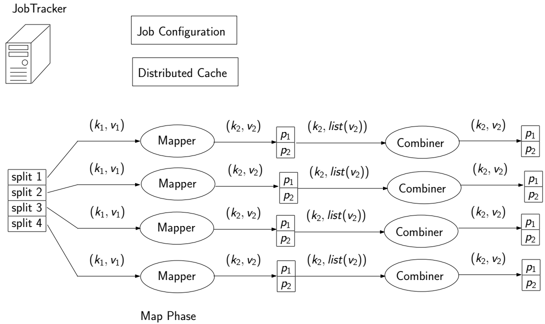
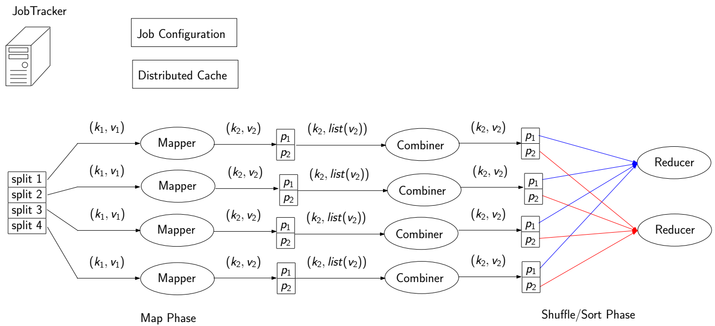


- The *Combiner* aggregates v_2 for a k_2 and a (k_2, v_2) is written to a partition on disk.

Background: MapReduce Job Overview



- The JobTracker assigns two TaskTrackers to run the Reducers, each Reducer copies and sorts it's inputs from corresponding partitions.

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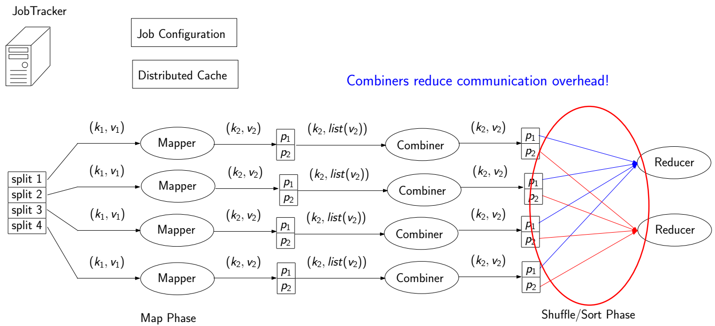

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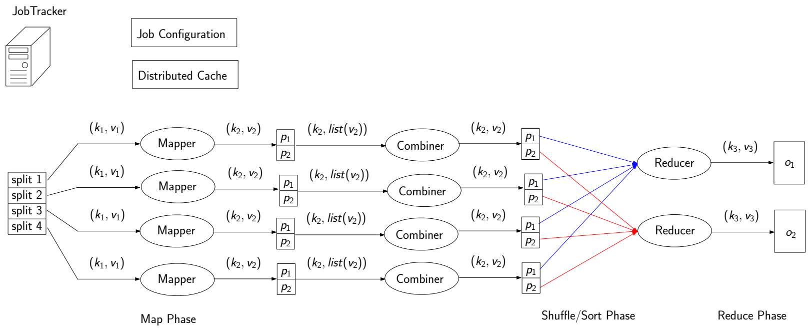
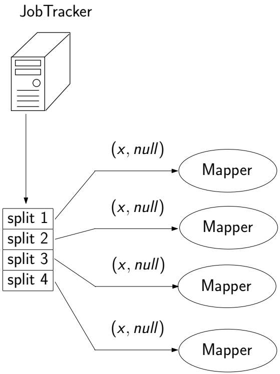


- Each Reducer reduces a $(k_2, list(v_2))$ to a single (k_3, v_3) and writes the results to a DFS file, o_i for $i \in [1, r]$.

Outline

Exact Top-k Wavelet Coefficients: Naive Solution



- Each of the m Mappers reads the input key x from its input split.

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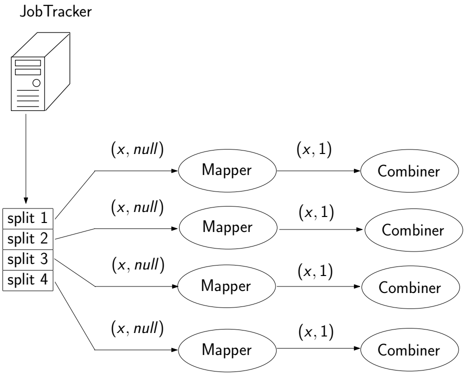

- Each Mapper emits $(x, 1)$ for combining by the Combiner.

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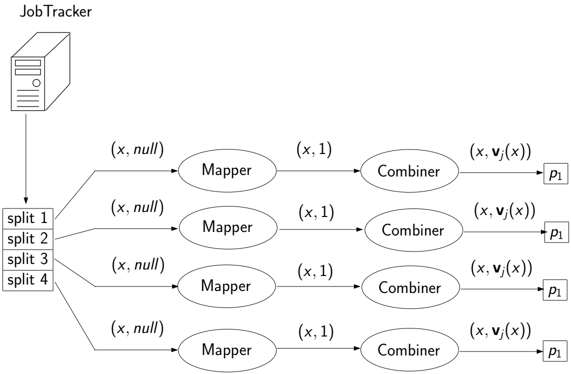

- Each Combiner emits $(x, v_j(x))$, where $v_j(x)$ is the local frequency of x.

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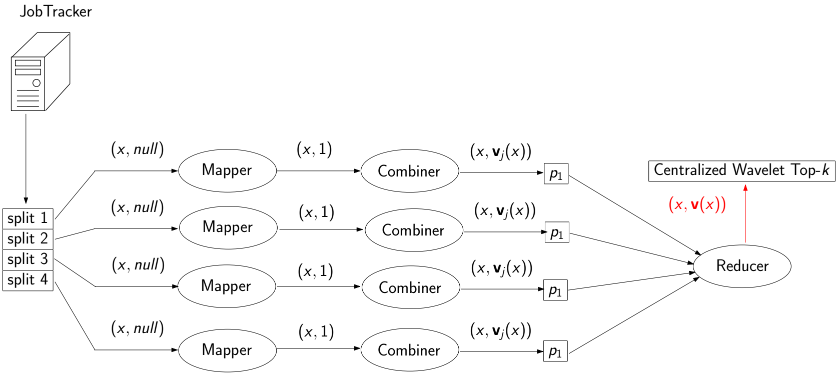

- The Reducer utilizes a Centralized Wavelet Top-k algorithm, supplying the $(x, v(x))$ in a streaming fashion.

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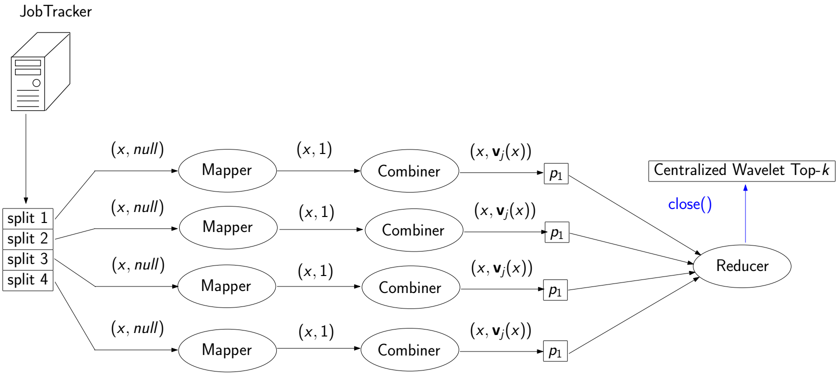

- At the end of the Reduce phase, the Reducer's *close()* method is invoked. The Reducer then requests the top-k $|w_i|$.

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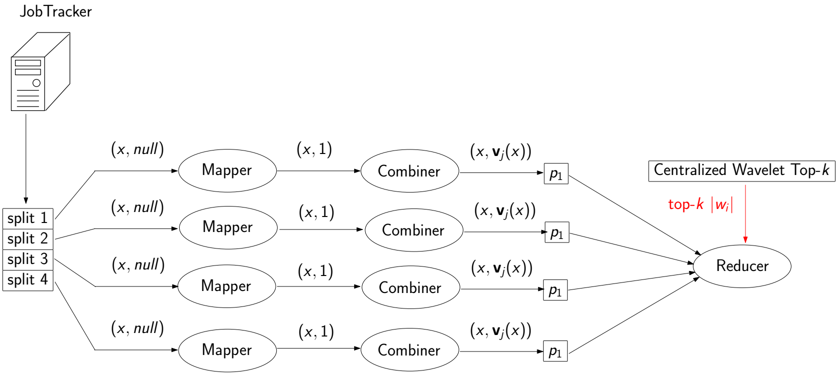

- The centralized algorithm computes the top-k $|w_i|$ and returns these to the Reducer.

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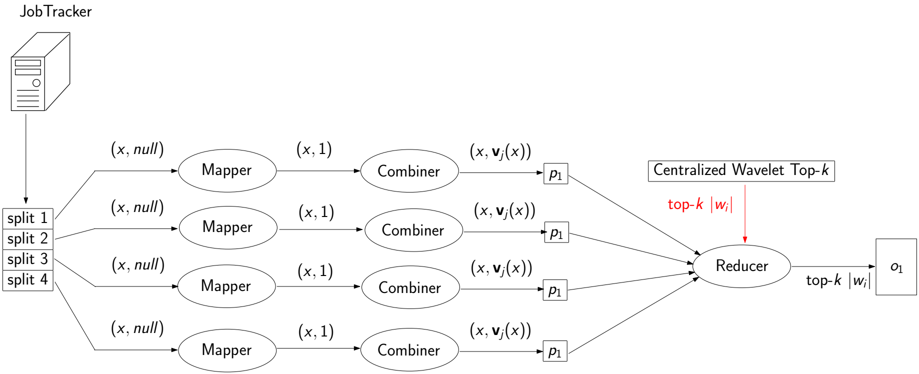

- Finally, the Reducer writes the top-k $|w_i|$ to its HDFS output file o_1.

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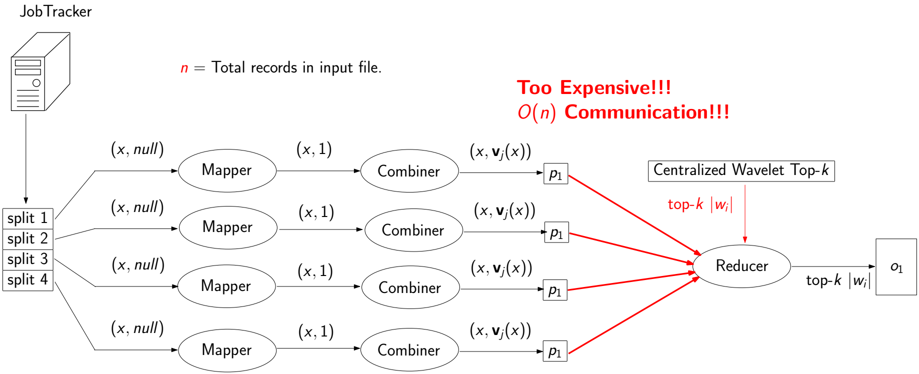

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- We can try to model the problem as a distributed top-k: $w_i = \mathbf{v} \cdot \psi_i = (\sum_{j=1}^{m} \mathbf{v}_j) \cdot \psi_i = \sum_{j=1}^{m} w_{i,j}$.

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$w_{i,j}$ is the local value of w_i in split j.

split 1	split 2	split 3	split 4
$w_{1,1}$	$w_{1,2}$	$w_{1,3}$	$w_{1,4}$
$w_{2,1}$	$w_{2,2}$	$w_{2,3}$	$w_{2,4}$
$w_{3,1}$	$w_{3,2}$	$w_{3,3}$	$w_{3,4}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$w_{u,1}$	$w_{u,2}$	$w_{u,3}$	$w_{u,4}$

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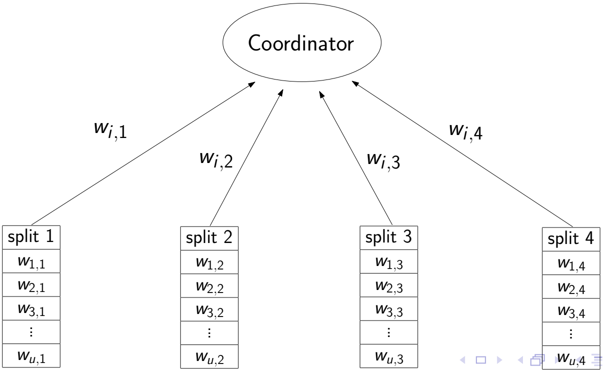

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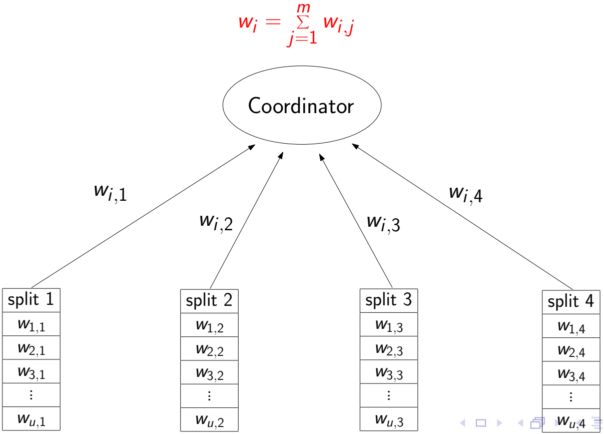

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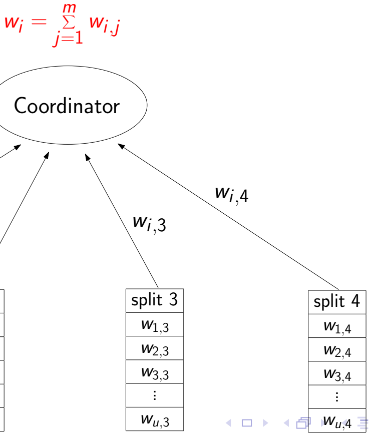

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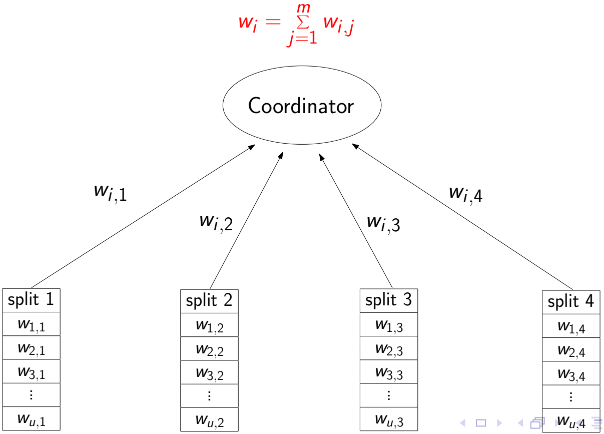
- We have $w_{i,j} < 0$ and $w_{i,j} \geq 0$ and want the largest $|w_i|$.



Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$

- An item x has a local score $s_i(x)$ at node i $\forall i \in [1 \dots m]$, where if x does not appear $s_i(x) = 0$

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$

- Each node sends:
  the top-k most positive scored items
  the top-k most negative scored items.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- The coordinator computes useful bounds for each received item.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- $\hat{s}(x)$ denotes the partial score sum for x

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- $\hat{s}(x)$ denotes the partial score sum for x

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- F_x is a receipt indication bit vector,
  if $s_i(x)$ is received $F_x(i) = 1$,
  else $F_x(i) = 0$.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

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  if $s_i(x)$ is received $F_x(i) = 1$,
  else $F_x(i) = 0$.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- $\tau^+(x)$ is an upper bound on the total score $s(x)$,
  if $s_i(x)$ received then $\tau^+(x) = \tau^+(x) + s_i(x)$
  else $\tau^+(x) = \tau^+(x) + k$'th most positive from node i

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- $\tau^+(x)$ is an upper bound on the total score $s(x)$,
  if $s_i(x)$ received then $\tau^+(x) = \tau^+(x) + s_i(x)$
  else $\tau^+(x) = \tau^+(x) + k$'th most positive from node i

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- $\tau^-(x)$ is a lower bound on the total score sum $s(x)$,
  if $s_i(x)$ received then $\tau^-(x) = \tau^-(x) + s_i(x)$
  else $\tau^-(x) = \tau^-(x) + k$'th most negative score from node i

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- $\tau^-(x)$ is a lower bound on the total score sum $s(x)$,
  if $s_i(x)$ received then $\tau^-(x) = \tau^-(x) + s_i(x)$
  else $\tau^-(x) = \tau^-(x) + k$'th most negative score from node i

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

- $\tau(x)$ is a lower bound on $|s(x)|$ computed as,
  $\tau(x) = 0$ if $sign(\tau^+(x)) \neq sign(\tau^-(x))$
  $\tau(x) = \min(|\tau^+(x)|, |\tau^-(x)|)$ otherwise.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

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  $\tau(x) = 0$ if $sign(\tau^+(x)) \neq sign(\tau^-(x))$
  $\tau(x) = \min(|\tau^+(x)|, |\tau^-(x)|)$ otherwise.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22,\ T_1/m = 22/3$

- We select the item with the kth largest $\tau(x)$.
  $\tau(x)$ is a lower bound T_1 on the top-k $|s(x)|$ for unseen item x.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22,\ T_1/m = 22/3$

- Any unseen item x must have at least:
  one $s_i(x) > T_1/m$ or
  one $s_i(x) < -T_1/m$
  To get into the top-k.

node 1		
id	x	$s_1(x)$
$e_{1,1}$	5	20
$e_{1,2}$	2	7
$e_{1,3}$	1	6
$e_{1,4}$	4	-2
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30

node 2		
id	x	$s_2(x)$
$e_{2,1}$	5	12
$e_{2,2}$	4	7
$e_{2,3}$	1	2
$e_{2,4}$	2	-5
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20

node 3		
id	x	$s_3(x)$
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,3}$	4	5
$e_{3,4}$	2	-3
$e_{3,5}$	5	-6
$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22,\ T_1/m = 22/3$

Round 1 End

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22, \; T_1/m = 22/3$

- T_1/m sent to each site.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22, \; T_1/m = 22/3$

- Each site finds items with
  $s_i(x) > T_1/m$ or
  $s_i(x) < T_1/m$.

node 1			
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

node 2			
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

node 3			
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22, \; T_1/m = 22/3$

- Items with $|s_i(x)| > T_1/m$ are sent to coordinator.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R					
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22,\ T_1/m = 22/3$

- Items with $|s_i(x)| > T_1/m$ are sent to coordinator.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

→

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

node 1			
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

node 2			
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

node 3			
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22, \ T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

- Partial score sum $s(5) = 20 + 12$

node 1			
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

node 2			
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

node 3			
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22, \; T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

- Receipt vector $F_5 = [110]$

node 1			
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

node 2			
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

node 3			
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22$, $T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

- $\tau^+(x)$ is now tighter,
  if $s_i(x)$ received then $\tau^+(x) = \tau^+(x) + s_i(x)$
  else $\tau^+(x) = \tau^+(x) + T_1/m$

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22$, $T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

- $\tau^-(x)$ is also tighter,
  if $s_i(x)$ received then $\tau^-(x) = \tau^-(x) + s_i(x)$
  else $\tau^-(x) = \tau^-(x) - T_1/m$

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

- Score absolute value bound $\tau(5) = \min(39.3, 24.6)$.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

- $\tau'(x)$ is an upper bound on $|s(x)|$, $\tau'(x) = \max\{|\tau^+(x)|, |\tau^-(x)|\}$

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22, \ T_1/m = 22/3$

- The coordinator updates the bounds for each item it has ever received.

- $\tau'(x)$ is an upper bound on $|s(x)|$, $\tau'(x) = \max\{|\tau^+(x)|, |\tau^-(x)|\}$

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22, \ T_1/m = 22/3$

- We select the item x with the kth largest $\tau(x)$, which serves as a new lower bound T_2 on $|s(x)|$ for any item.

node 1			
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

node 2			
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

node 3			
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

- We select the item x with the kth largest $\tau(x)$, which serves as a new lower bound T_2 on $|s(x)|$ for any item.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

- Any item with $\tau'(x) < T_2$ cannot be in the top-k and is pruned from R.

node 1			
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

node 2			
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

node 3			
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

- Any remaining items with a 0 in vector F_x are selected.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

Round 2 End

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
√	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

$s_3(3) = ?$

- The coordinator asks for missing scores for items still in R.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
√	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

$s_3(3) = ?$

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
√	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,6}$	6	-10

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

$T_1 = 22, \; T_1/m = 22/3$

$T_2 = 45$

$s_3(3) = ?$

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
√	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,6}$	6	-10

→

R						
x	$\hat{s}(x)$	F_x	$\tau^+(x)$	$\tau^-(x)$	$\tau(x)$	$\tau'(x)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

$s_3(3) = ?$

node 1			
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

node 2			
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
√	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

node 3			
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,6}$	6	-10

R		
x	$\hat{s}(x)$	F_i
~~1~~	~~10~~	~~001~~
3	-38	111
~~5~~	~~32~~	~~110~~
6	-45	111

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

$s_3(3) = ?$

- After collecting all scores, the coordinator can determine top-k $|s(x)|$.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
√	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	**3**	**6**
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,6}$	6	-10

R		
x	$\hat{s}(x)$	F_i
~~1~~	~~10~~	~~001~~
3	-38	111
~~5~~	~~32~~	~~110~~
6	-45	111

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

$s_3(3) = ?$

- After collecting all scores, the coordinator can determine top-k $|s(x)|$.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
	$e_{2,4}$	2	-5
√	$e_{2,5}$	3	-14
√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
√	$e_{3,1}$	1	10
	$e_{3,2}$	3	6
	$e_{3,3}$	4	5
	$e_{3,4}$	2	-3
	$e_{3,5}$	5	-6
√	$e_{3,6}$	6	-10

Exact Top-k Wavelet Coefficients: Our Solution

$k = 1$

R		
id	x	$s_j(x)$
$e_{1,1}$	5	20
$e_{1,5}$	6	-15
$e_{1,6}$	3	-30
$e_{2,1}$	5	12
$e_{2,5}$	3	-14
$e_{2,6}$	6	-20
$e_{3,1}$	1	10
$e_{3,2}$	3	6
$e_{3,6}$	6	-10

R		
x	$\hat{s}(x)$	F_i
~~1~~	~~10~~	~~001~~
3	-38	111
~~5~~	~~32~~	~~110~~
6	-45	111

$T_1 = 22,\ T_1/m = 22/3$

$T_2 = 45$

$\star$ $\mathbf{s(6) = -45}$

$s_3(3) = ?$

- After collecting all scores, the coordinator can determine top-k $|s(x)|$.

	node 1		
	id	x	$s_1(x)$
√	$e_{1,1}$	5	20
	$e_{1,2}$	2	7
	$e_{1,3}$	1	6
	$e_{1,4}$	4	-2
√	$e_{1,5}$	6	-15
√	$e_{1,6}$	3	-30

	node 2		
	id	x	$s_2(x)$
√	$e_{2,1}$	5	12
	$e_{2,2}$	4	7
	$e_{2,3}$	1	2
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√	$e_{2,6}$	6	-20

	node 3		
	id	x	$s_3(x)$
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Round 3 End

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    3. amount of I/O incurred

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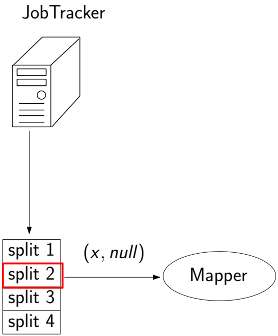
Outline

Approximate Top-k Wavelet Coefficients: Sketch



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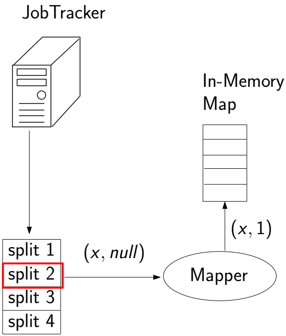

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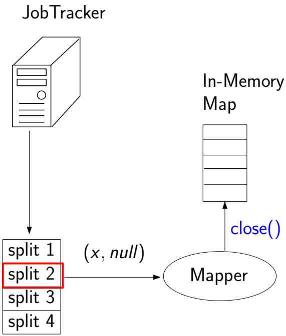

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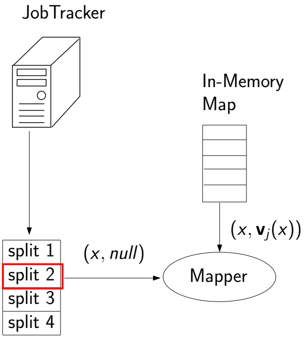

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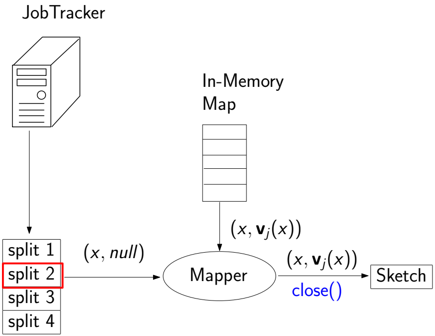

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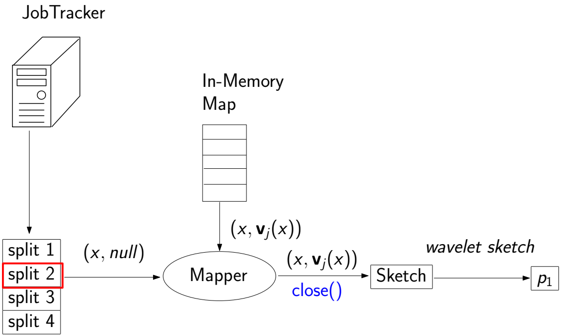

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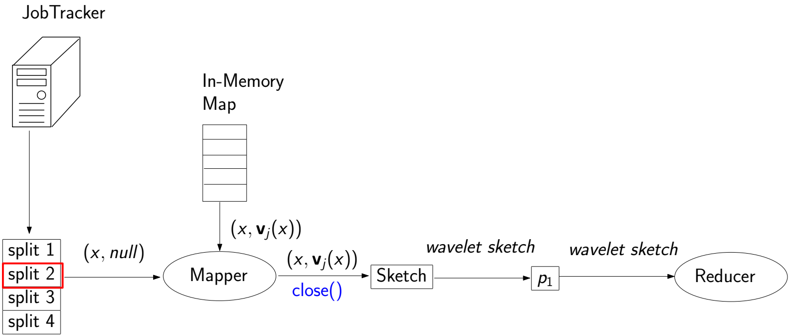

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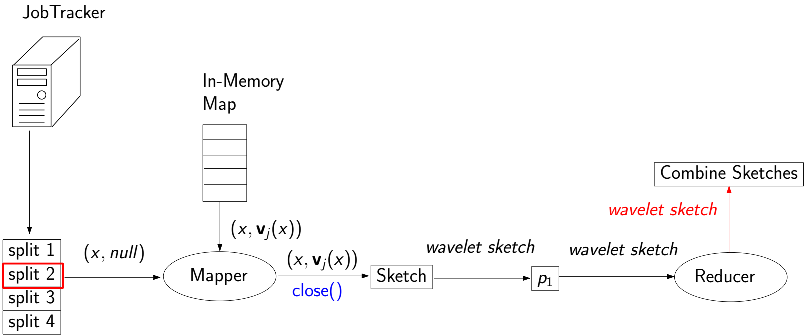

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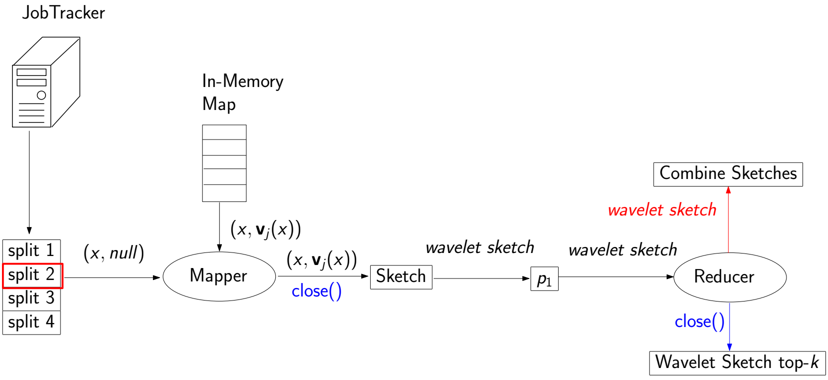

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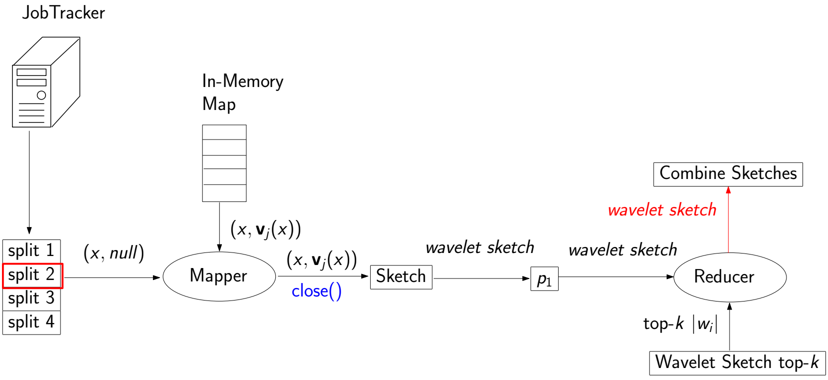

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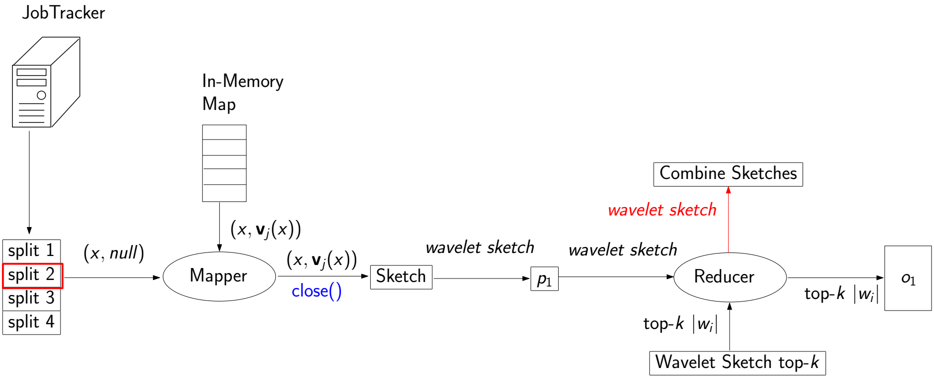
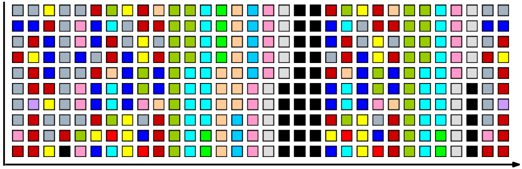


Outline

Approximate Top-k Wavelet Coefficients: Basic Random Sampling



n_j Records in split j

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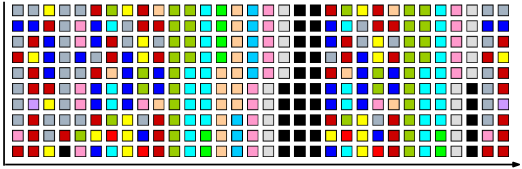

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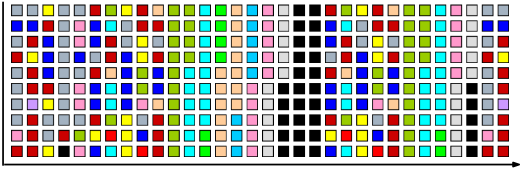

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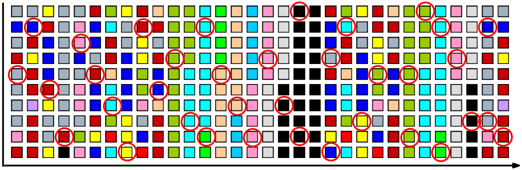

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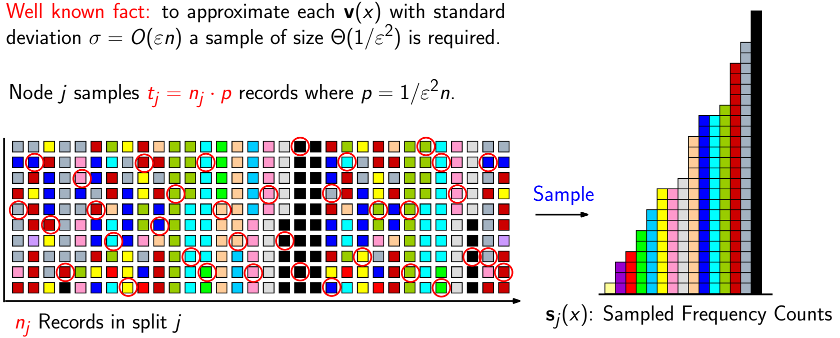

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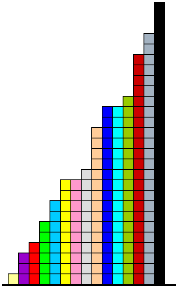

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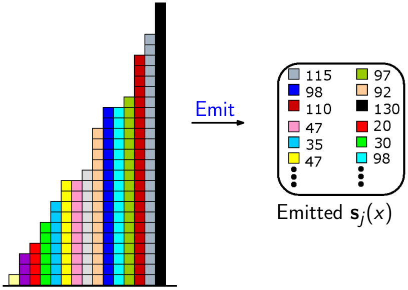

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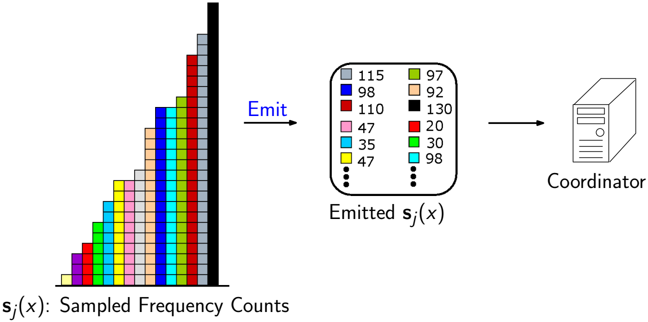

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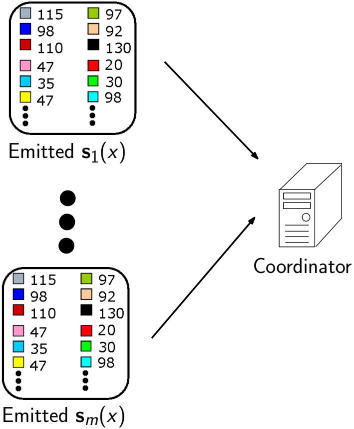

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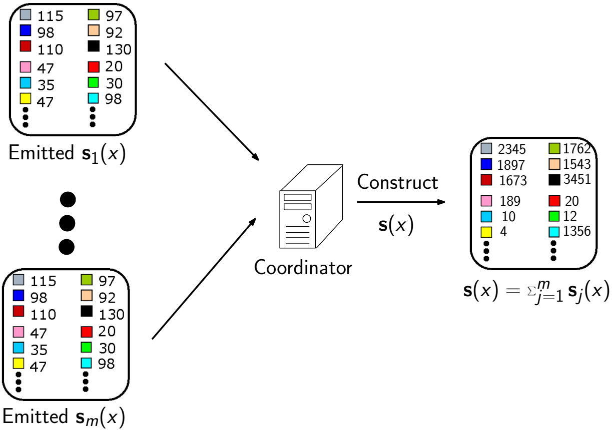

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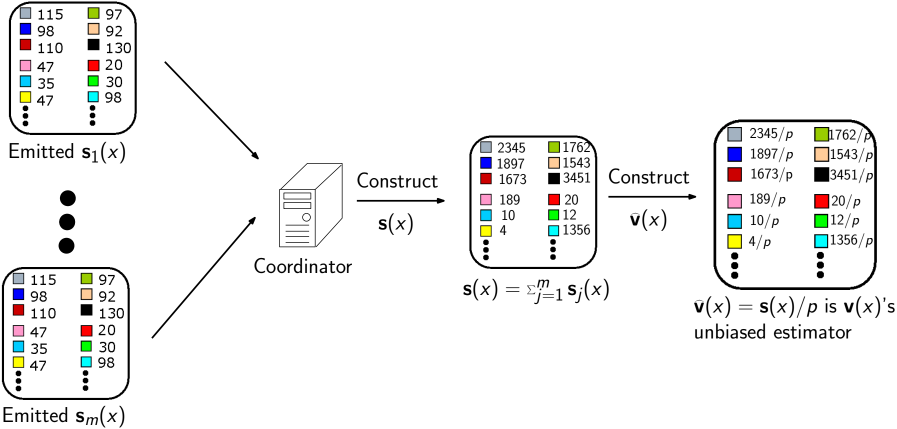

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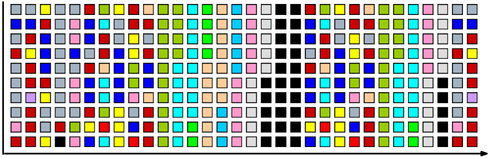
  - Key idea: ignore sampled keys with small frequencies in a split.

Outline

Approximate Top-k Wavelet Coefficients: Improved Sampling



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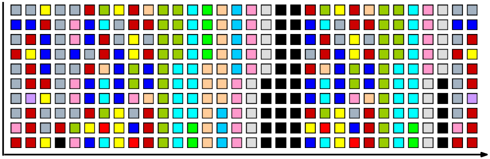

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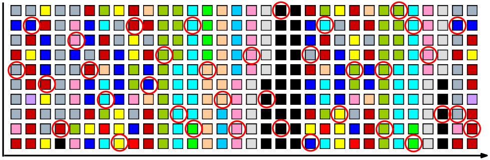

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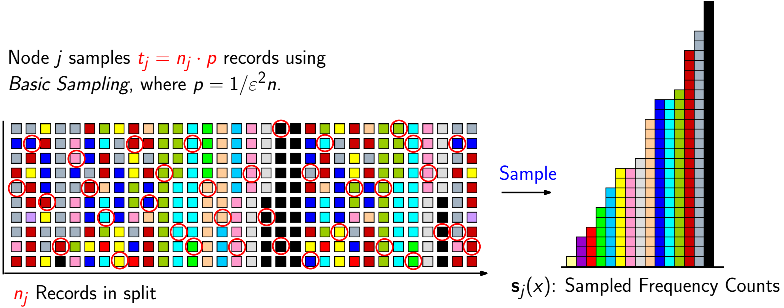

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Node j sends $(x, \mathbf{s}_j(x))$ only if $\mathbf{s}_j(x) > \varepsilon t_j$.

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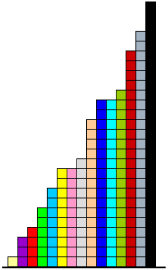

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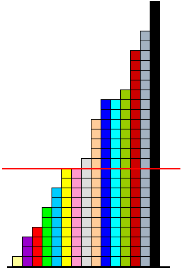

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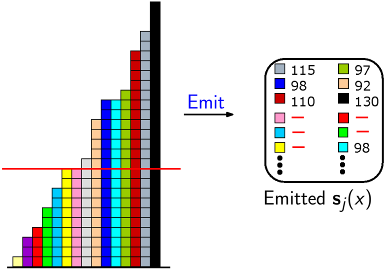

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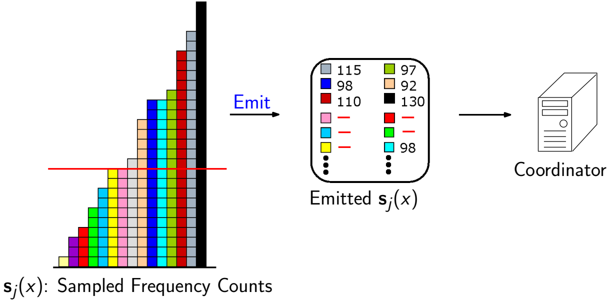

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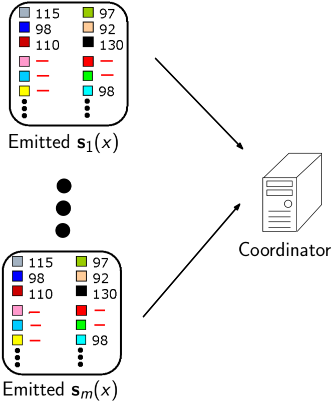

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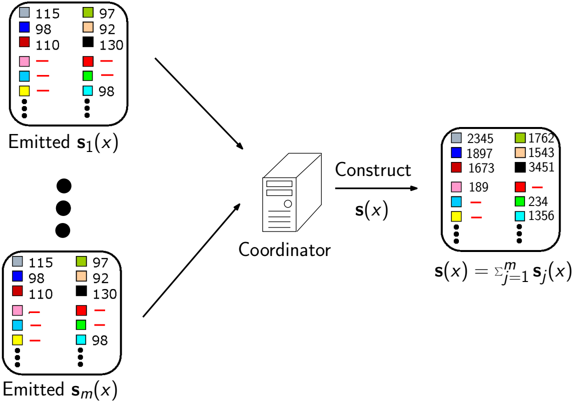

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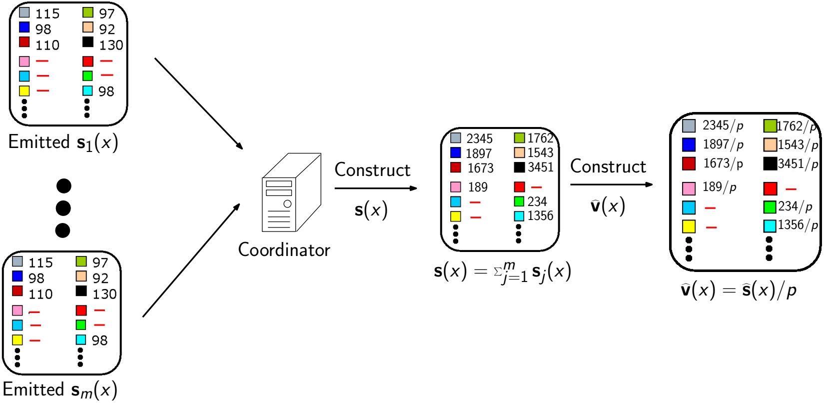

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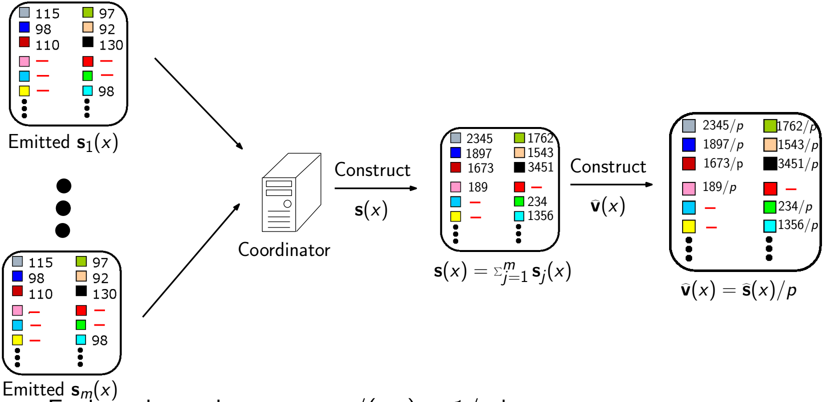

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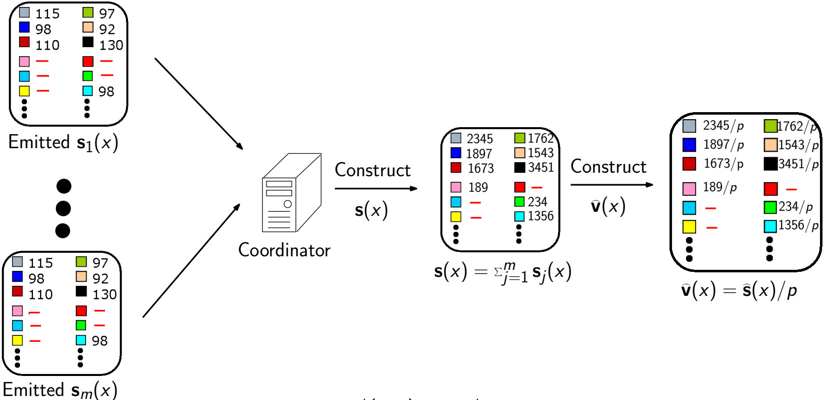

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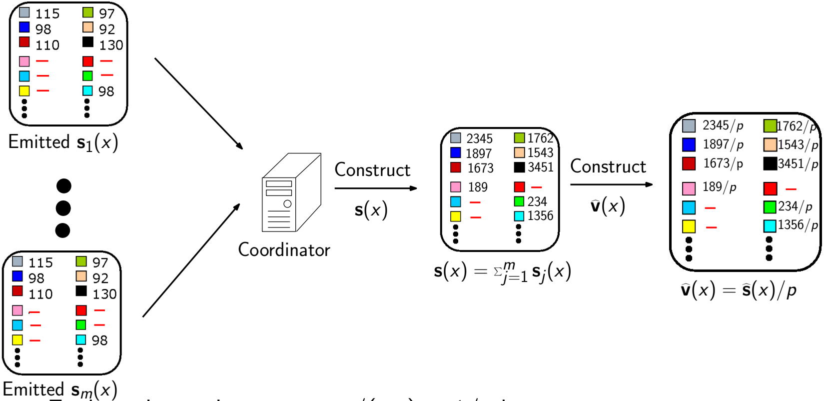
    - The total communication is $O(m/\varepsilon)$.

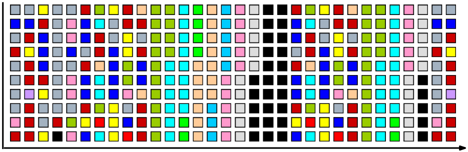
Approximate Top-k Wavelet Coefficients: Improved Sampling



- Each node sends at most $t_j/(\varepsilon t_j) = 1/\varepsilon$ keys.
  - The total communication is $O(m/\varepsilon)$.
- $\mathbf{E}[\widehat{\mathbf{v}}(x)]$ may be εn away from $\mathbf{v}(x)$ as $s_j(x) < \varepsilon t_j$ are ignored.

Outline

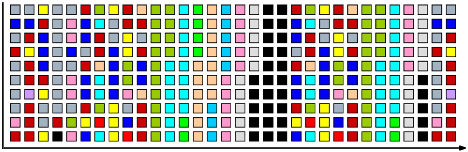
Approximate Top-k Wavelet Coefficients: Two-Level Sampling



n_j Records in split

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

Node j samples $t_j = n_j \cdot p$ records using *Basic Sampling*, where $p = 1/\varepsilon^2 n$.



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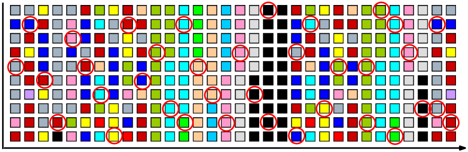

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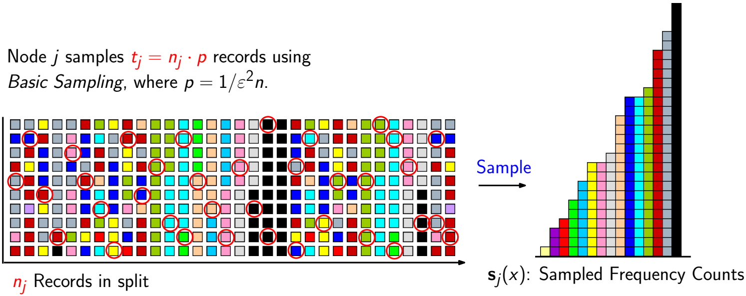

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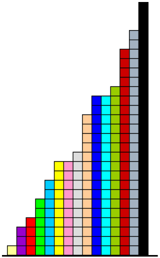

$\mathbf{s}_j(x)$: Sampled Frequency Counts

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

Sample record x with probability $\min\{\varepsilon\sqrt{m} \cdot \mathbf{s}_j(x), 1\}$.

- If $\mathbf{s}_j(x) \geq 1/(\varepsilon\sqrt{m})$, emit $(x, \mathbf{s}_j(x))$.
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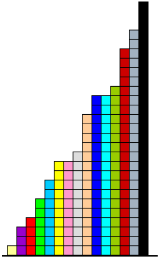

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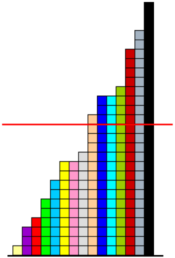

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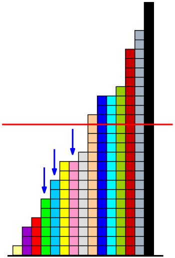

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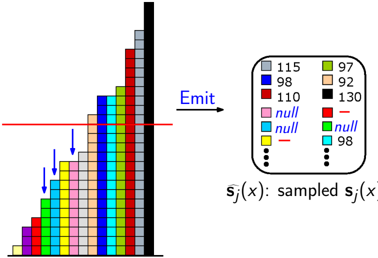

$\widehat{\mathbf{s}_j}(x)$: sampled $\mathbf{s}_j(x)$

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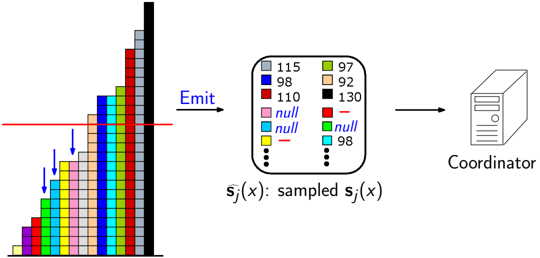

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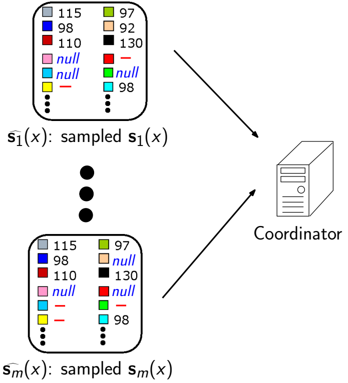

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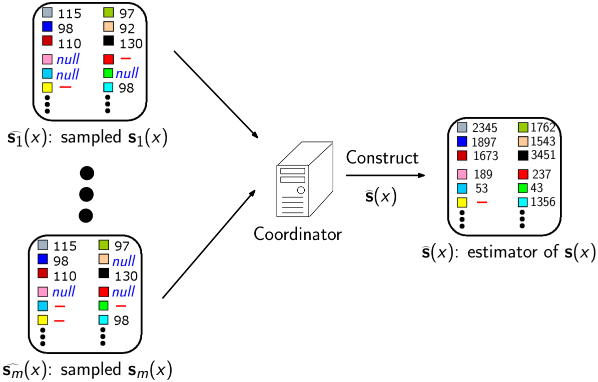

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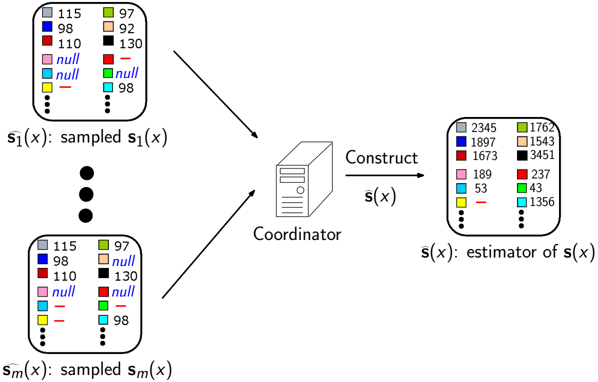

- To construct $\widehat{\mathbf{s}}(x)$.
  - If $(x, \mathbf{s}_j(x))$ received, $\rho(x) = \rho(x) + \mathbf{s}_j(x)$.

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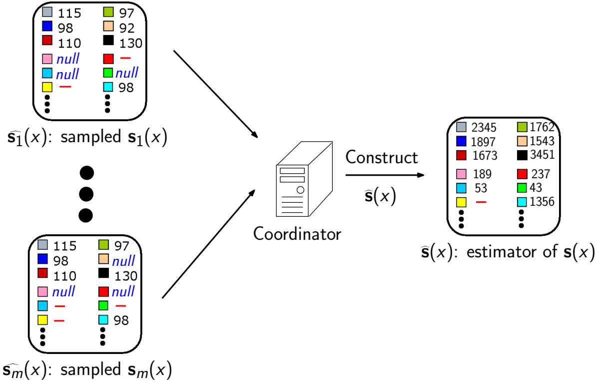

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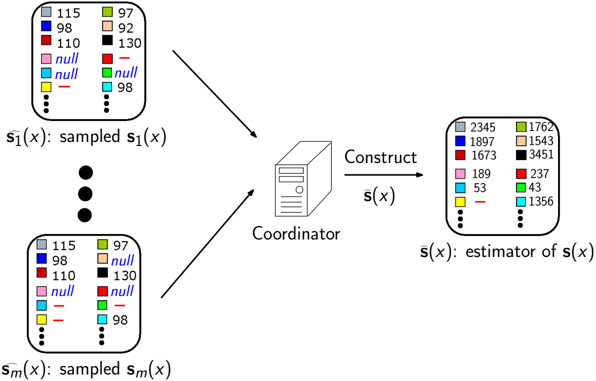

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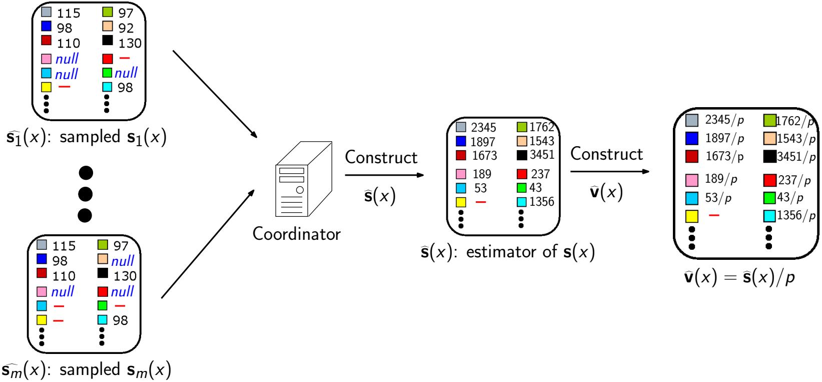

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- Finally, $\widehat{\mathbf{s}}(x) = \rho(x) + M(x)/\varepsilon\sqrt{m}$.
- Then, $\widehat{\mathbf{v}}(x) = \widehat{\mathbf{s}}(x)/p$ is an unbiased estimator for $\mathbf{v}(x)$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

Theorem

$\widehat{\mathbf{s}}(x)$ *is an unbiased estimator of* $\mathbf{s}(x)$ *with standard deviation at most* $1/\varepsilon$.

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Theorem

- $\widehat{w}_i$ *is an unbiased estimator for any* w_i.
- *Recall* $w_i = \langle \mathbf{v}, \psi_i \rangle$, *for* $\psi_i = (-\phi_{j+1,2k} + \phi_{j+1,2k+1})/\sqrt{u/2^j}$ *where* ϕ *is a* $[0,1]$ *vector defined for* $j = 1, \ldots, \log u$ *and* $k = 0, \ldots, 2^j - 1$. *The variance of* $\widehat{w}_i$ *is bounded by* $\frac{\varepsilon 2^j n}{u\sqrt{m}} \sum_{x=2ku/2^{j+1}+1}^{(2k+2)u/2^{j+1}} \mathbf{s}(x)$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

Theorem

$\widehat{\mathbf{s}}(x)$ is an unbiased estimator of $\mathbf{s}(x)$ with standard deviation at most $1/\varepsilon$.

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$\widehat{\mathbf{v}}(x)$ is an unbiased estimator of $\mathbf{v}(x)$ with standard deviation at most εn.

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- *$\widehat{w}_i$ is an unbiased estimator for any w_i.*
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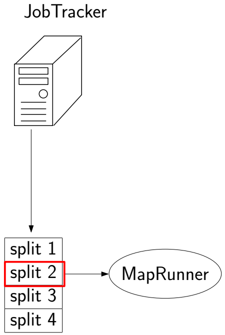
Theorem

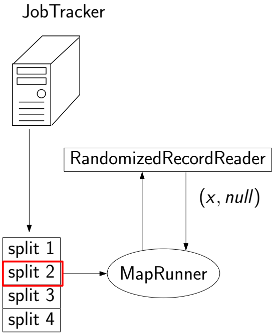
The expected total communication cost of our two-level sampling algorithm is $O(\sqrt{m}/\varepsilon)$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling



n_j= records in split j
s_j= split j sample frequency vector

Approximate Top-k Wavelet Coefficients: Two-Level Sampling



n_j= records in split j
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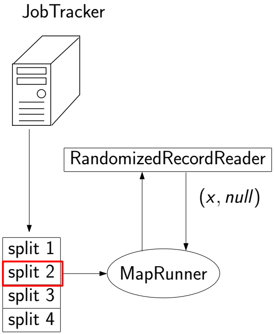
1. RandomizedRecordReader j (RR_j) samples $n_j/\varepsilon^2 n$ records.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling



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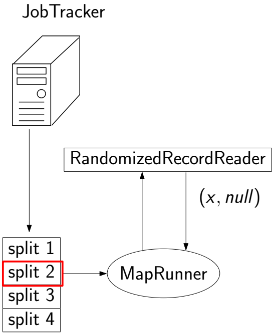

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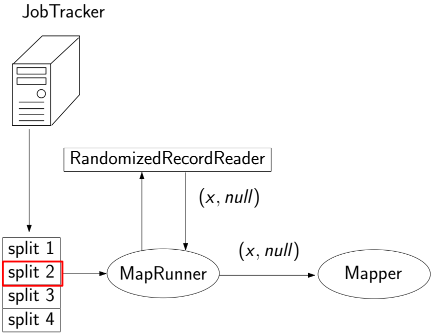

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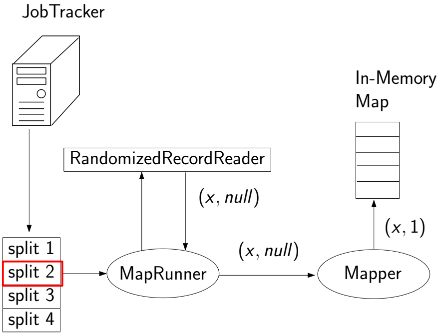

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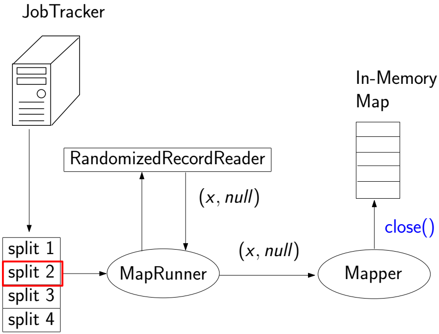

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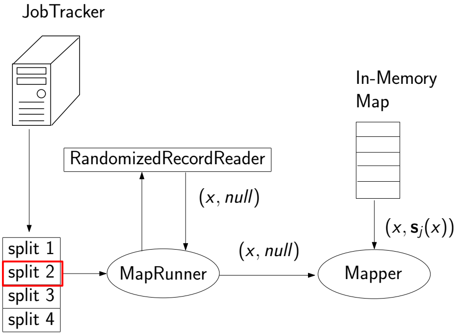

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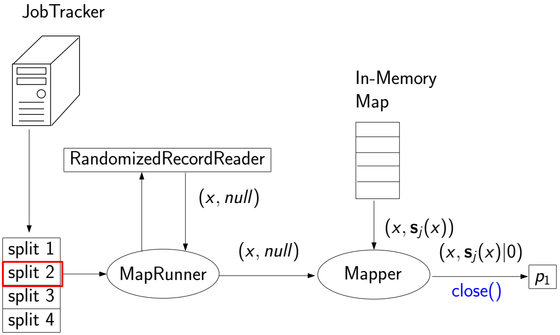

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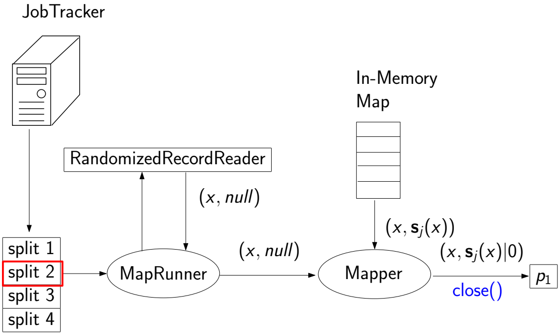

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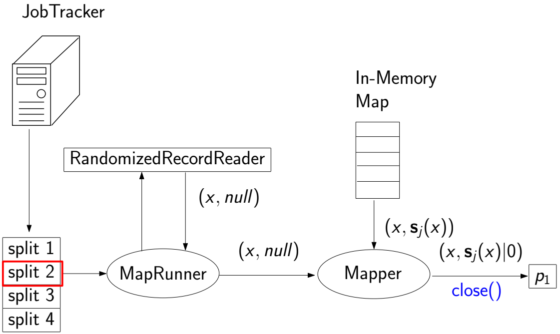

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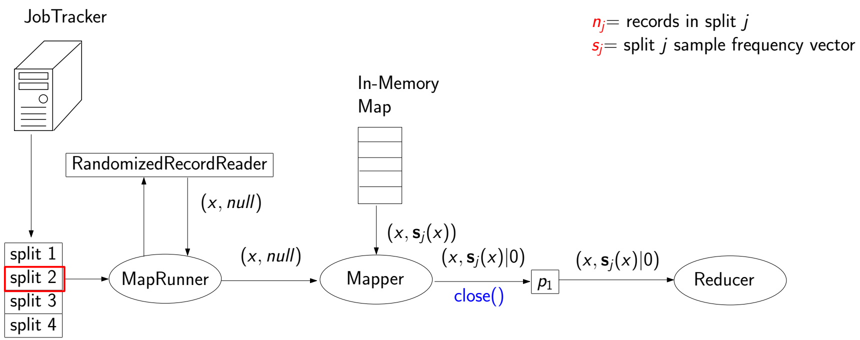

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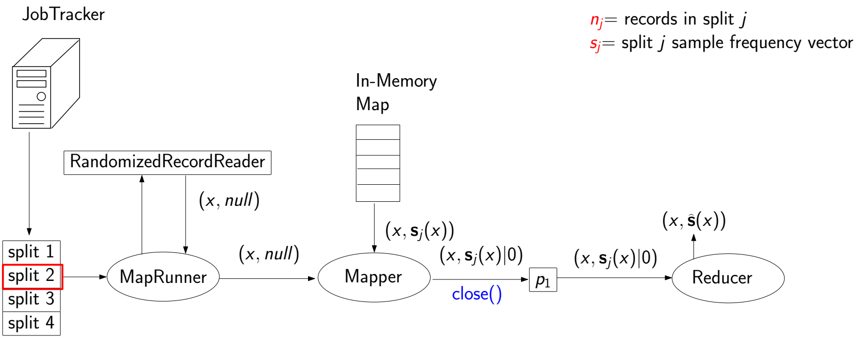
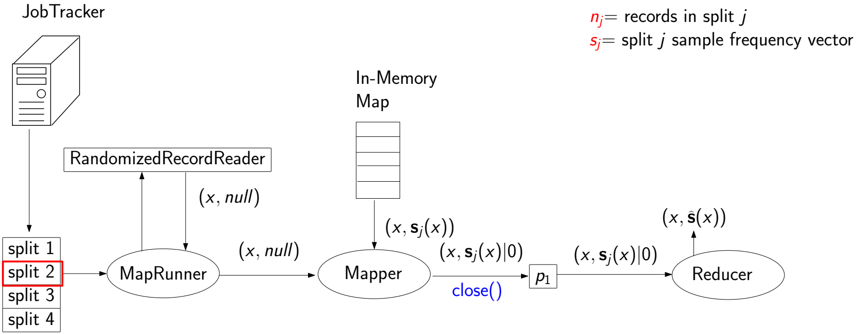


3. Construct $\widehat{\mathbf{s}}(x)$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling



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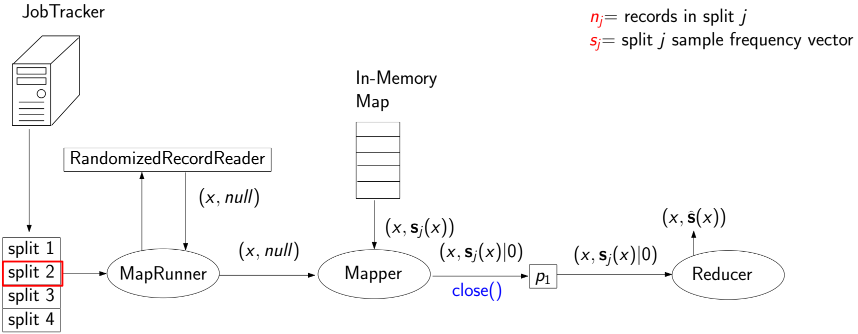

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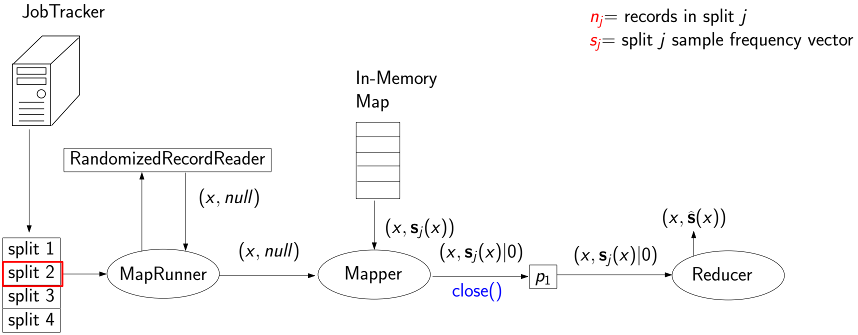

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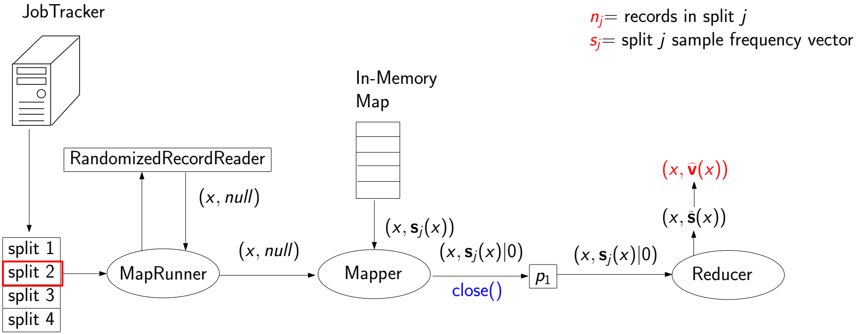

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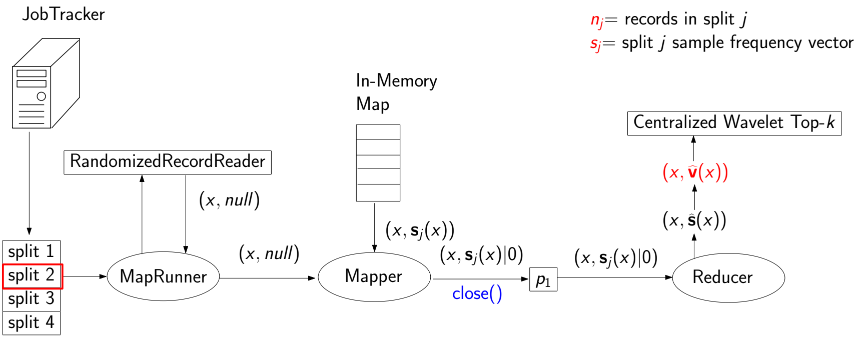

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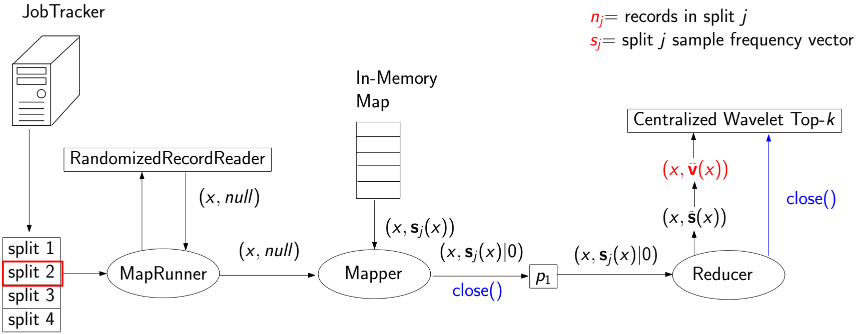

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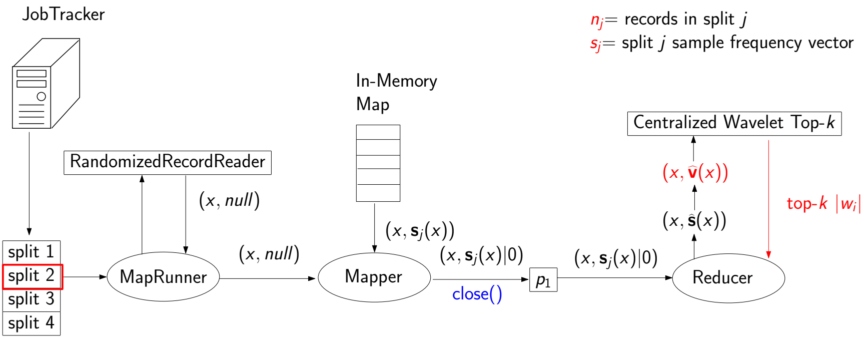

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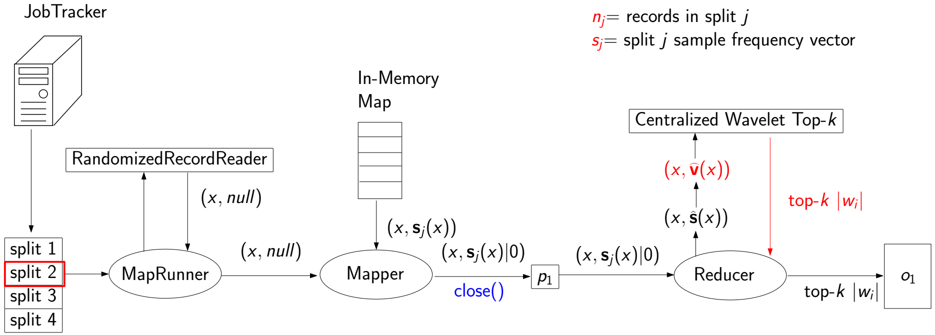

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Outline

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- All machines are directly connected to a 1000Mbps switch.

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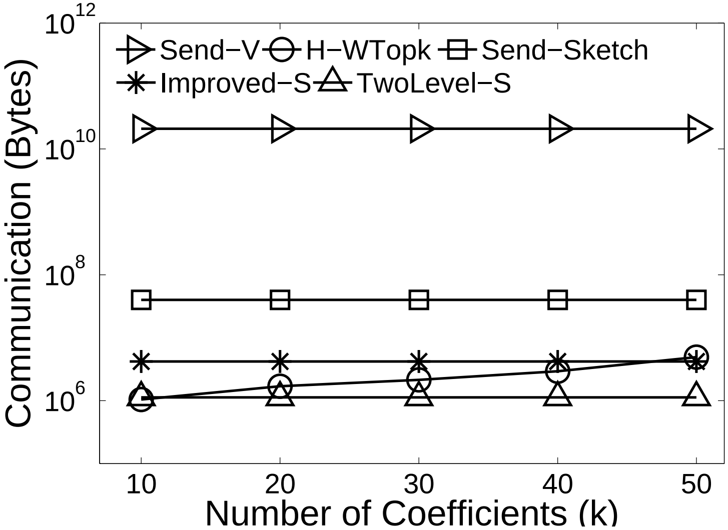
Experiments: Defaults

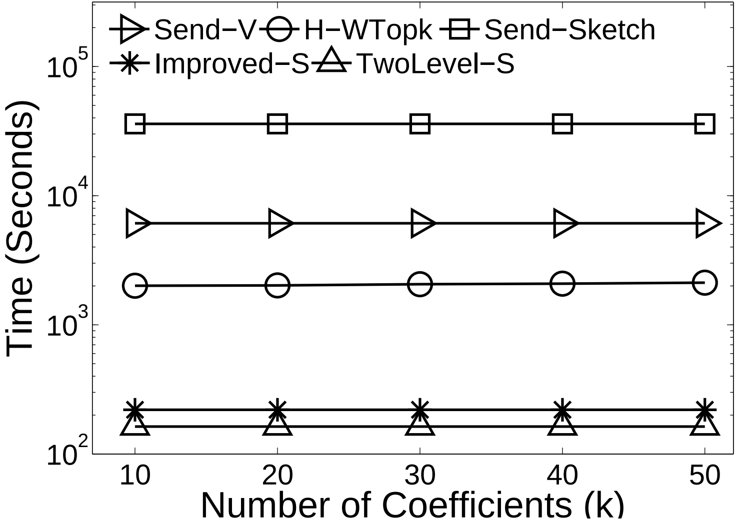
- Default values:

Symbol	Definition	Default
α	Zipfian skewness	1.1
u	max key in domain	$\log_2 u = 29$
n	total records	13.4 billion
	dataset size	50GB
β	split size	256MB
m	number of splits	200
B	network bandwidth	500Mbps

Experiments: Vary k



Experiments: Vary k



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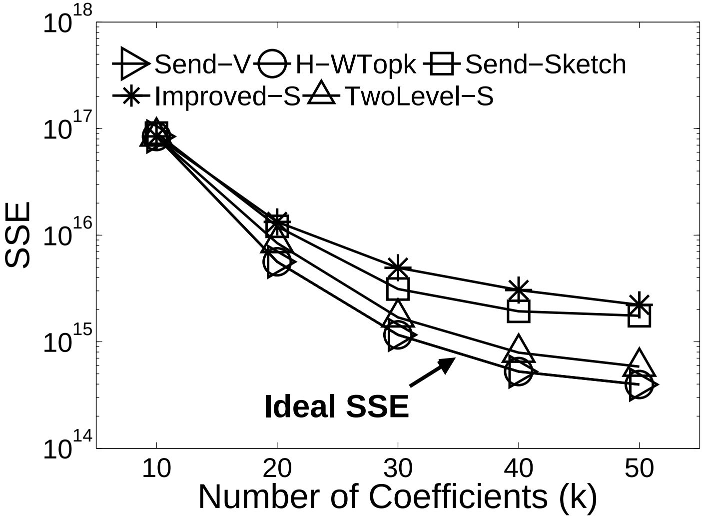
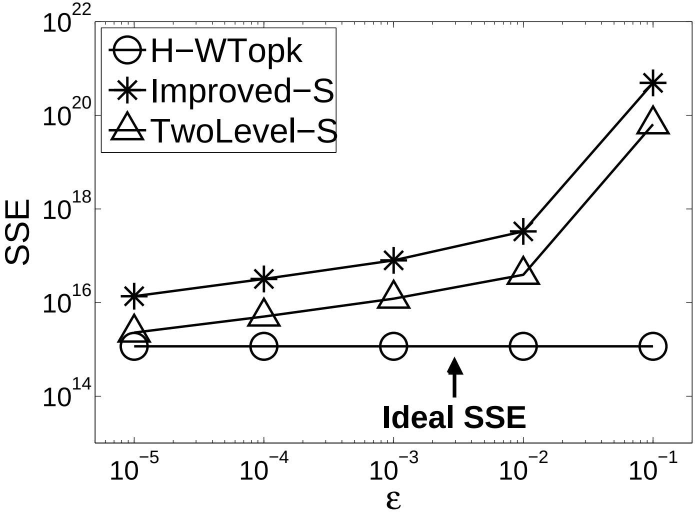


Experiments: Vary ε



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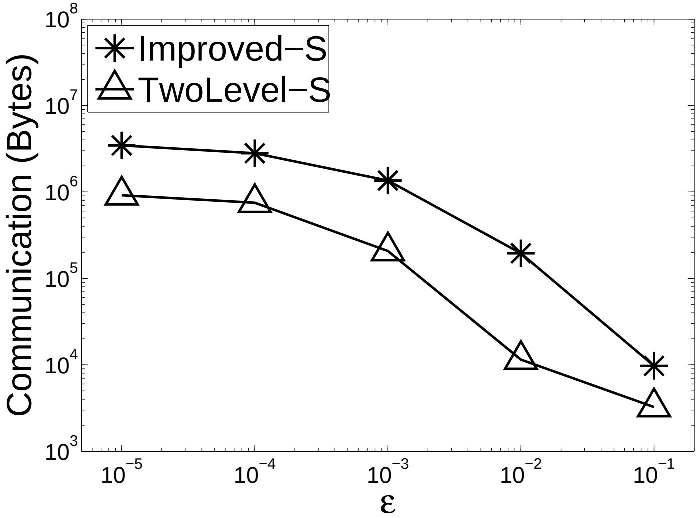

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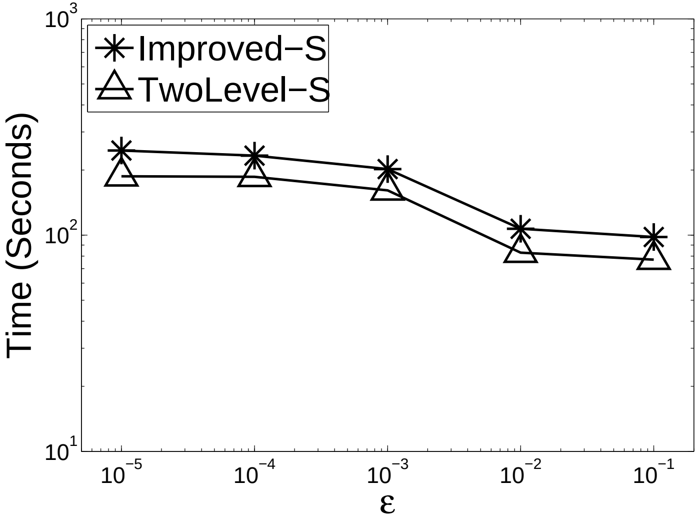
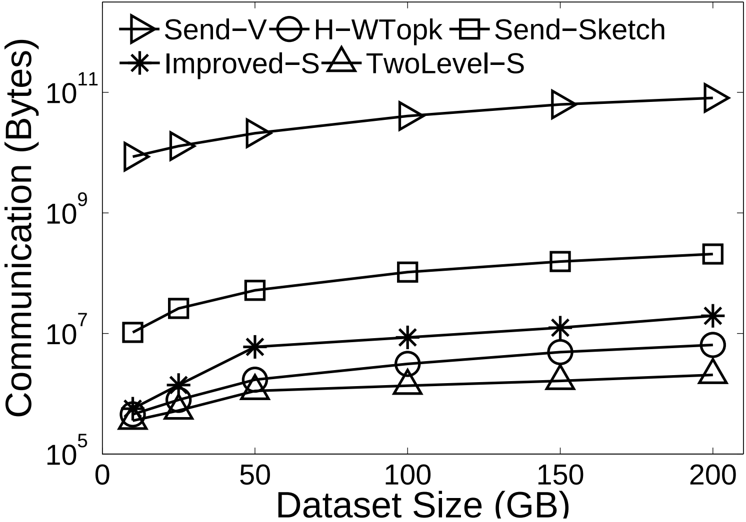


Experiments: Vary n



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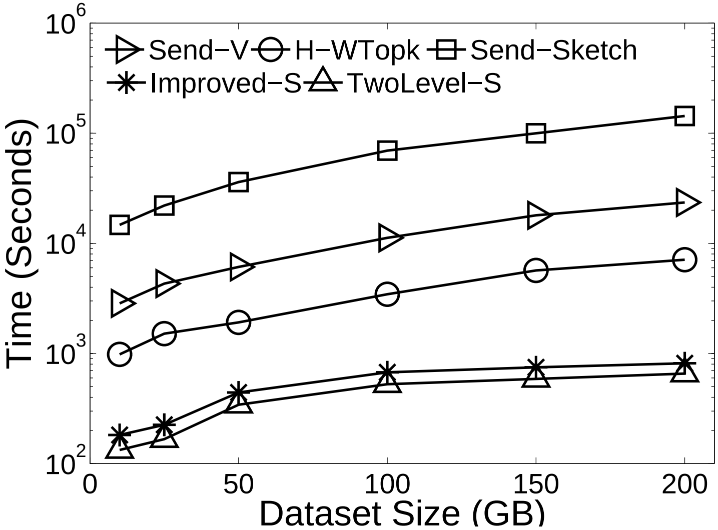
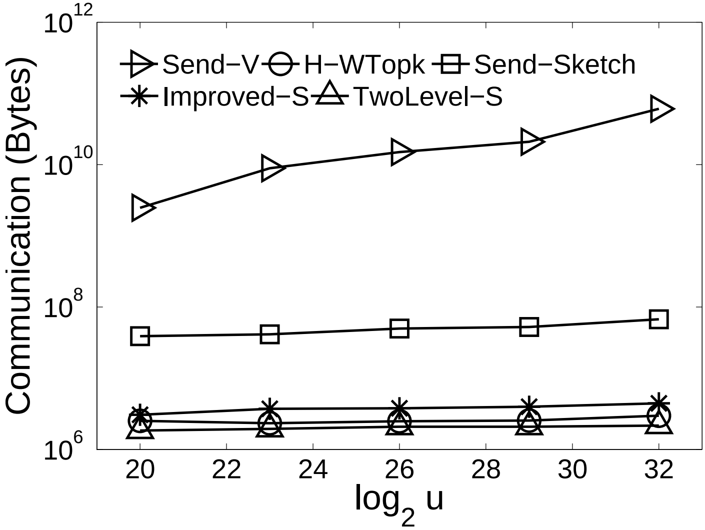


Experiments: Vary u



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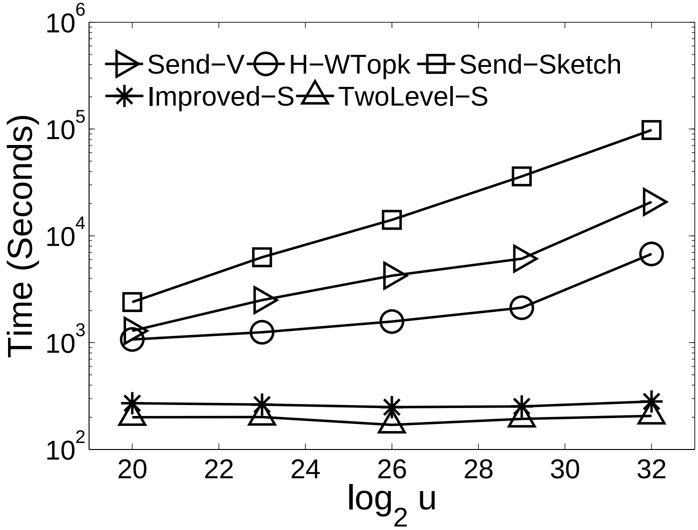
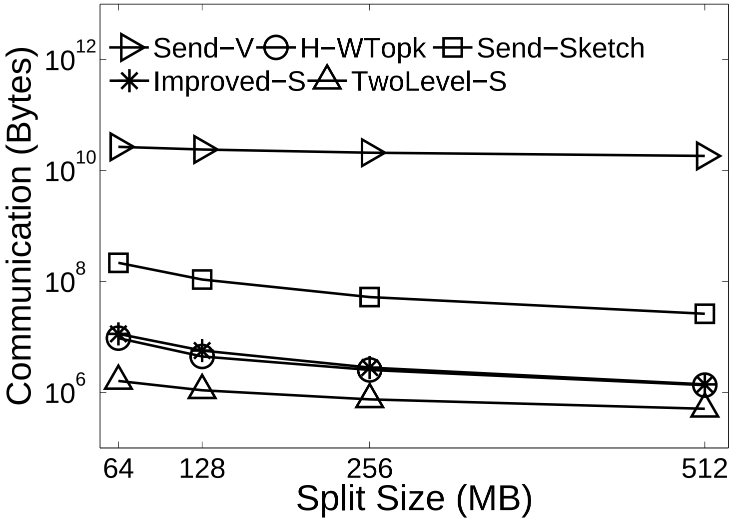


Experiments: Vary β



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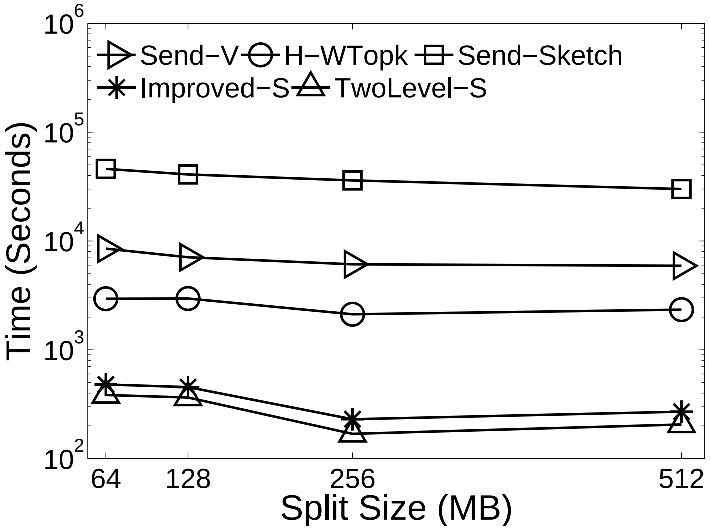
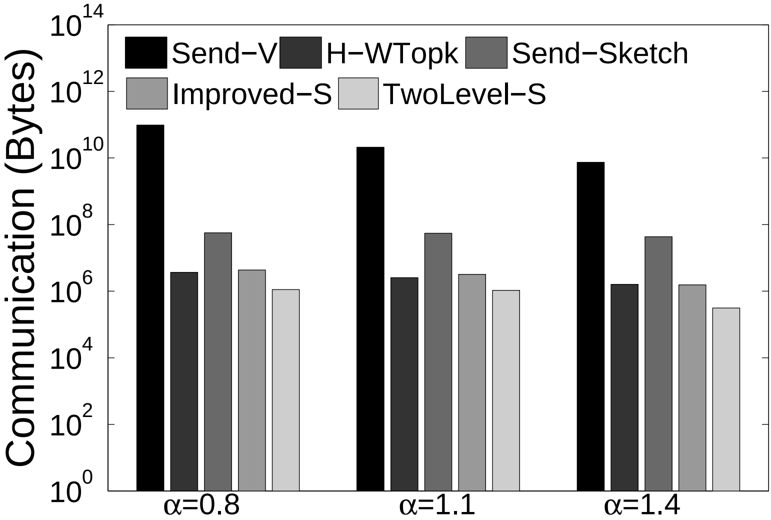


Experiments: Vary α



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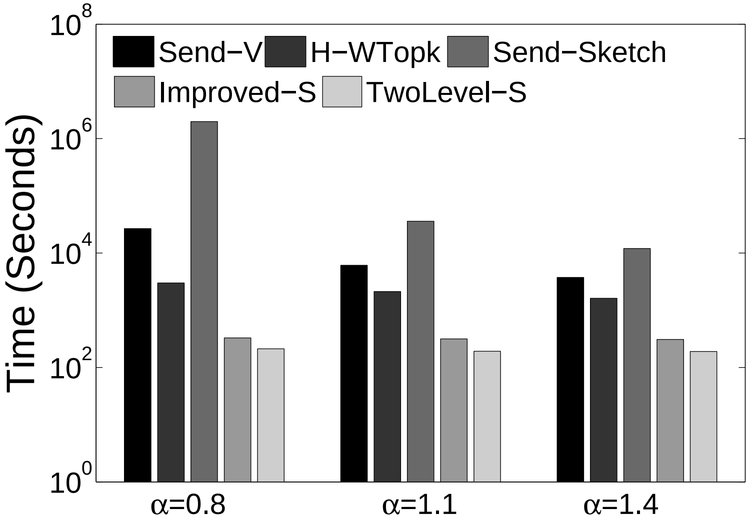
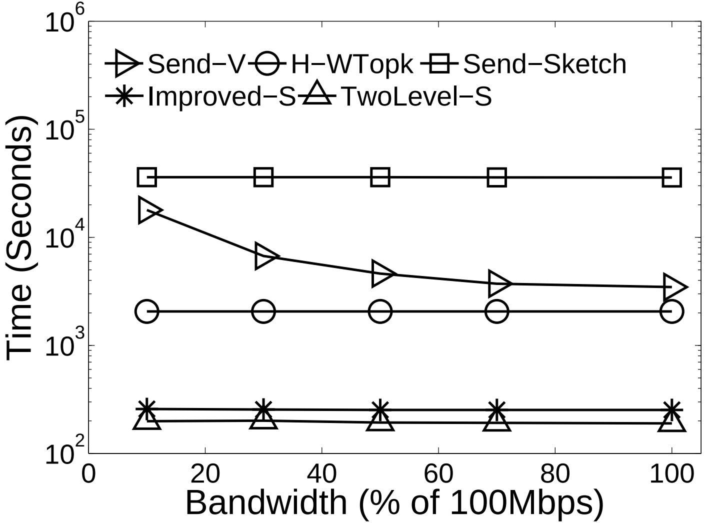


Experiments: Vary B



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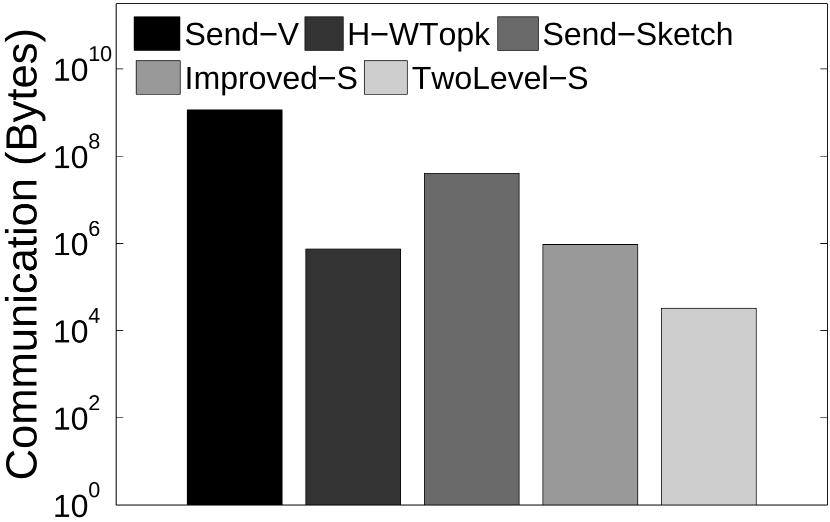

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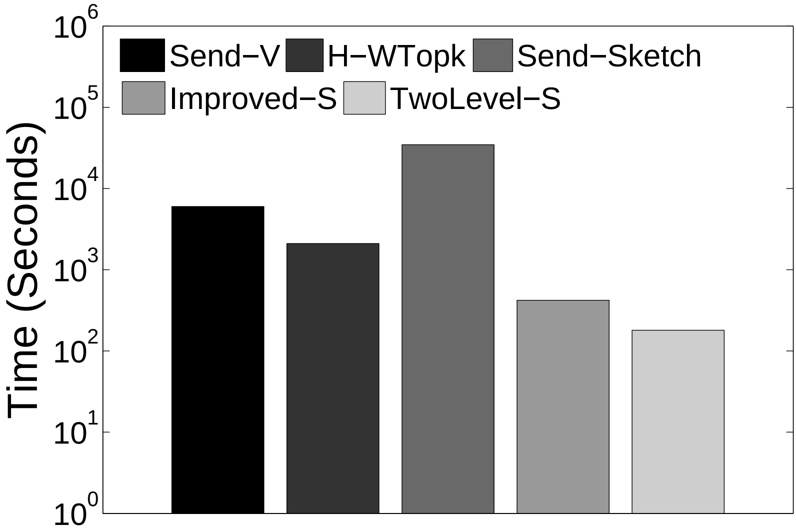

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  - other histograms including the V-optimal histogram,
  - sketches and synopsis,
  - geometric summaries (ε-approximations and coresets),
  - graph summaries (distance oracles).

The End

Thank You

Q and A

Introduction: Histograms

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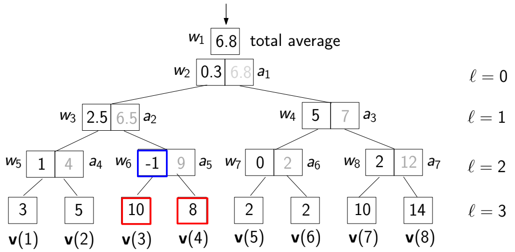

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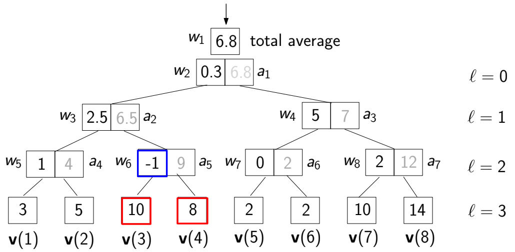

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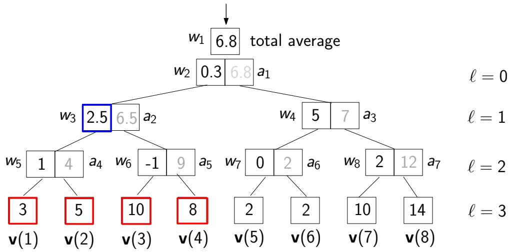

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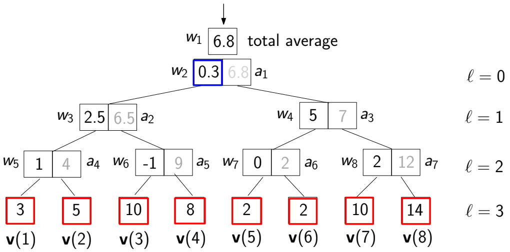

$$w_2 = \langle \mathbf{v}, \psi_2 \rangle$$

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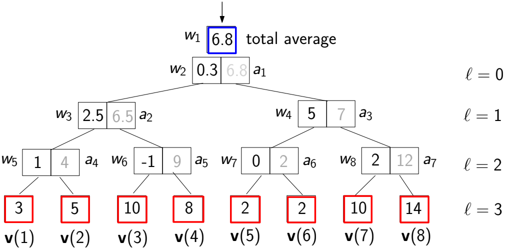
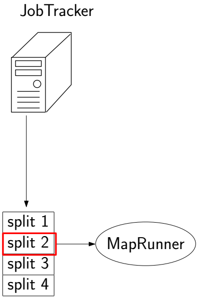


$$w_1 = \langle \mathbf{v}, \psi_1 \rangle$$

Background: Hadoop MapReduce, Map Phase



- The JobTracker assigns an InputSplit to a TaskTracker, a MapRunner task runs on the TaskTracker to process the split.

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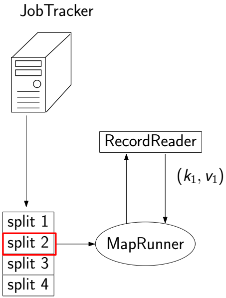

- The MapRunner acquires a RecordReader from the InputFormat for the file to view the InputSplit as a stream of records, (k_1, v_1).

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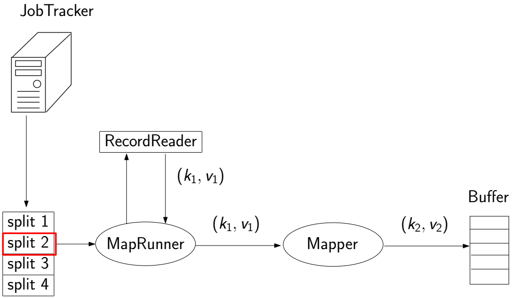
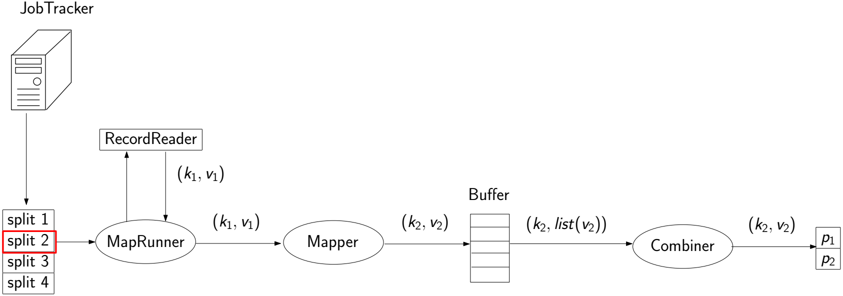


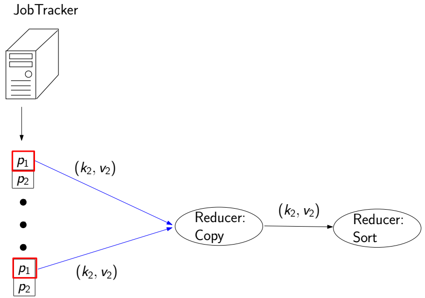
- The MapRunner invokes the user specified *Mapper* for each (k_1, v_1), the Mapper emits (k_2, v_2) and stores in an in-memory buffer.

Background: Hadoop MapReduce, Map Phase



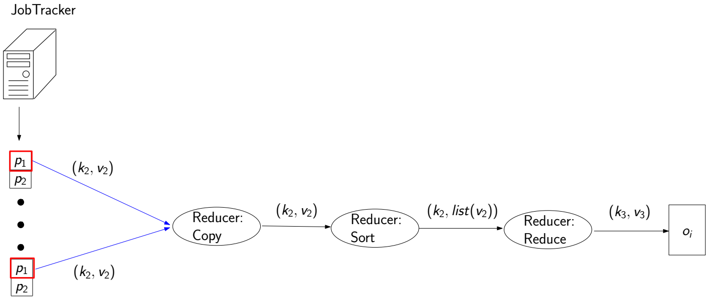
- When the buffer fills, the optional *Combiner* is executed over $(k_2, list(v_2))$, and a (k_2, v_2) is dumped to a partition on disk.

Background: Hadoop MapReduce, Shuffle and Sort Phase



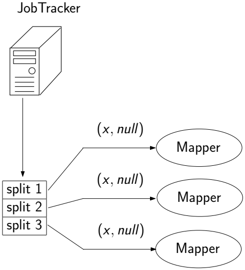
- The JobTracker assigns Reducers to TaskTrackers for each partition, each reducer first copies on (k_2, v_2) and then sorts on k_2.

Background: Hadoop MapReduce, Reduce Phase

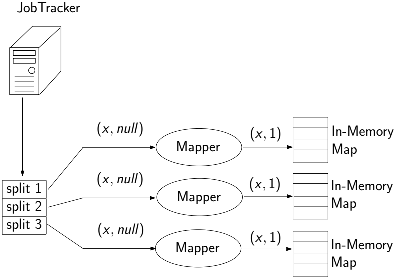


- The sorting output $(k_2, list(v_2))$ is processed one k_2 at a time and reduced, the reduced output (k_3, v_3) is written to reducer output o_i.

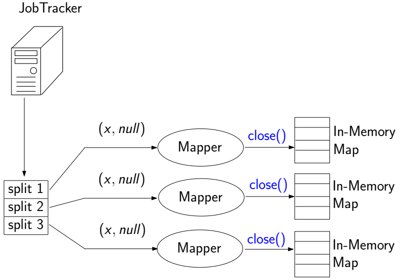
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



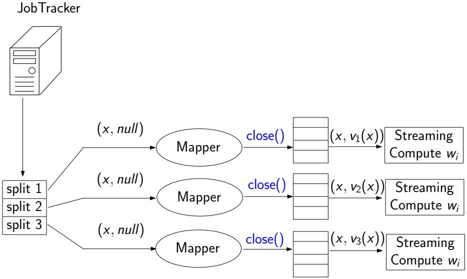
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



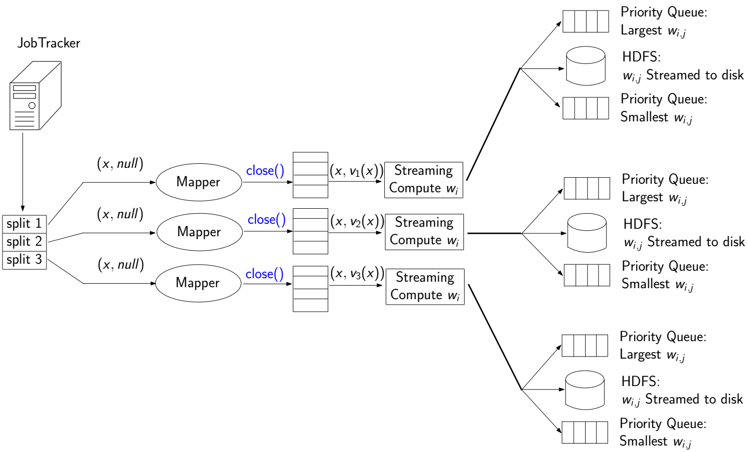
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



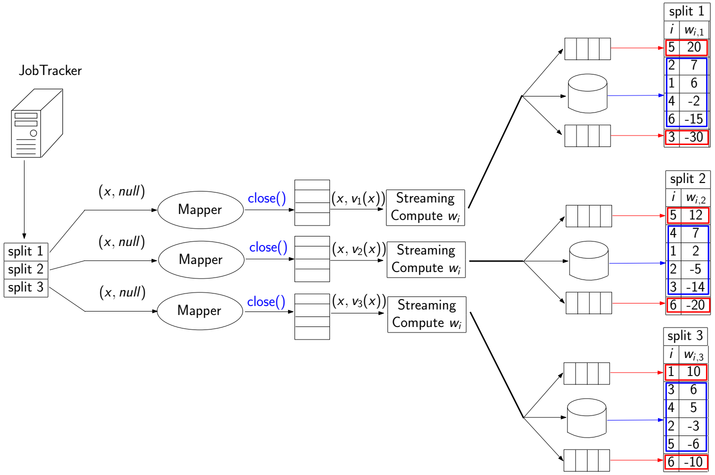
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



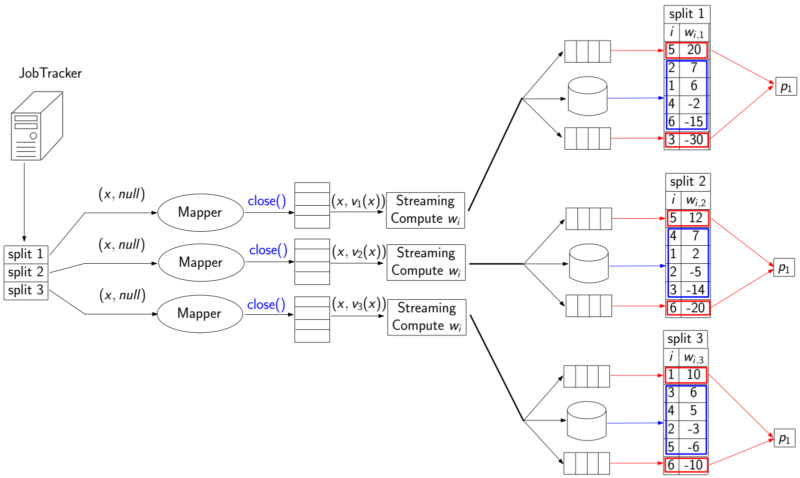
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



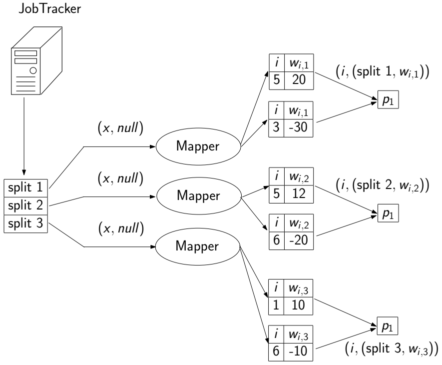
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



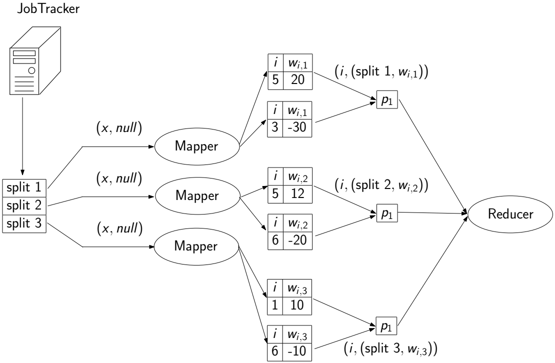
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



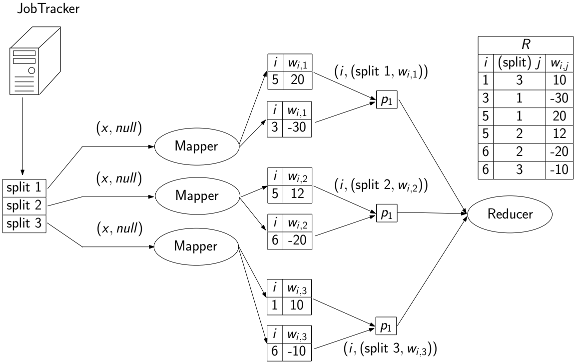
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



Exact Top-k Wavelet Coefficients: Hadoop Phase 1

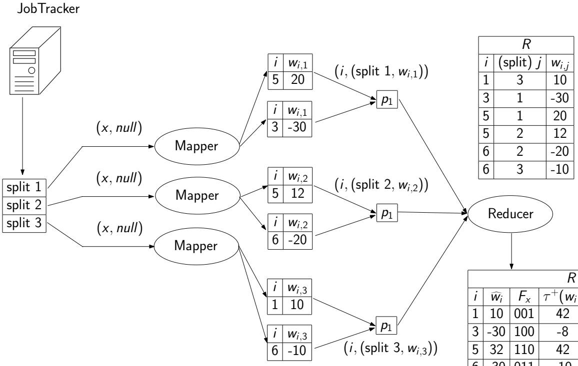


Exact Top-k Wavelet Coefficients: Hadoop Phase 1



R		
i	(split) j	$w_{i,j}$
1	3	10
3	1	-30
5	1	20
5	2	12
6	2	-20
6	3	-10

Exact Top-k Wavelet Coefficients: Hadoop Phase 1



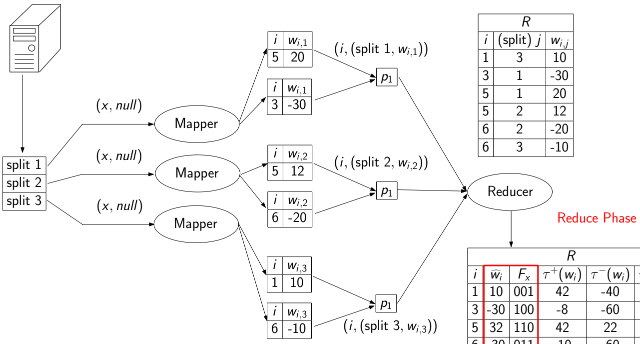
R

i	(split) j	$w_{i,j}$
1	3	10
3	1	-30
5	1	20
5	2	12
6	2	-20
6	3	-10

R

i	$\widehat{w_i}$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

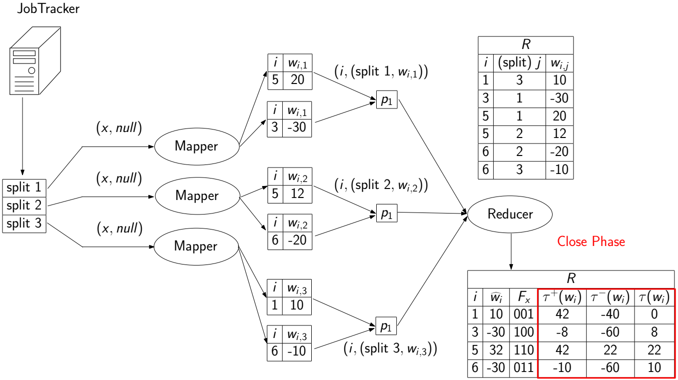
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



R		
i	(split) j	$w_{i,j}$
1	3	10
3	1	-30
5	1	20
5	2	12
6	2	-20
6	3	-10

R					
i	$\widehat{w_i}$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

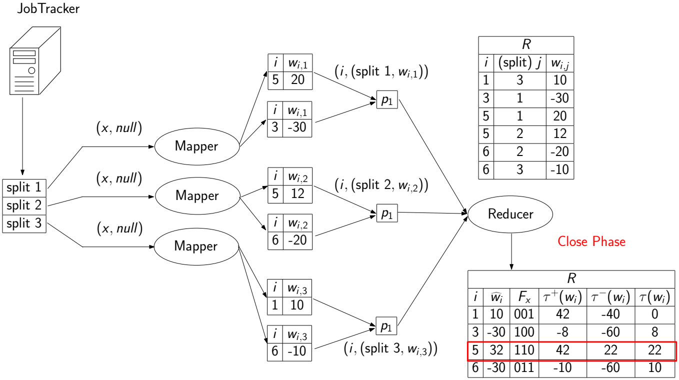
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



R		
i	(split) j	$w_{i,j}$
1	3	10
3	1	-30
5	1	20
5	2	12
6	2	-20
6	3	-10

R					
i	$\widehat{w_i}$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

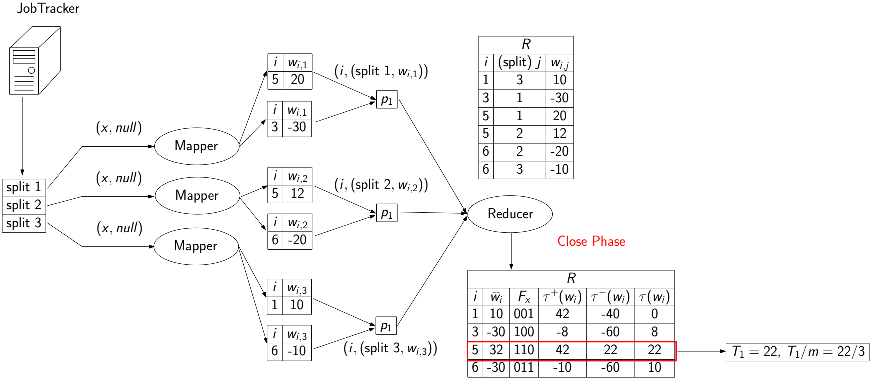
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



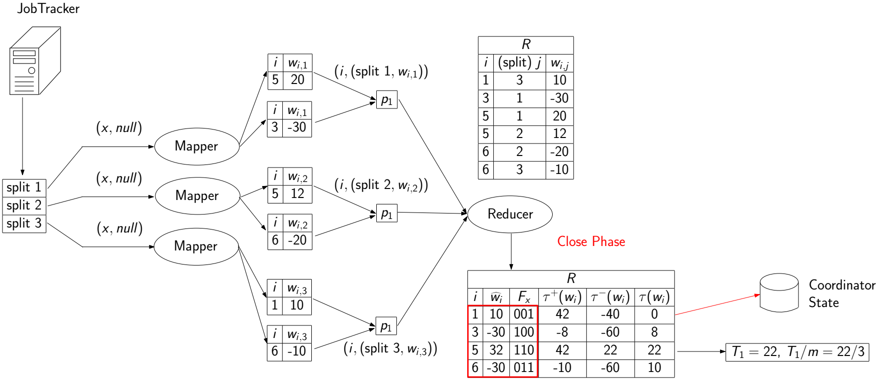
R		
i	(split) j	$w_{i,j}$
1	3	10
3	1	-30
5	1	20
5	2	12
6	2	-20
6	3	-10

R					
i	$\widehat{w_i}$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

Exact Top-k Wavelet Coefficients: Hadoop Phase 1



Exact Top-k Wavelet Coefficients: Hadoop Phase 1

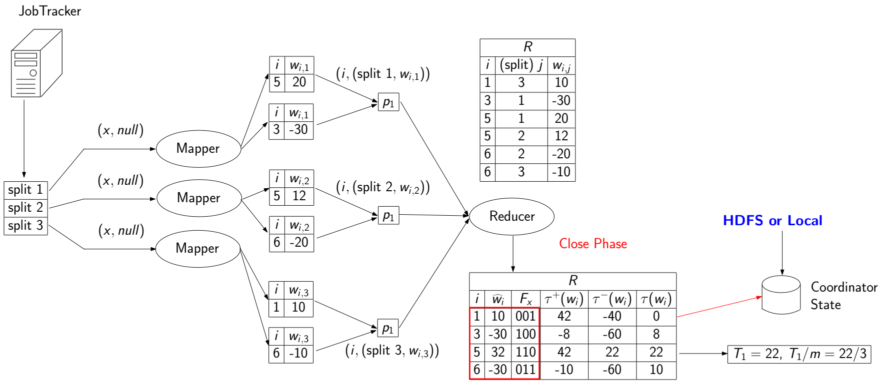


R		
i	(split) j	$w_{i,j}$
1	3	10
3	1	-30
5	1	20
5	2	12
6	2	-20
6	3	-10

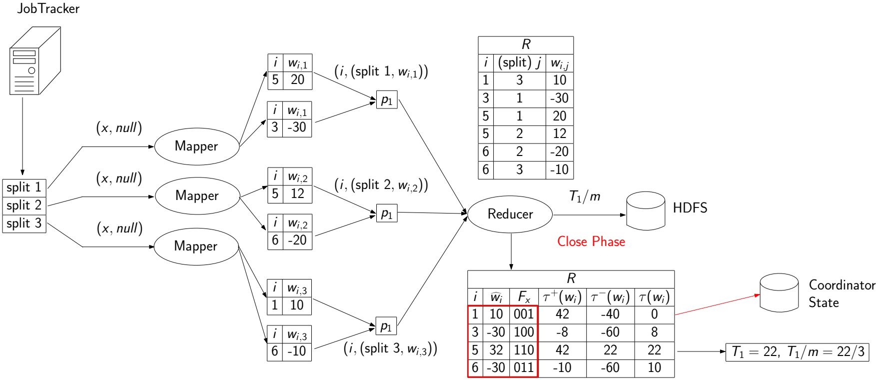
R					
i	$\widehat{w_i}$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

$T_1 = 22,\ T_1/m = 22/3$

Exact Top-k Wavelet Coefficients: Hadoop Phase 1



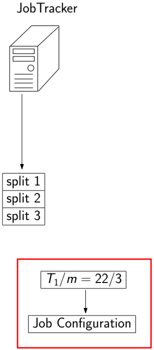
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



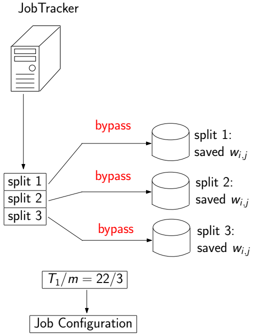
R		
i	(split) j	$w_{i,j}$
1	3	10
3	1	-30
5	1	20
5	2	12
6	2	-20
6	3	-10

R					
i	$\widehat{w_i}$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$
1	10	001	42	-40	0
3	-30	100	-8	-60	8
5	32	110	42	22	22
6	-30	011	-10	-60	10

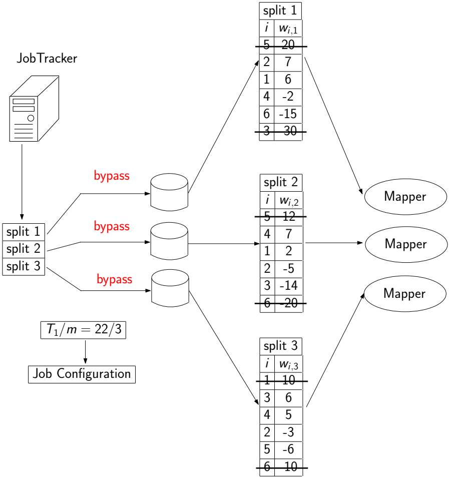
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



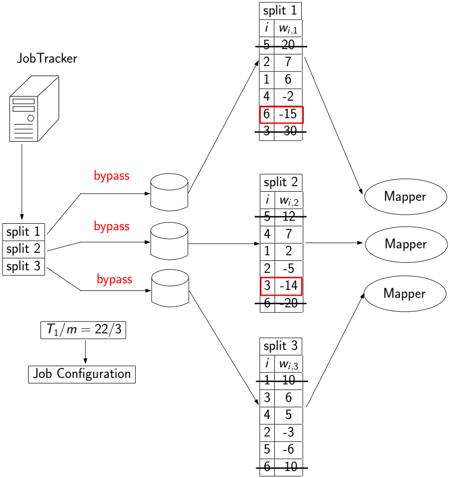
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



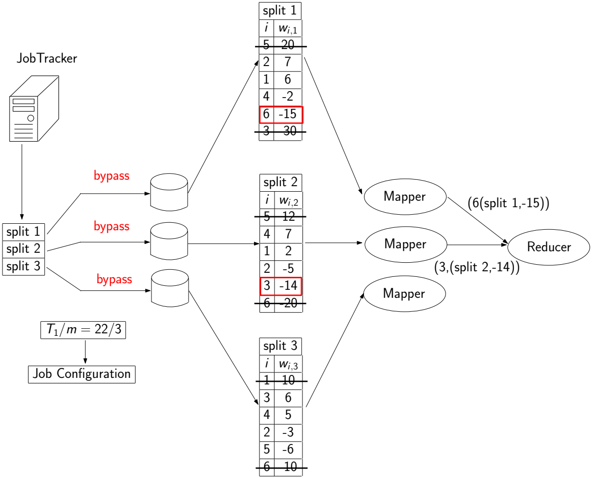
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



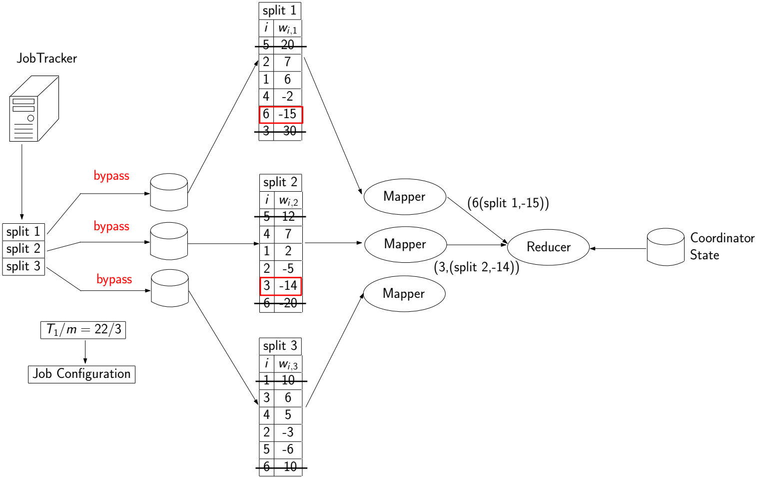
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



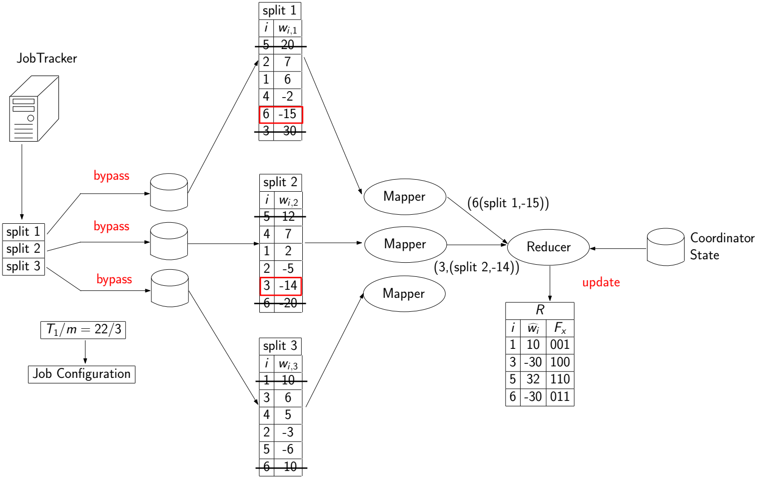
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



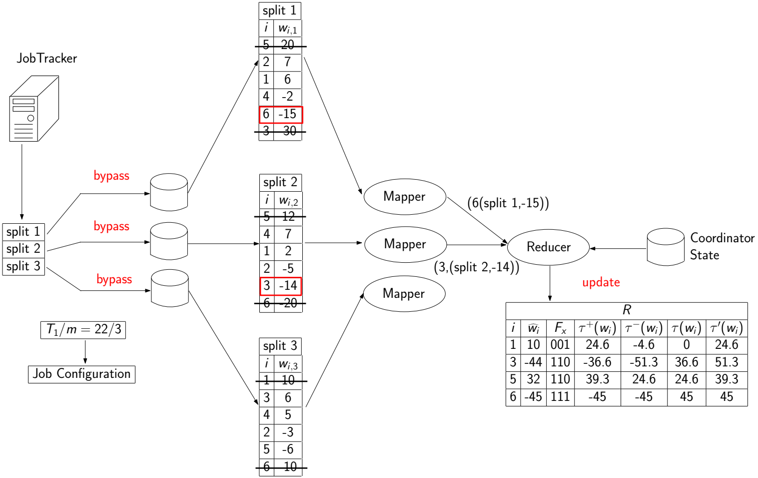
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



Exact Top-k Wavelet Coefficients: Hadoop Phase 3



Exact Top-k Wavelet Coefficients: Hadoop Phase 3



split 1

i	$w_{i,1}$
~~5~~	~~20~~
2	7
1	6
4	-2
6	-15
~~3~~	~~30~~

split 2

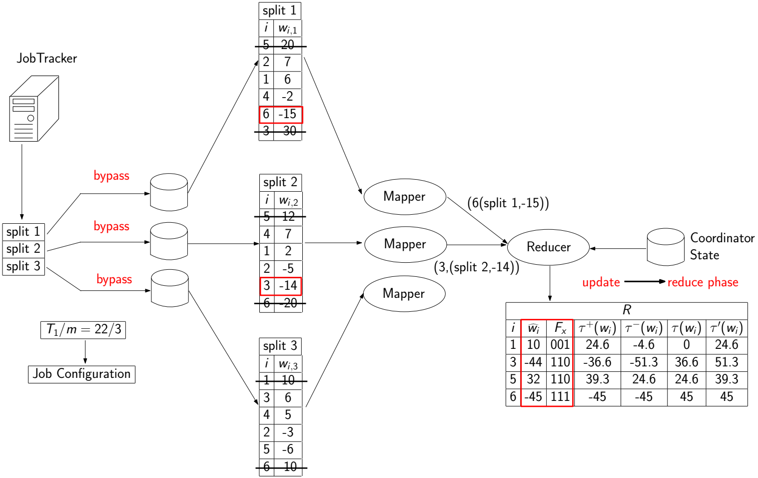
i	$w_{i,2}$
~~5~~	~~12~~
4	7
1	2
2	-5
3	-14
~~6~~	~~-20~~

split 3

i	$w_{i,3}$
~~1~~	~~10~~
3	6
4	5
2	-3
5	-6
~~6~~	~~10~~

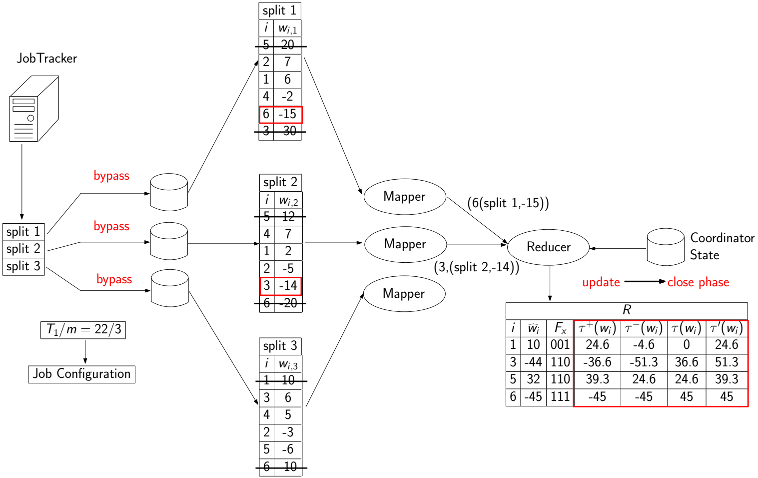
R

i	$\widehat{w}_i$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$	$\tau'(w_i)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45



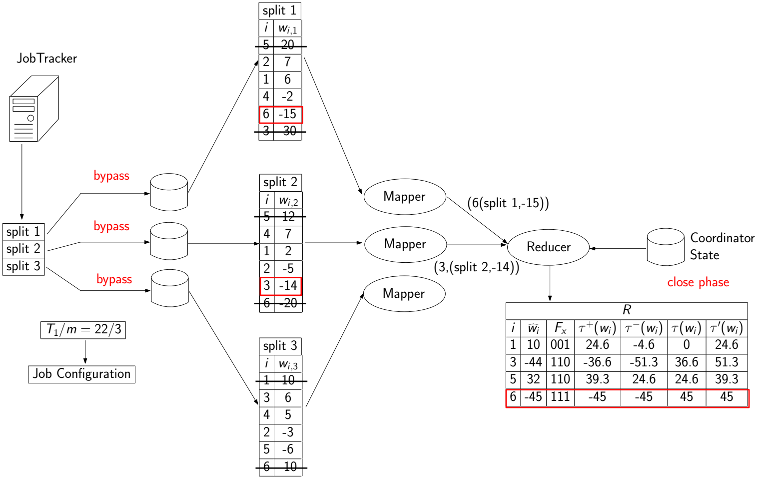
R						
i	$\widehat{w}_i$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$	$\tau'(w_i)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

Exact Top-k Wavelet Coefficients: Hadoop Phase 3



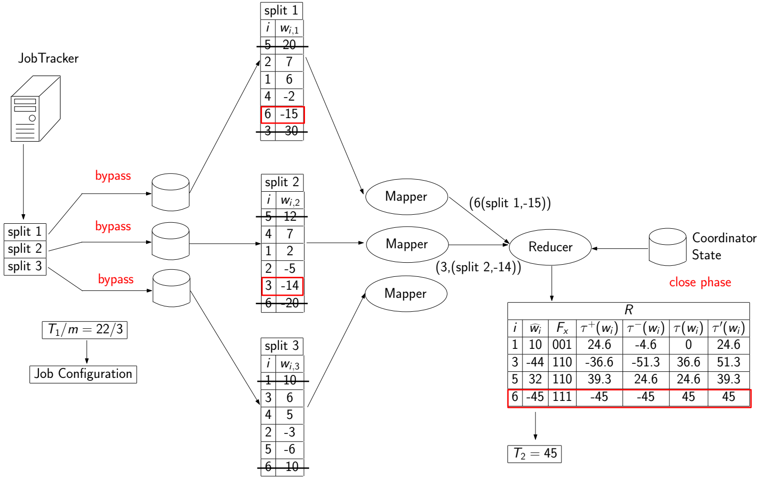
R						
i	$\widehat{w}_i$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$	$\tau'(w_i)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

Exact Top-k Wavelet Coefficients: Hadoop Phase 3

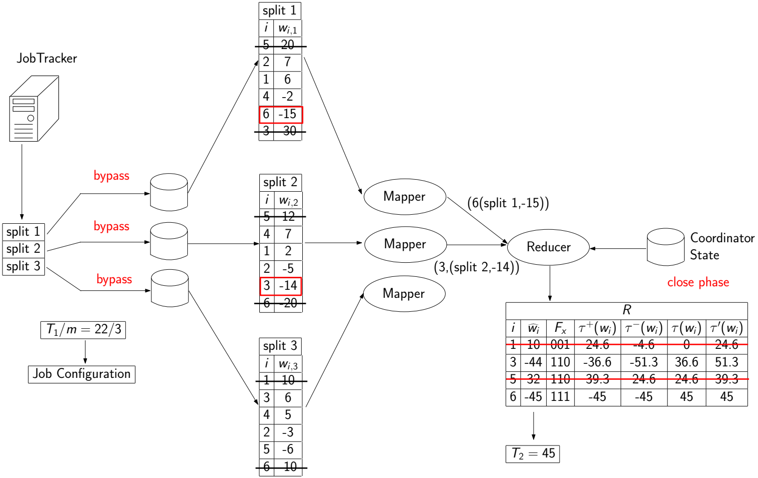


i	$\widehat{w}_i$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$	$\tau'(w_i)$
1	10	001	24.6	-4.6	0	24.6
3	-44	110	-36.6	-51.3	36.6	51.3
5	32	110	39.3	24.6	24.6	39.3
6	-45	111	-45	-45	45	45

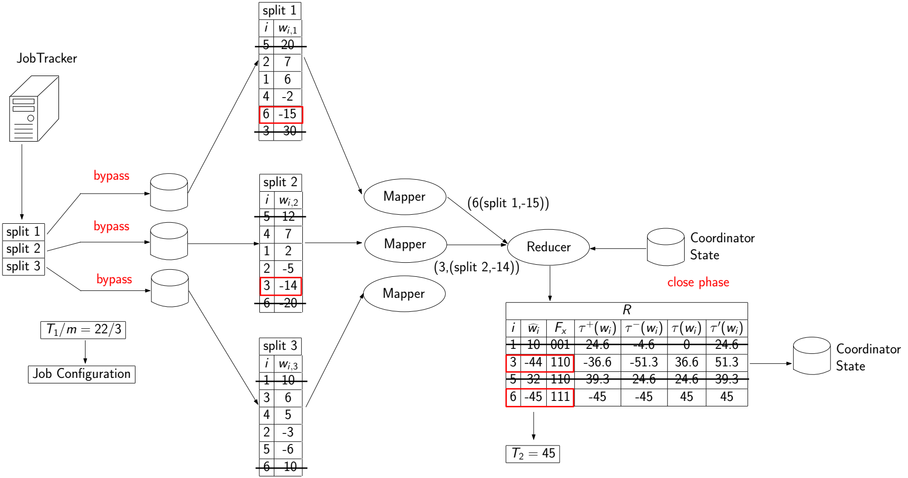
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



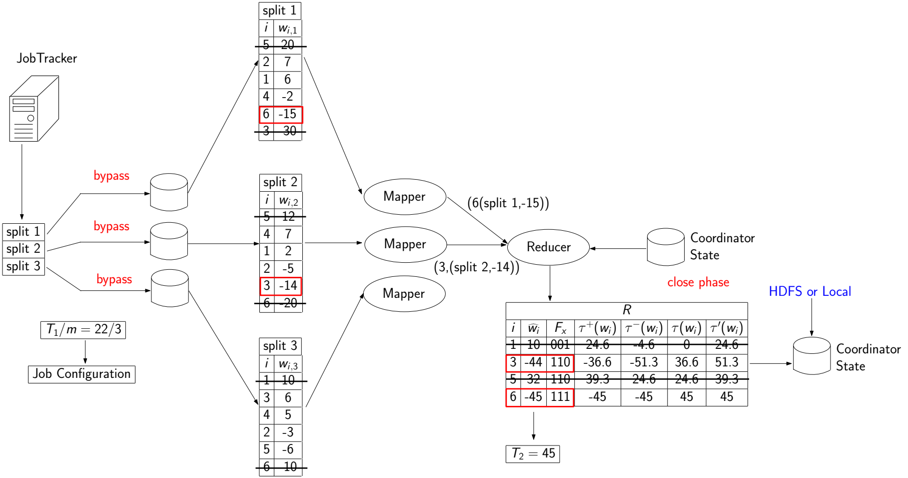
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



Exact Top-k Wavelet Coefficients: Hadoop Phase 3



Exact Top-k Wavelet Coefficients: Hadoop Phase 3



split 1

i	$w_{i,1}$
~~5~~	~~20~~
2	7
1	6
4	-2
6	-15
~~3~~	~~30~~

split 2

i	$w_{i,2}$
~~5~~	~~12~~
4	7
1	2
2	-5
3	-14
~~6~~	~~-20~~

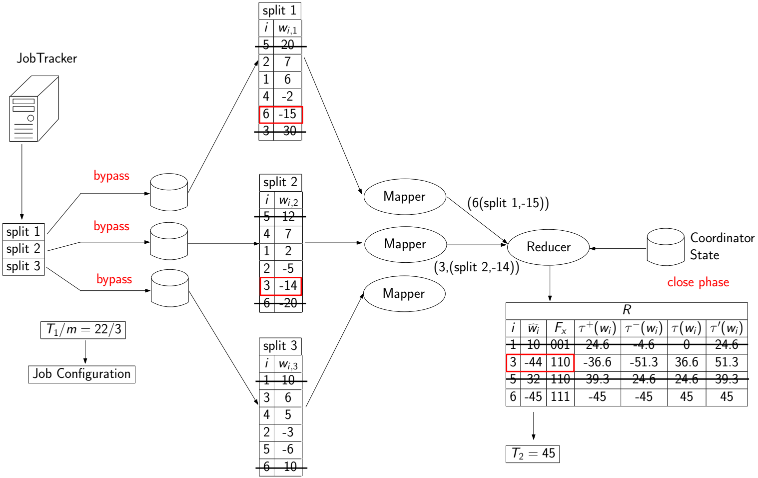
split 3

i	$w_{i,3}$
~~1~~	~~10~~
3	6
4	5
2	-3
5	-6
~~6~~	~~10~~

R

i	$\widehat{w}_i$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$	$\tau'(w_i)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

Exact Top-k Wavelet Coefficients: Hadoop Phase 3



split 1	
i	$w_{i,1}$
~~5~~	~~20~~
2	7
1	6
4	-2
6	-15
~~3~~	~~30~~

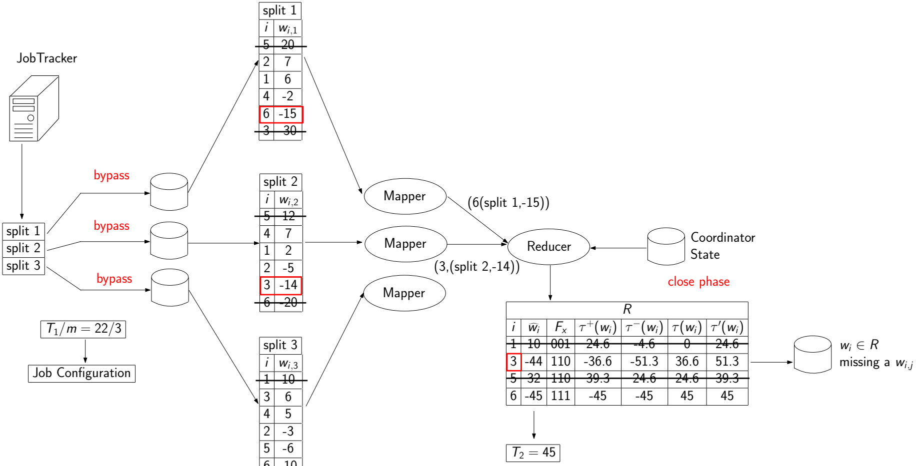
split 2	
i	$w_{i,2}$
~~5~~	~~12~~
4	7
1	2
2	-5
3	-14
~~6~~	~~-20~~

split 3	
i	$w_{i,3}$
~~1~~	~~10~~
3	6
4	5
2	-3
5	-6
~~6~~	~~10~~

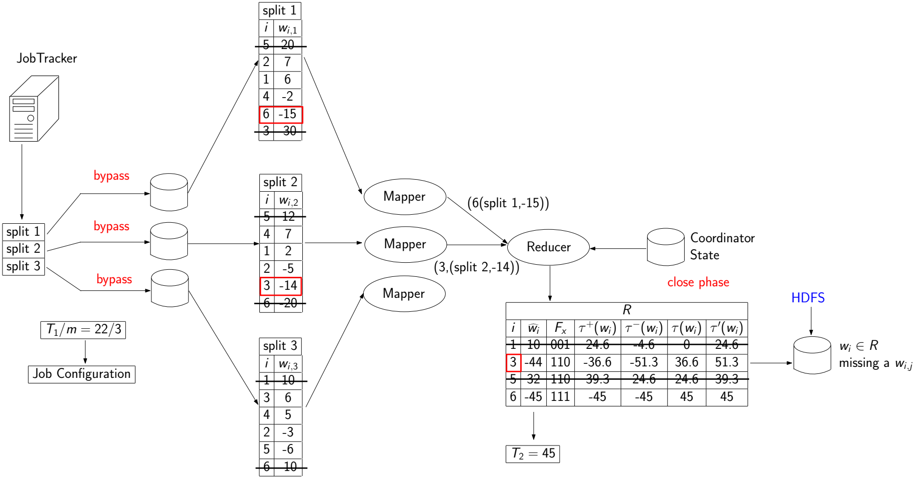
R

i	$\widehat{w_i}$	F_x	$\tau^+(w_i)$	$\tau^-(w_i)$	$\tau(w_i)$	$\tau'(w_i)$
~~1~~	~~10~~	~~001~~	~~24.6~~	~~-4.6~~	~~0~~	~~24.6~~
3	-44	110	-36.6	-51.3	36.6	51.3
~~5~~	~~32~~	~~110~~	~~39.3~~	~~24.6~~	~~24.6~~	~~39.3~~
6	-45	111	-45	-45	45	45

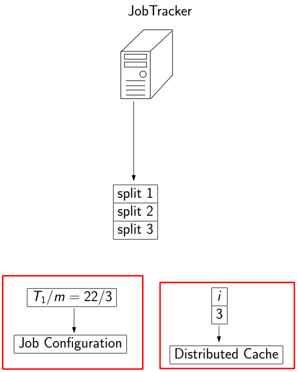
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



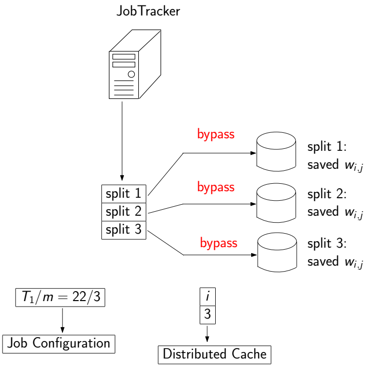
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



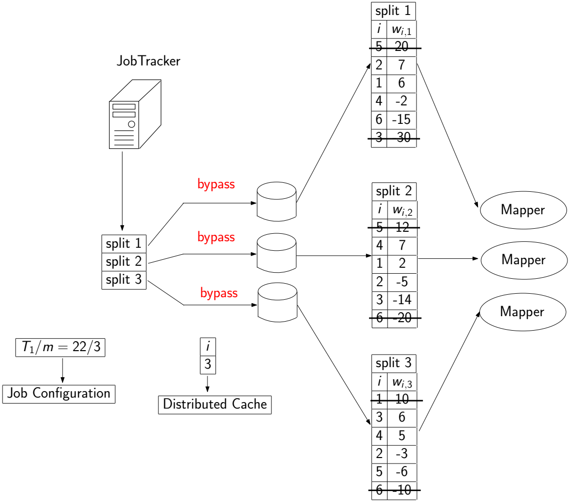
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



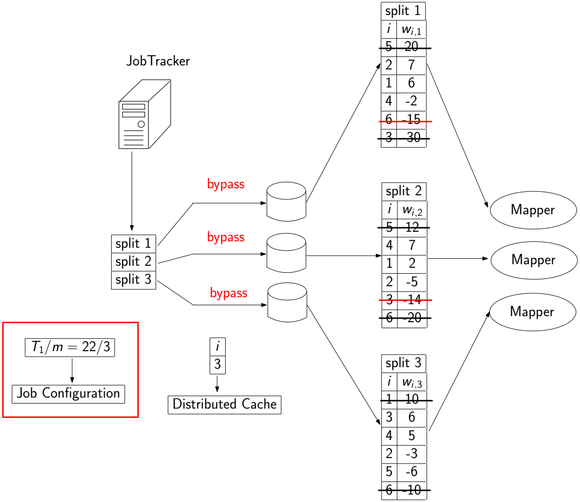
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



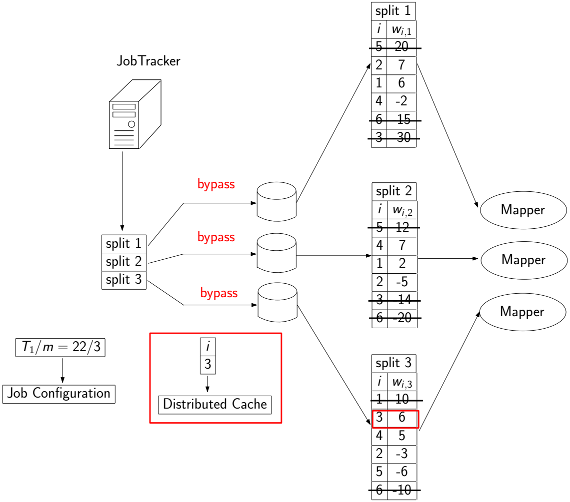
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



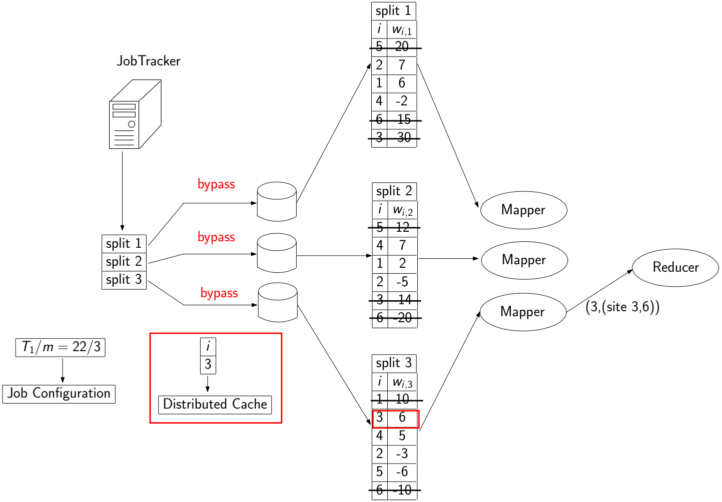
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



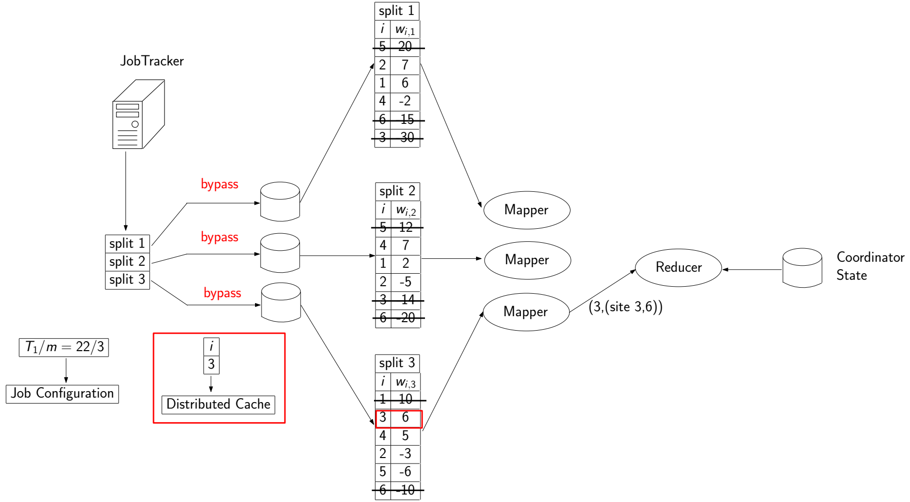
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



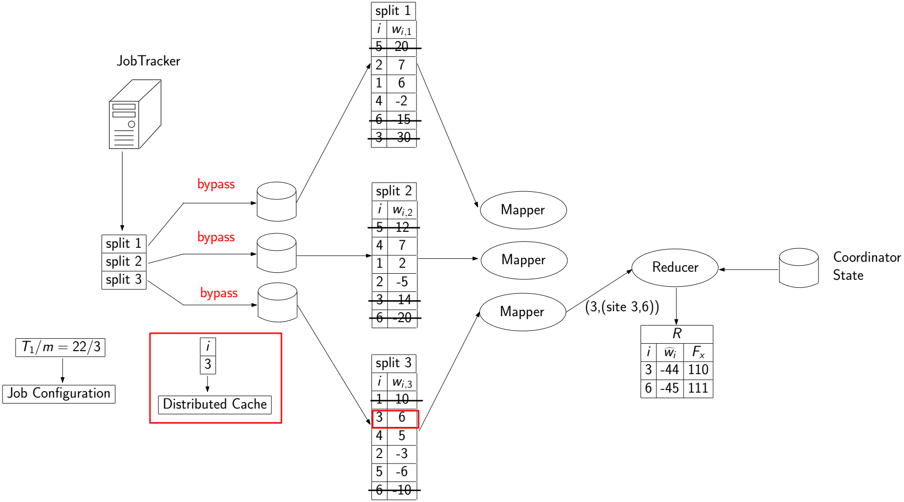
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



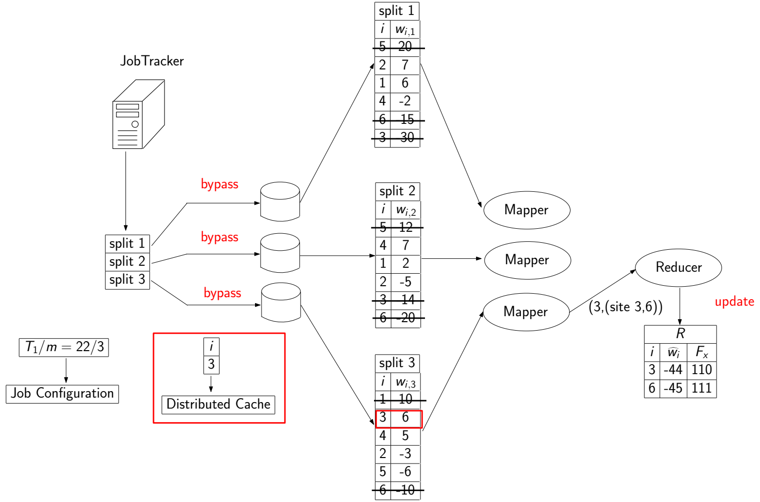
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



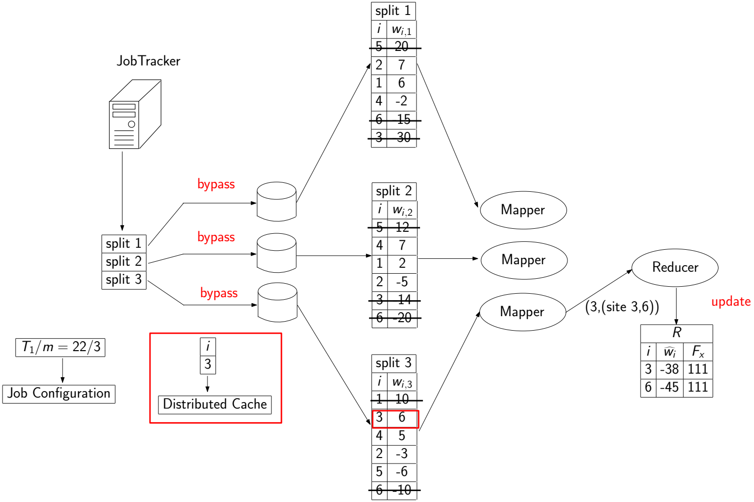
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



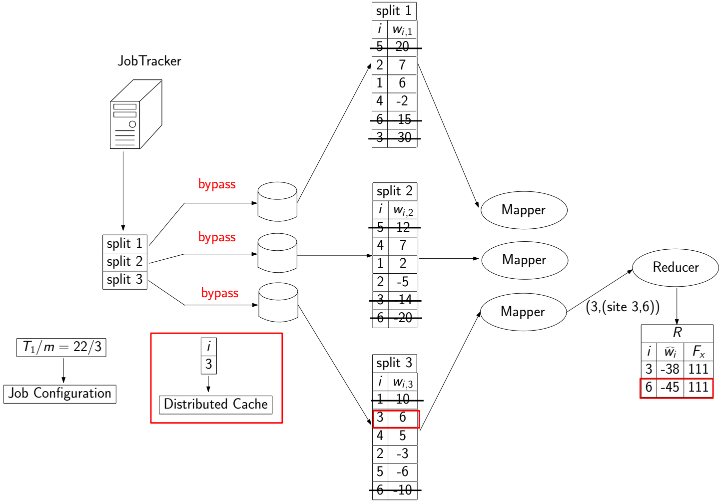
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



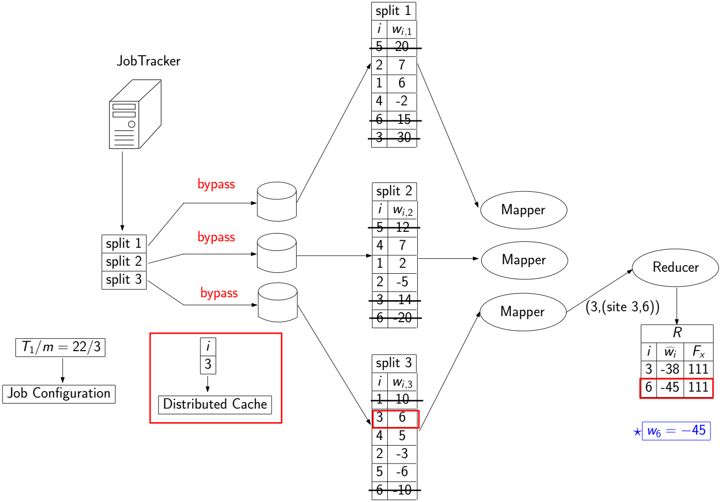
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



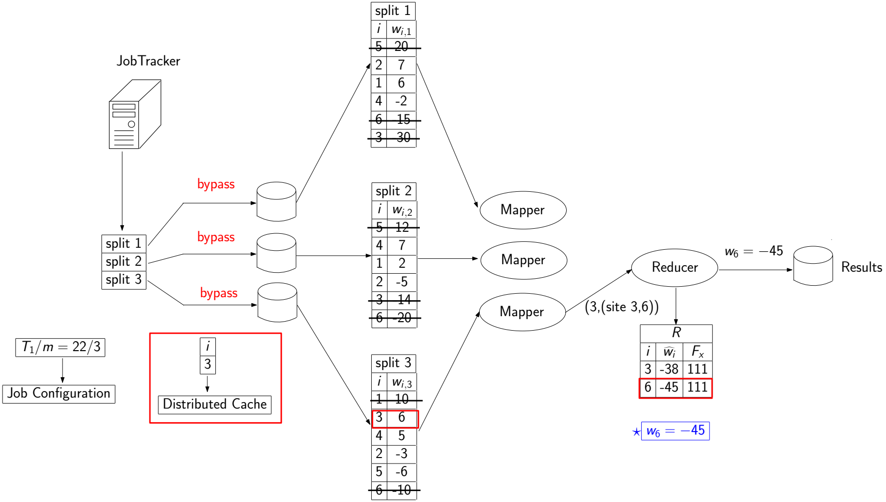
Exact Top-k Wavelet Coefficients: Hadoop Phase 3



Exact Top-k Wavelet Coefficients: Hadoop Phase 3

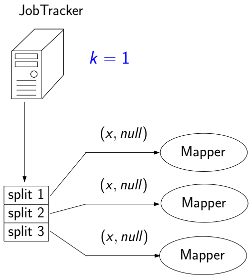


Exact Top-k Wavelet Coefficients: Hadoop Phase 3

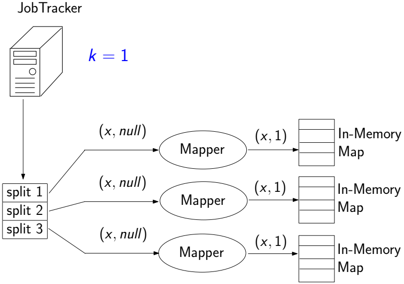


Outline

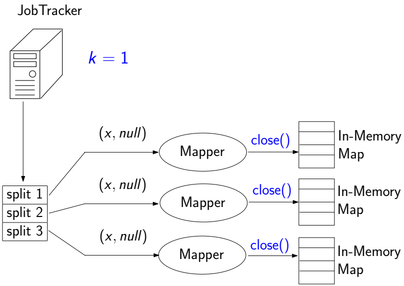
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



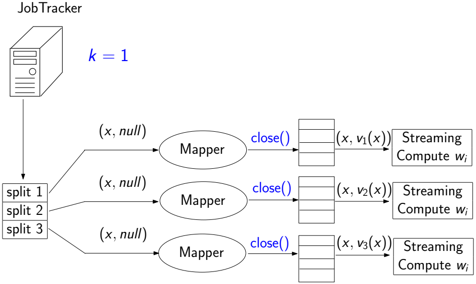
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



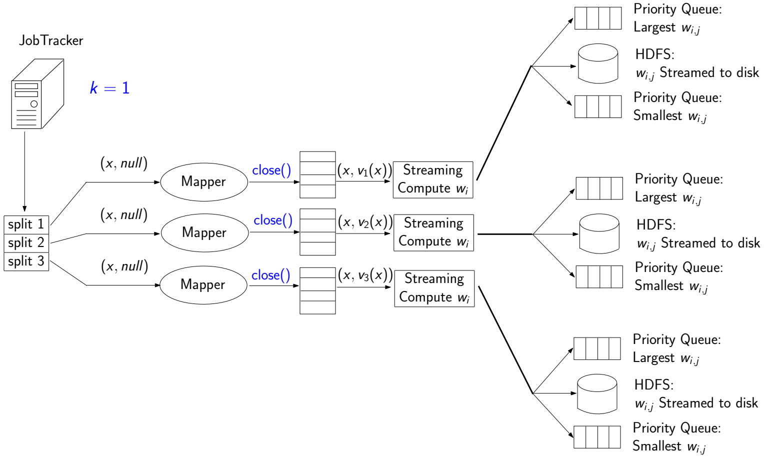
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



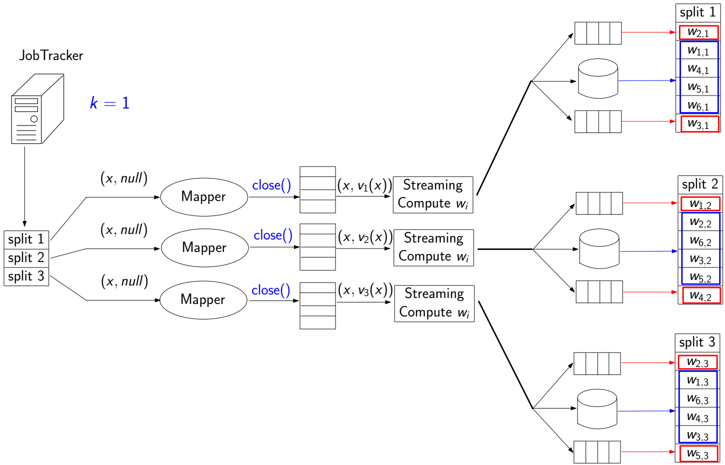
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



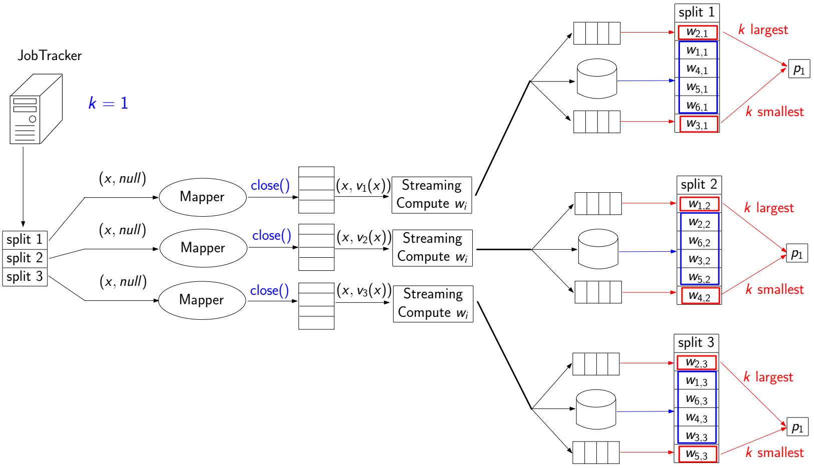
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



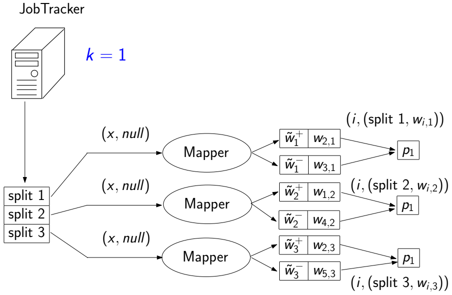
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



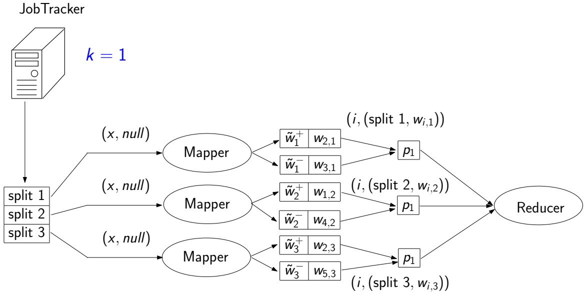
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



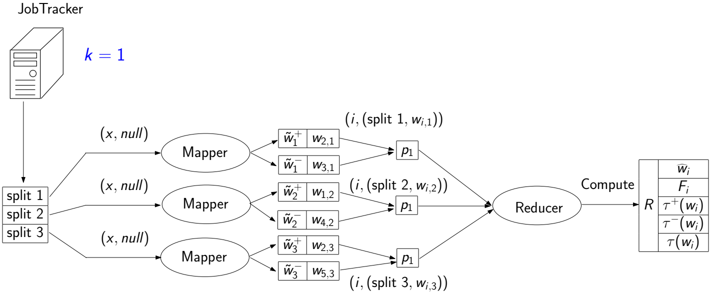
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



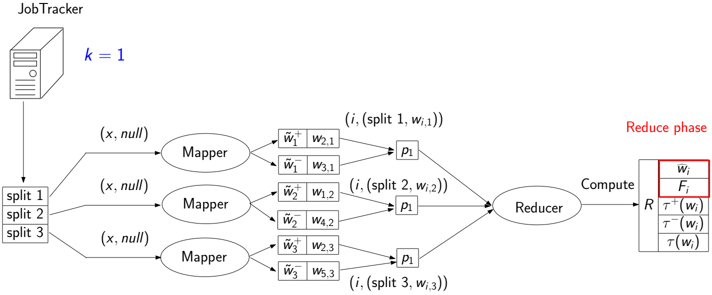
Exact Top-k Wavelet Coefficients: Hadoop Phase 1



Exact Top-k Wavelet Coefficients: Hadoop Phase 1



Exact Top-k Wavelet Coefficients: Hadoop Phase 1



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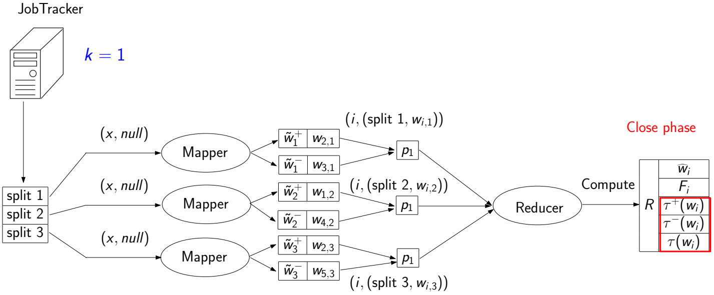

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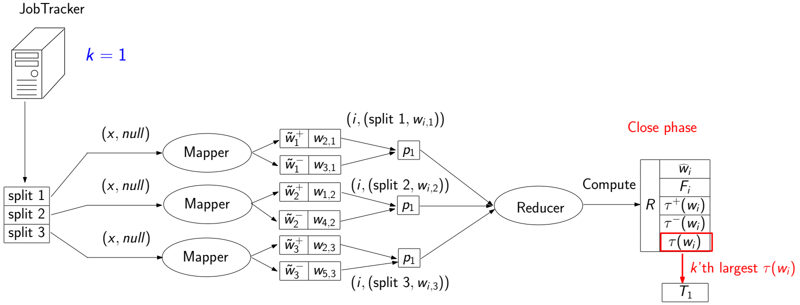

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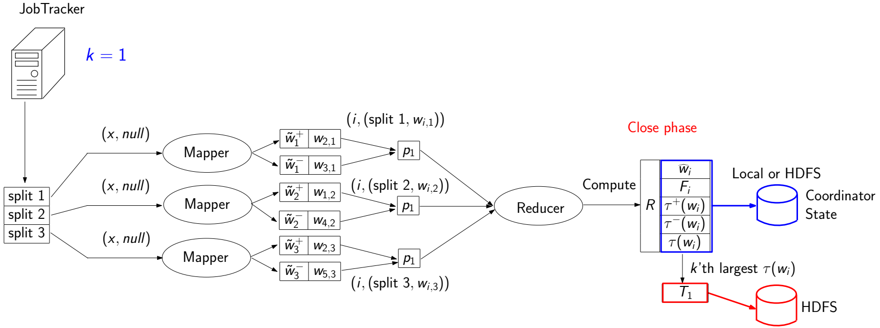
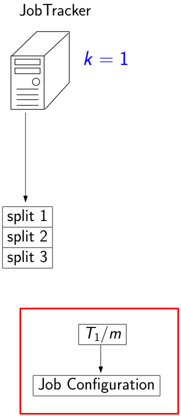
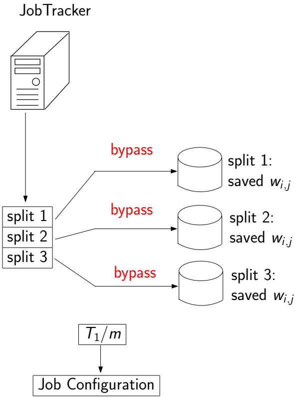


Exact Top-k Wavelet Coefficients: Hadoop Phase 2



Exact Top-k Wavelet Coefficients: Hadoop Phase 2



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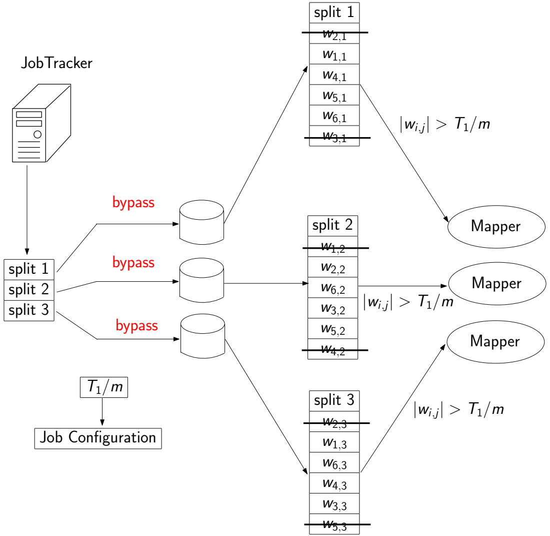

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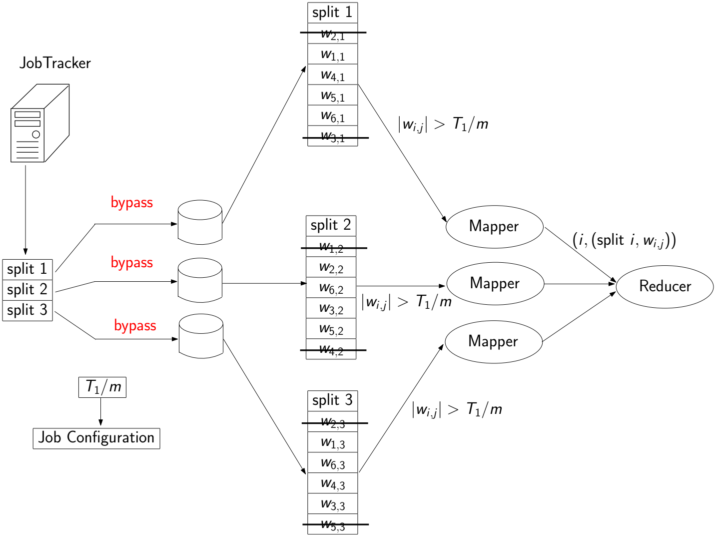

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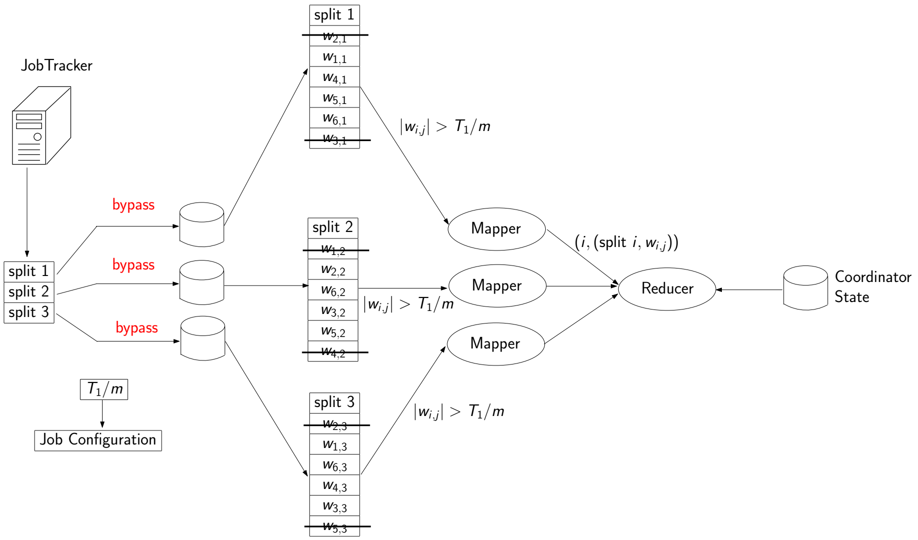

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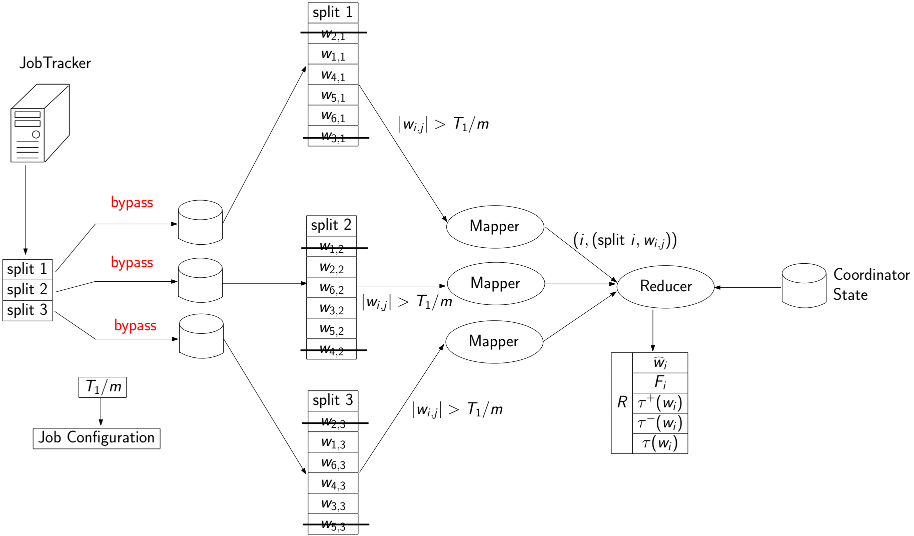

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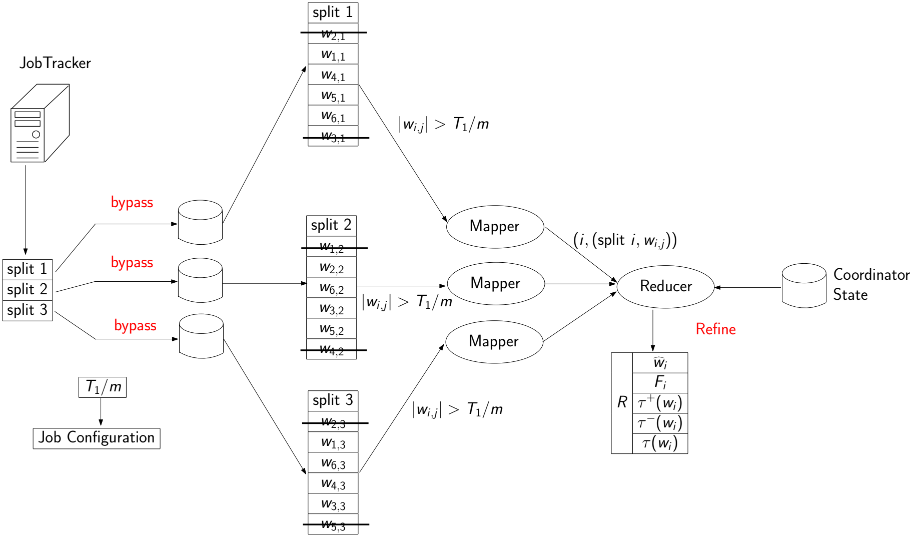

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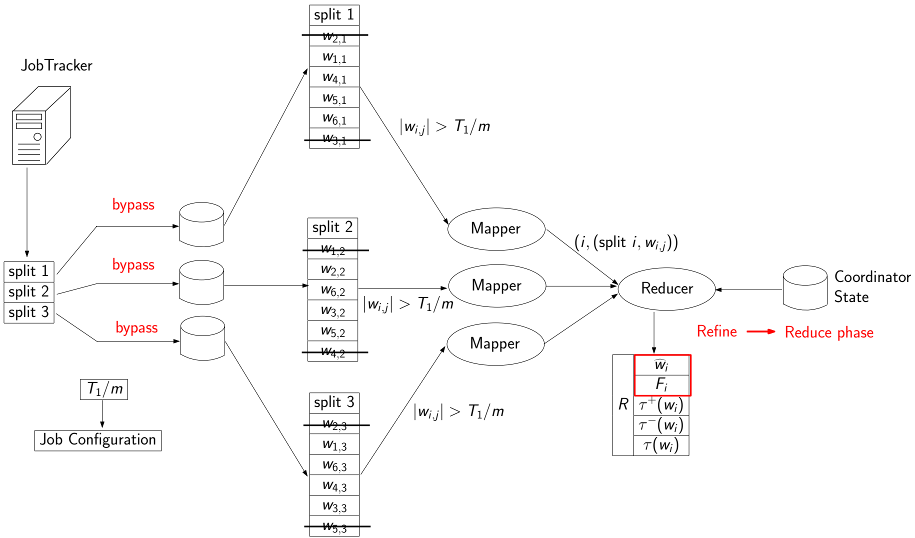

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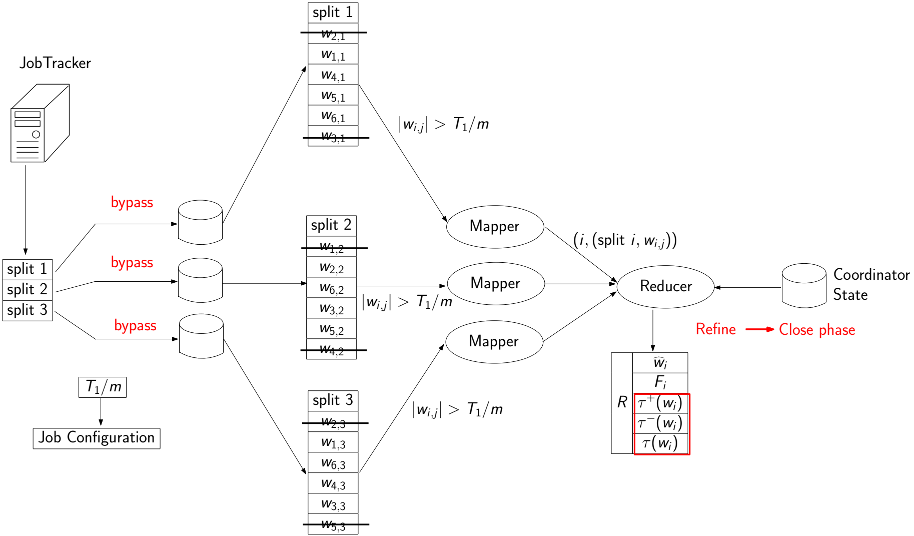

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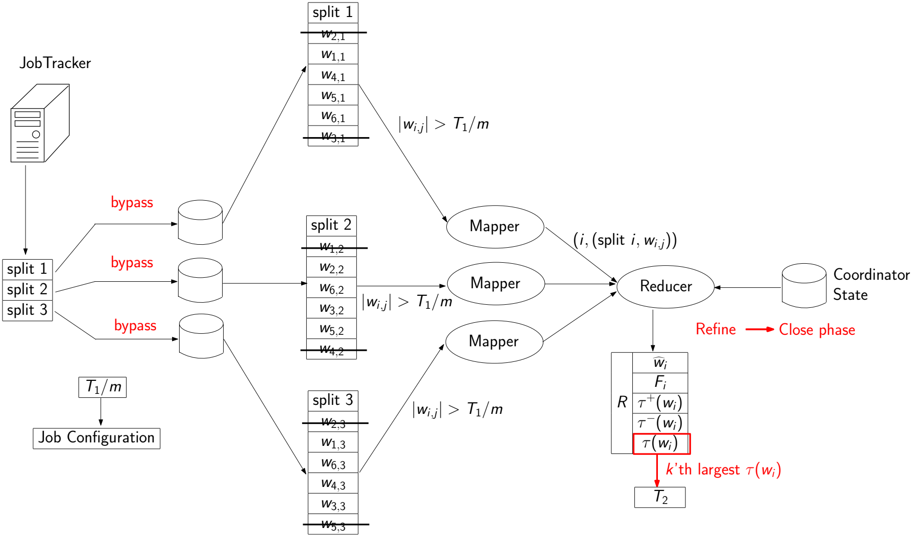

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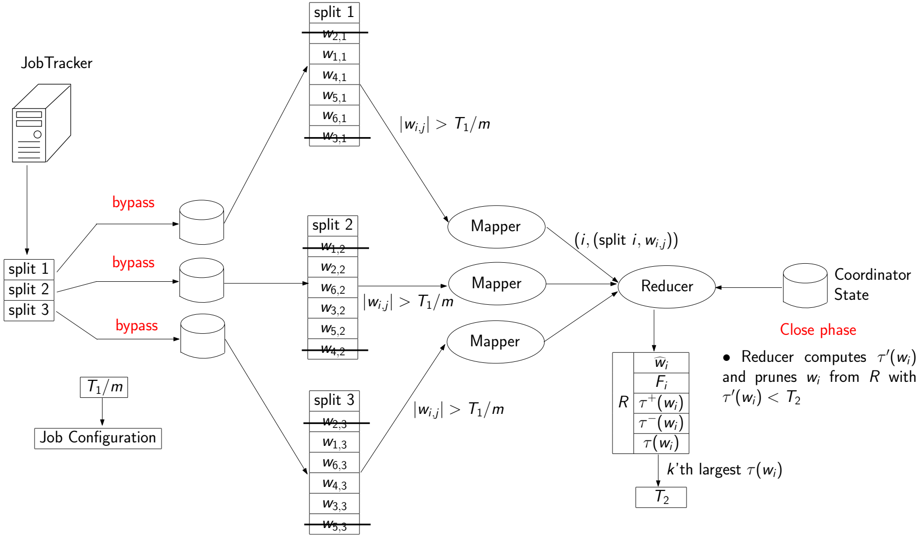
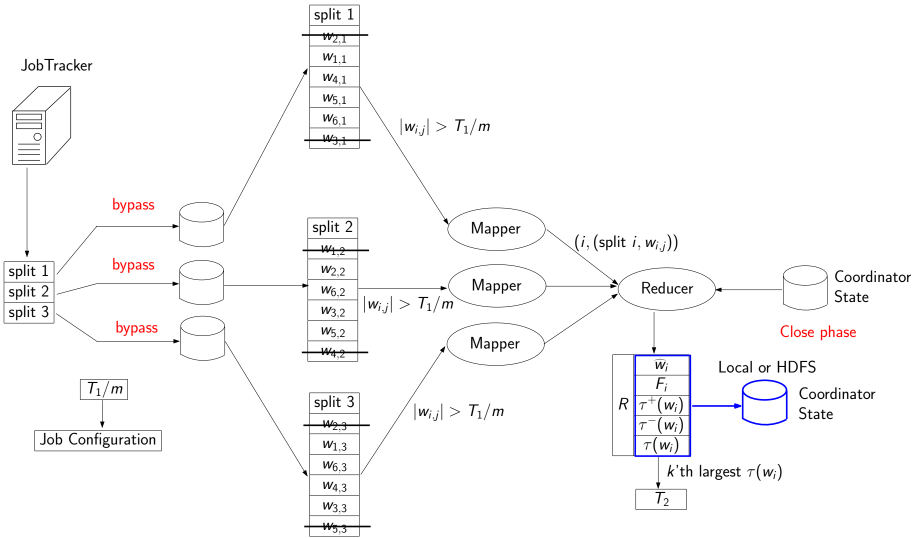


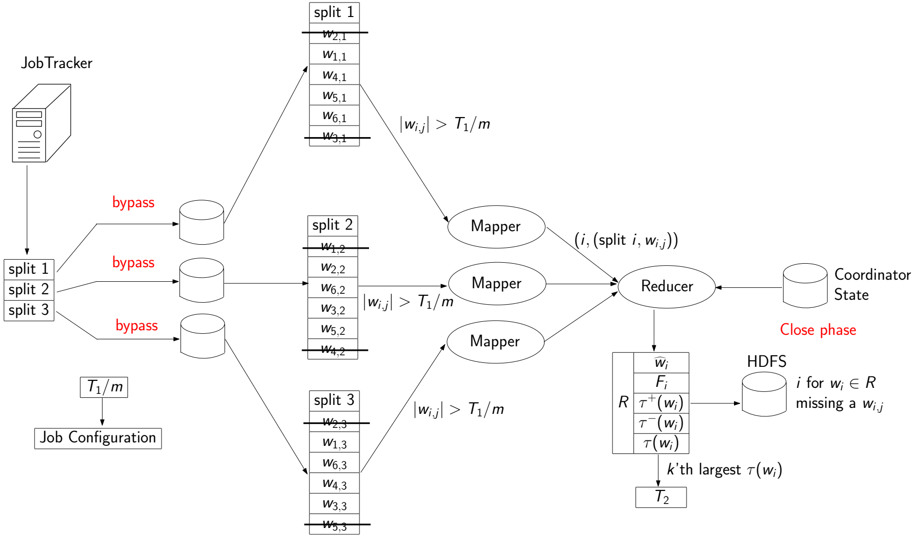
Close phase

- Reducer computes $\tau'(w_i)$ and prunes w_i from R with $\tau'(w_i) < T_2$

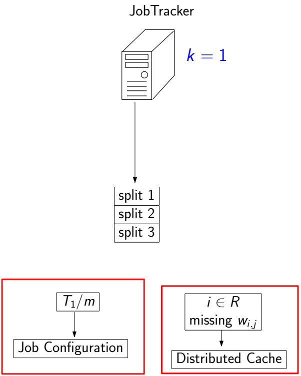
Exact Top-k Wavelet Coefficients: Hadoop Phase 2



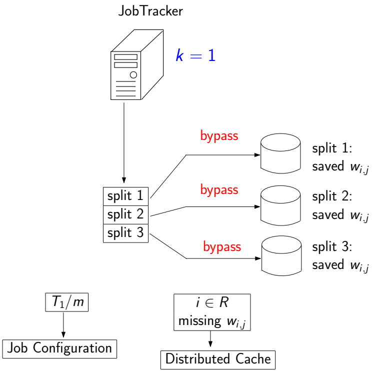
Exact Top-k Wavelet Coefficients: Hadoop Phase 2



Exact Top-k Wavelet Coefficients: Hadoop Phase 3



Exact Top-k Wavelet Coefficients: Hadoop Phase 3



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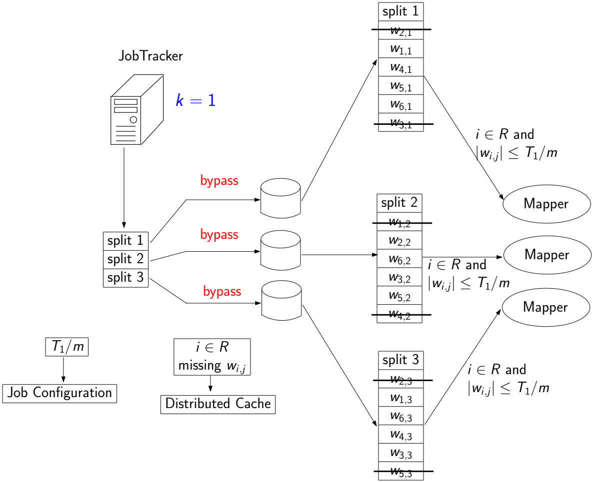

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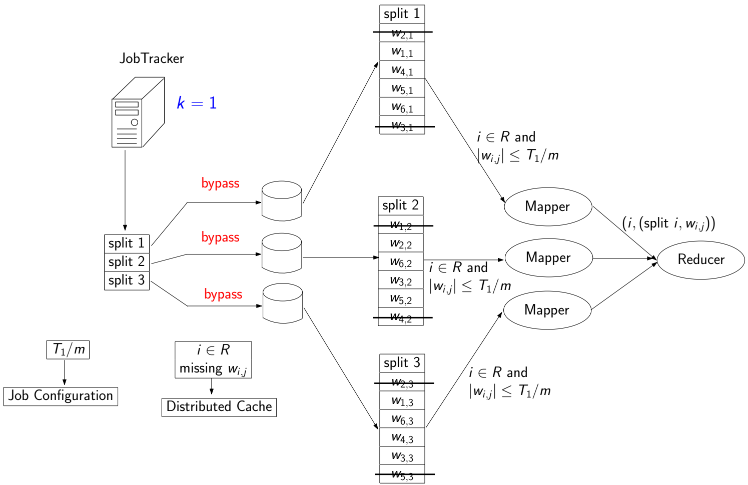

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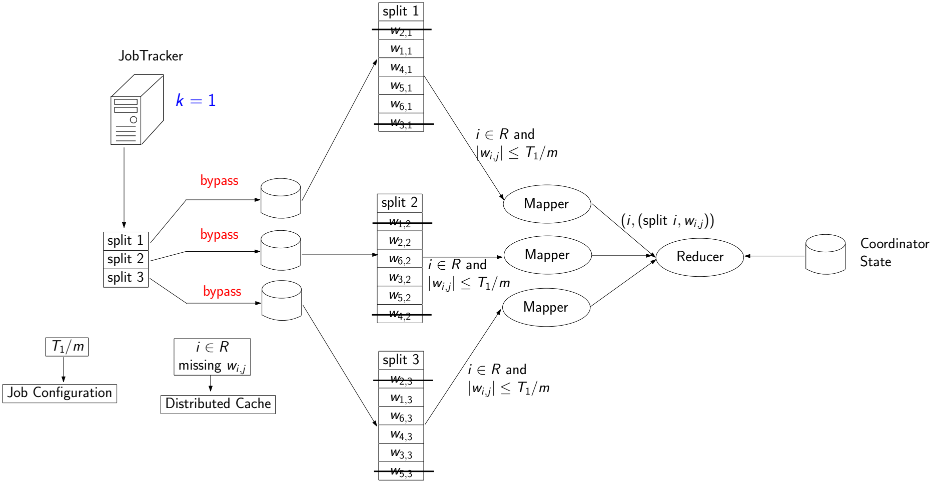

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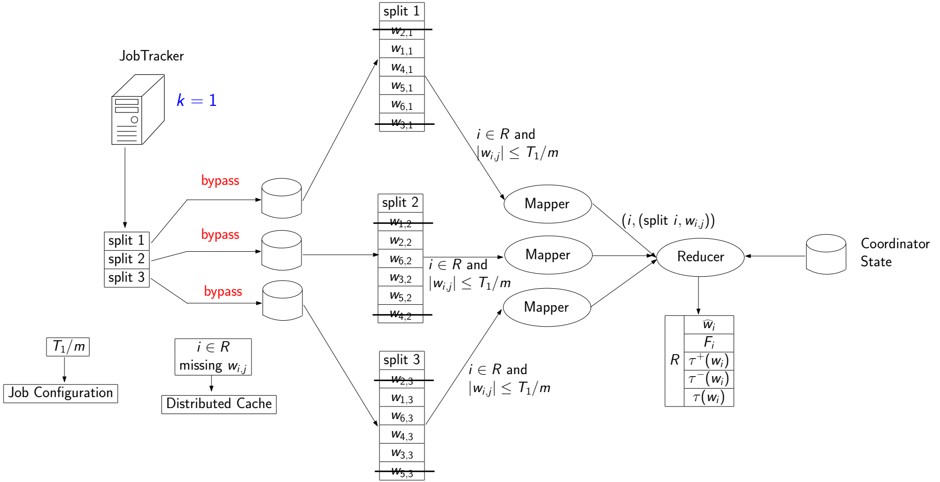

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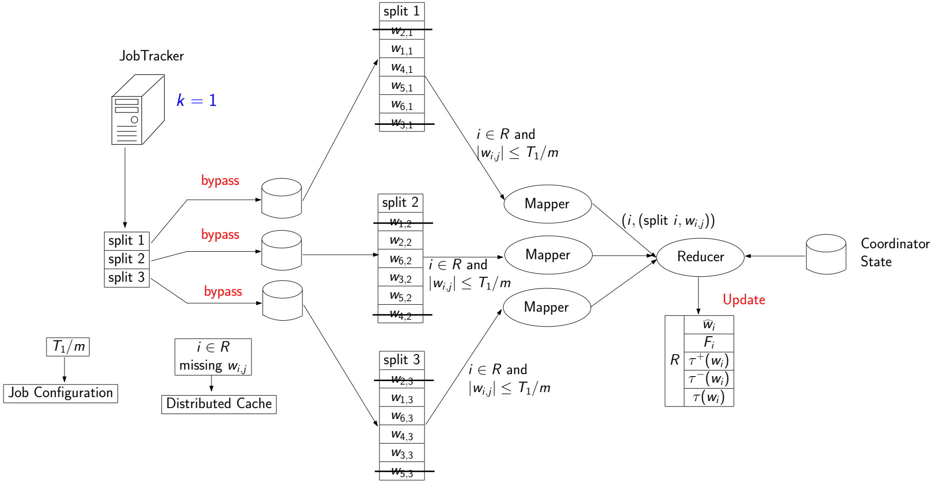

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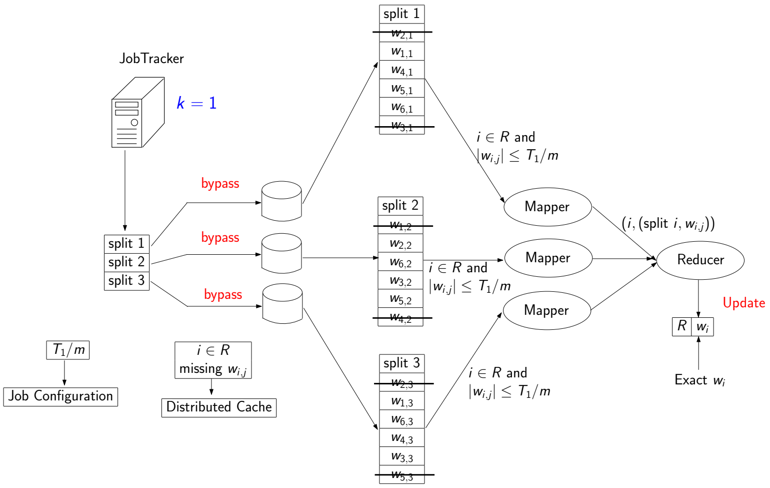
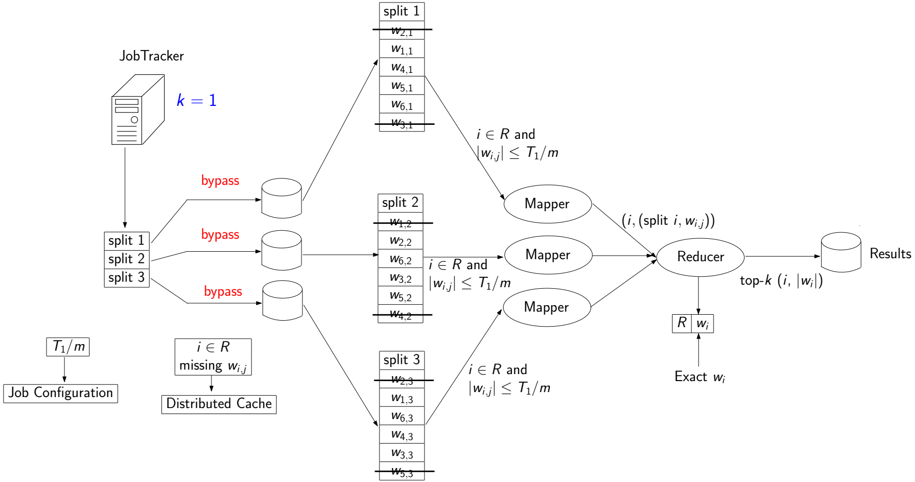
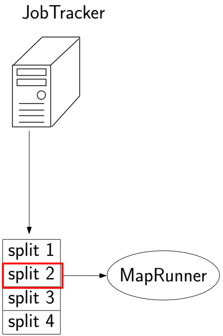


Exact Top-k Wavelet Coefficients: Hadoop Phase 3



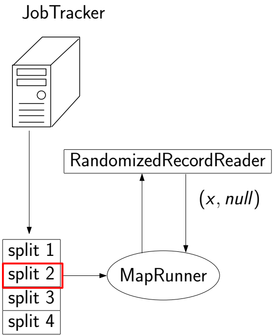
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



n_j= records in split j

s_j= split j sample frequency vector

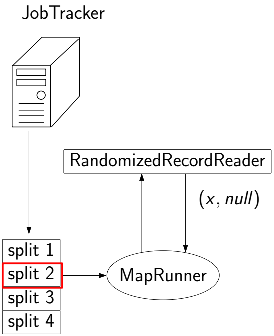
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



n_j= records in split j
s_j= split j sample frequency vector

1. RandomizedRecordReader j (RR_j) samples $n_j/\varepsilon^2 n$ records.

Approximate Top-k Wavelet Coefficients: Basic Random Sampling



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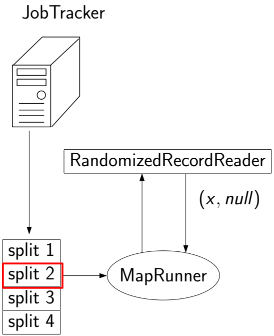

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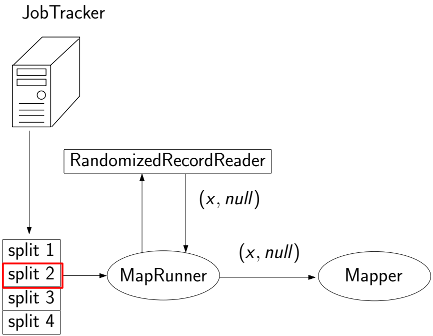
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



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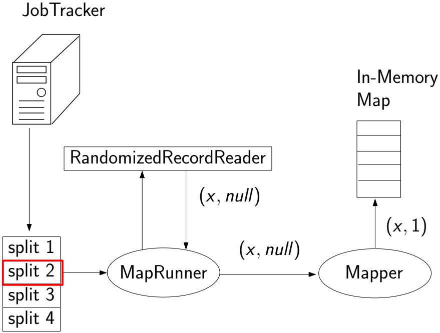

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Approximate Top-k Wavelet Coefficients: Basic Random Sampling

n_j= records in split j
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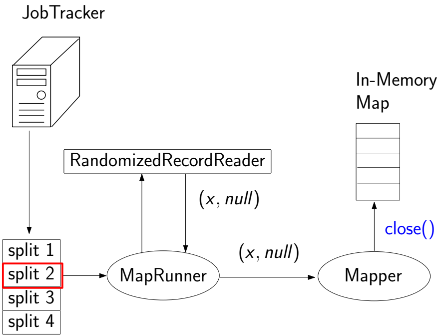

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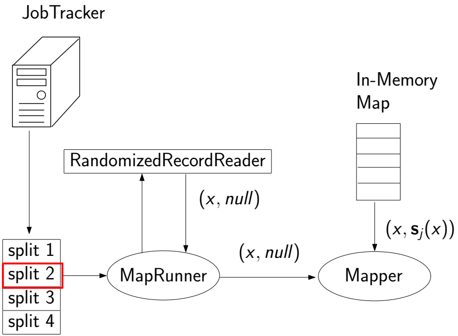

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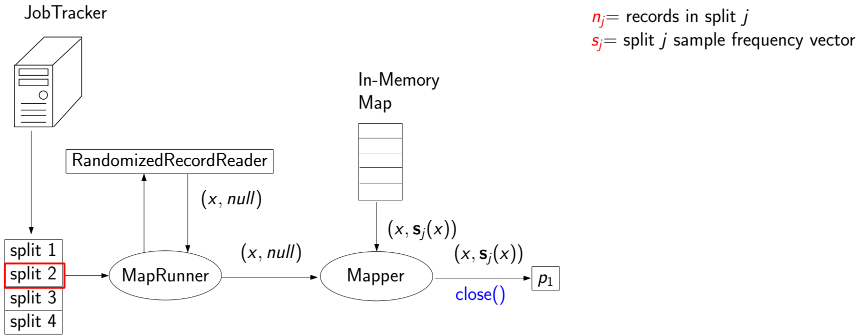
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



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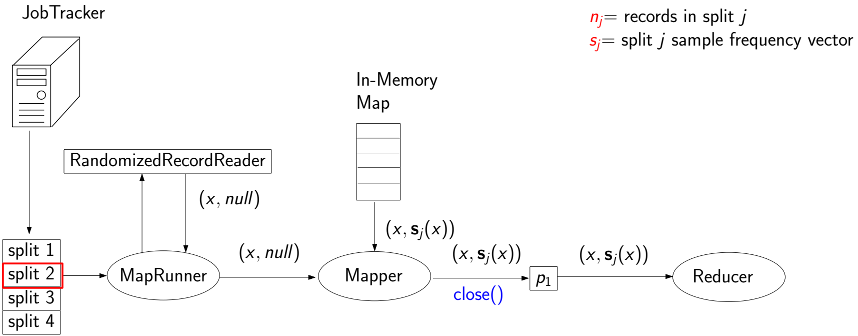

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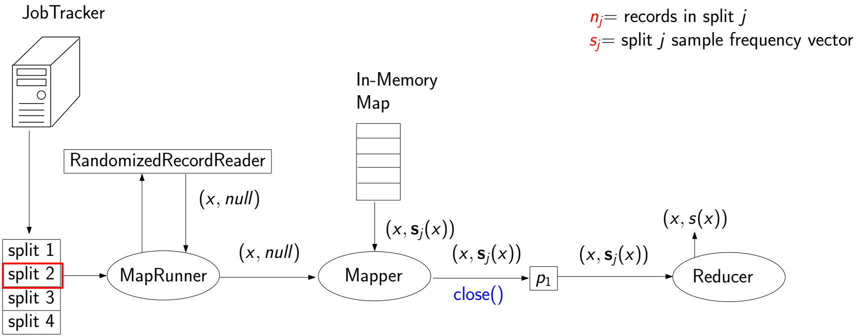

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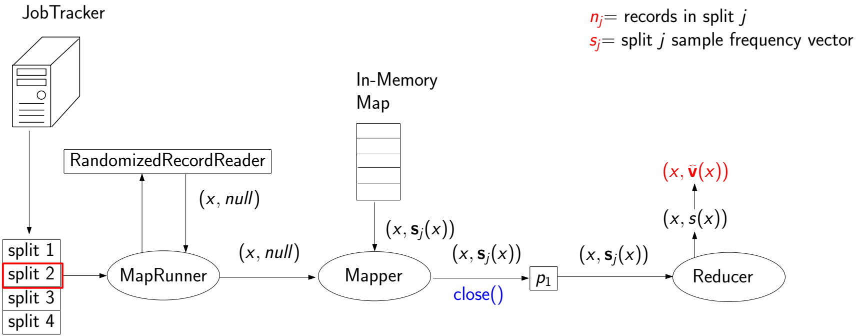
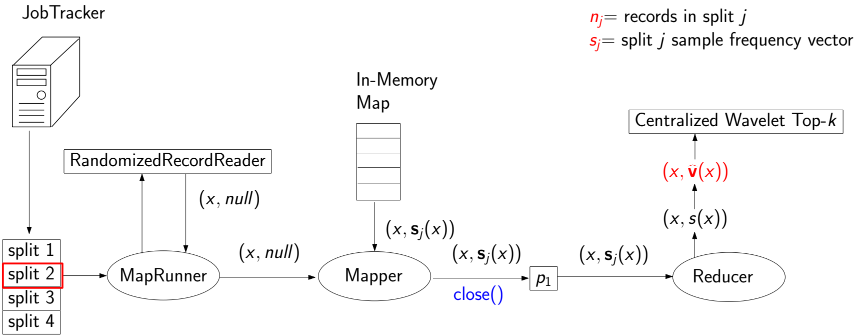
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



1. RandomizedRecordReader j (RR_j) samples $n_j/\varepsilon^2 n$ records.
   1. RR_j randomly selects $n_j/\varepsilon^2 n$ offsets in split j.
   2. RR_j sorts the offsets in ascending order then seeks the record at each sampled offset.
2. Reducer uses $\widehat{\mathbf{v}}(x) = \mathbf{s}(x)/p$, our unbiased estimator for $\mathbf{v}(x)$.

Approximate Top-k Wavelet Coefficients: Basic Random Sampling



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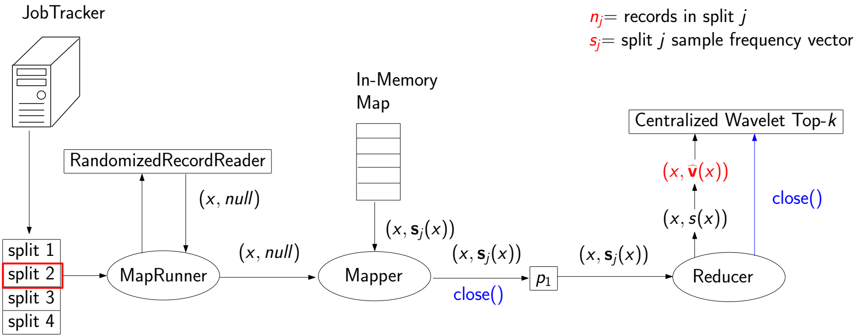
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



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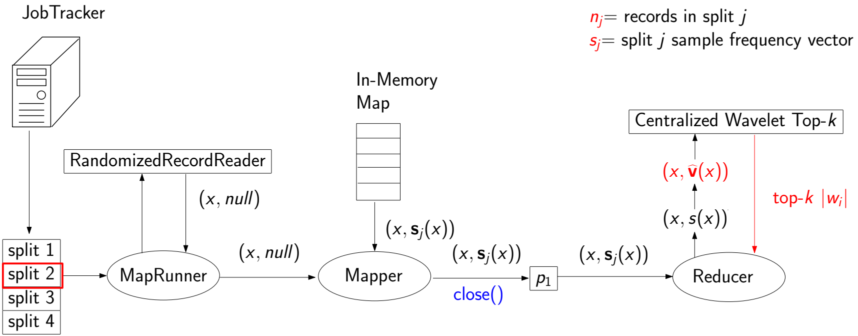
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



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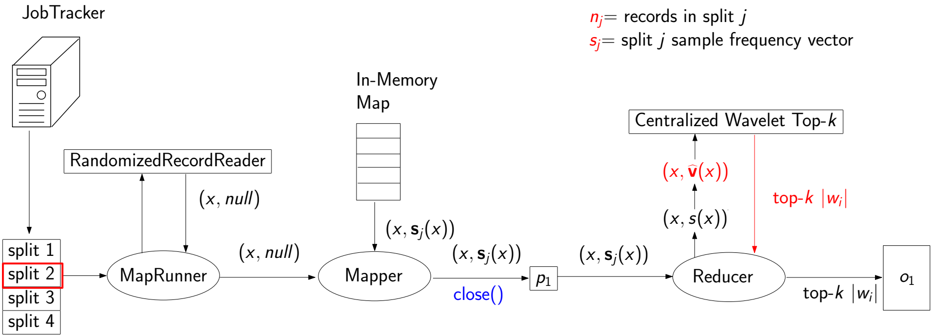
Approximate Top-k Wavelet Coefficients: Basic Random Sampling



1. RandomizedRecordReader j (RR_j) samples $n_j/\varepsilon^2 n$ records.
   1. RR_j randomly selects $n_j/\varepsilon^2 n$ offsets in split j.
   2. RR_j sorts the offsets in ascending order then seeks the record at each sampled offset.
2. Reducer uses $\widehat{\mathbf{v}}(x) = \mathbf{s}(x)/p$, our unbiased estimator for $\mathbf{v}(x)$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

Theorem

$\widehat{\mathbf{s}}(x)$ *is an unbiased estimator of* $\mathbf{s}(x)$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

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Proof.

1. Our estimator is $\widehat{\mathbf{s}}(x) = \rho(x) + M/\varepsilon\sqrt{m}$.

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  3. $\mathbf{E}[M] = \sum_{j=1}^{m'} \varepsilon\sqrt{m} \cdot \mathbf{s}_j(x) = \varepsilon\sqrt{m}(\mathbf{s}(x) - \rho(x))$.

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   - Let $M = \sum_{j=1}^{m'} X_j$.
     3. $\mathbf{E}[M] = \sum_{j=1}^{m'} \varepsilon\sqrt{m} \cdot \mathbf{s}_j(x) = \varepsilon\sqrt{m}(\mathbf{s}(x) - \rho(x))$.
4. $\mathbf{E}[\widehat{\mathbf{s}}(x)] = \mathbf{E}[\rho(x) + M/\varepsilon\sqrt{m}]$

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□

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

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$\widehat{\mathbf{s}}(x)$ *is an estimator of* $\mathbf{s}(x)$ *with standard deviation at most* $1/\varepsilon$.

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□

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

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The expected total communication cost of our two-level sampling algorithm is $O(\sqrt{m}/\varepsilon)$.

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The expected total communication cost of our two-level sampling algorithm is $O(\sqrt{m}/\varepsilon)$.

Proof.

1. Our estimator is $\widehat{\mathbf{s}}(x) = \rho(x) + M/\varepsilon\sqrt{m}$.

- The first-level sample size is $pn = 1/\varepsilon^2$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

Theorem

The expected total communication cost of our two-level sampling algorithm is $O(\sqrt{m}/\varepsilon)$.

Proof.

1. Our estimator is $\widehat{\mathbf{s}}(x) = \rho(x) + M/\varepsilon\sqrt{m}$.

- The first-level sample size is $pn = 1/\varepsilon^2$.
- If $\mathbf{s}_j(x) \geq 1/(\varepsilon\sqrt{m})$ we emit $(x, s_j(x))$.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

Theorem

The expected total communication cost of our two-level sampling algorithm is $O(\sqrt{m}/\varepsilon)$.

Proof.

1. Our estimator is $\widehat{\mathbf{s}}(x) = \rho(x) + M/\varepsilon\sqrt{m}$.

- The first-level sample size is $pn = 1/\varepsilon^2$.
- If $\mathbf{s}_j(x) \geq 1/(\varepsilon\sqrt{m})$ we emit $(x, s_j(x))$.
   2. There are $\leq (1/\varepsilon^2)/(1/\varepsilon\sqrt{m}) = \sqrt{m}/\varepsilon$ such keys.

Approximate Top-k Wavelet Coefficients: Two-Level Sampling

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The expected total communication cost of our two-level sampling algorithm is $O(\sqrt{m}/\varepsilon)$.

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  2. There are $\leq (1/\varepsilon^2)/(1/\varepsilon\sqrt{m}) = \sqrt{m}/\varepsilon$ such keys.
- If $s_j(x) < 1/(\varepsilon\sqrt{m})$,
  we emit $(x, null)$ with probability $\varepsilon\sqrt{m} \cdot \mathbf{s}_j x$.

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- If $s_j(x) < 1/(\varepsilon\sqrt{m})$,
  we emit $(x, null)$ with probability $\varepsilon\sqrt{m} \cdot \mathbf{s}_j x$.
  3. On expectation there are,
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  2. There are $\leq (1/\varepsilon^2)/(1/\varepsilon\sqrt{m}) = \sqrt{m}/\varepsilon$ such keys.
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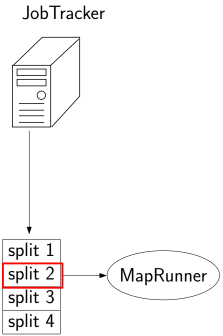
     $\sum_j \sum_x \varepsilon\sqrt{m} \cdot \mathbf{s}_j(x) \leq \varepsilon\sqrt{m} \cdot 1/\varepsilon^2 = \sqrt{m}/\varepsilon$.
- By (2) and (3), the total number of emitted keys is $O(\sqrt{m}/\varepsilon)$.

□

Approximate Top-k Wavelet Coefficients: Improved Sampling



n_j= records in split j
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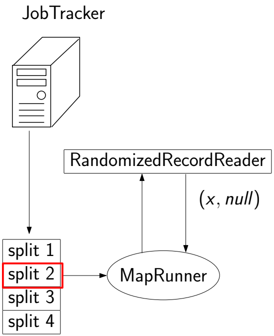

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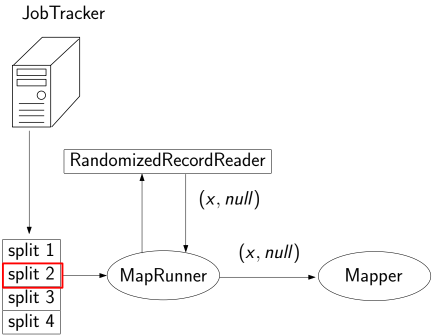

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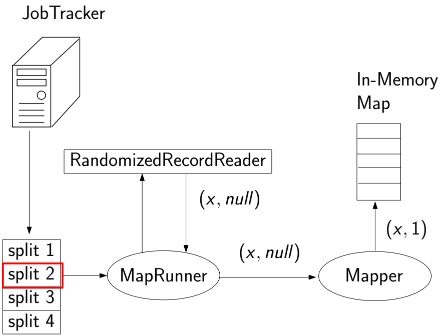

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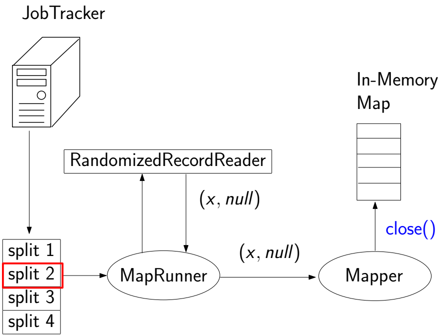

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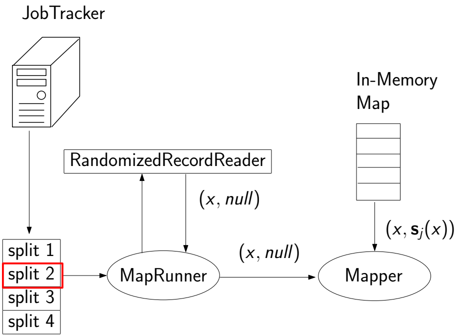

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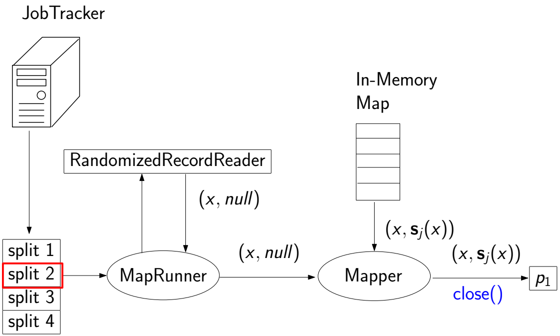

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2. If $\mathbf{s}_j(x) > \varepsilon t_j$, the Mapper emits $(x, \mathbf{s}_j(x))$.

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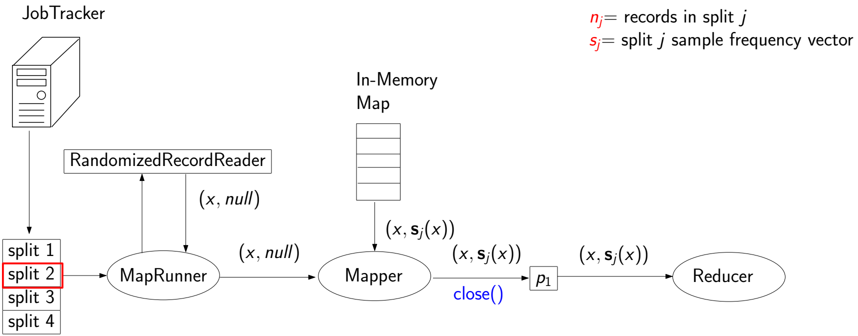

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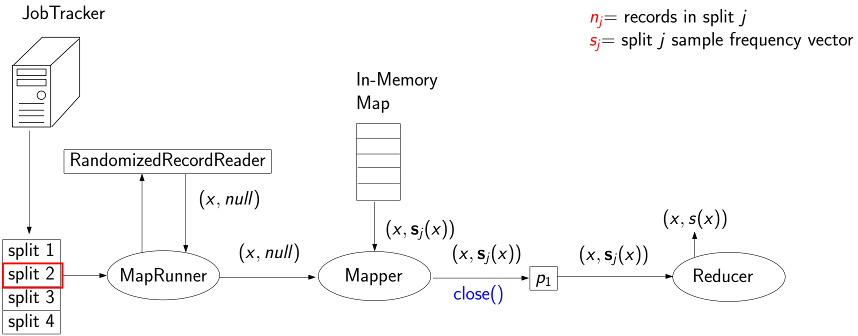

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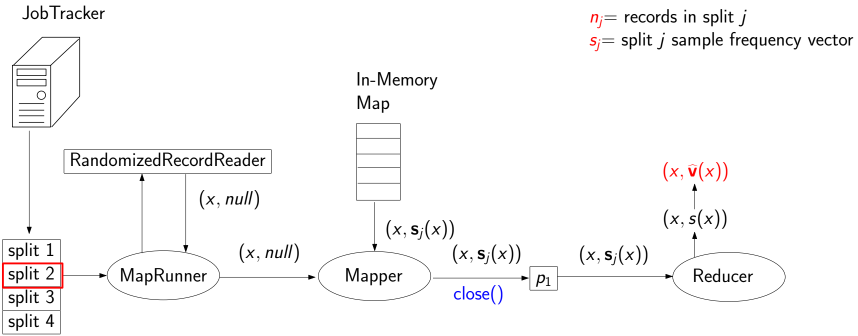

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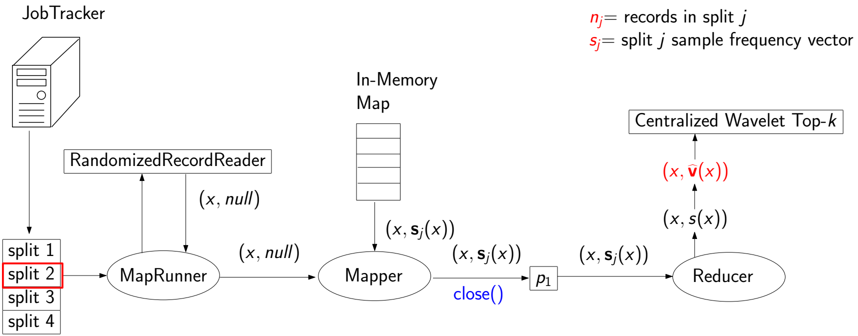

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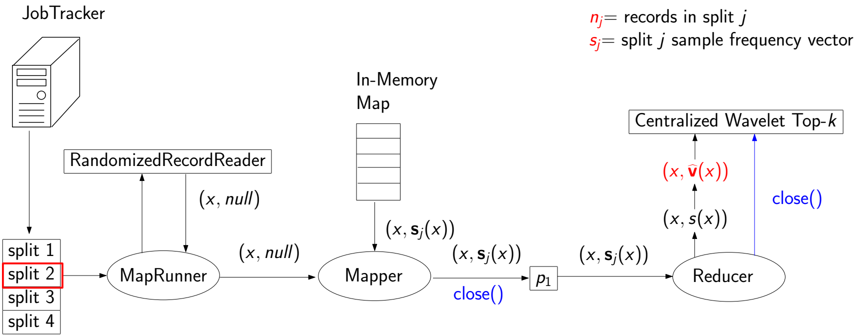

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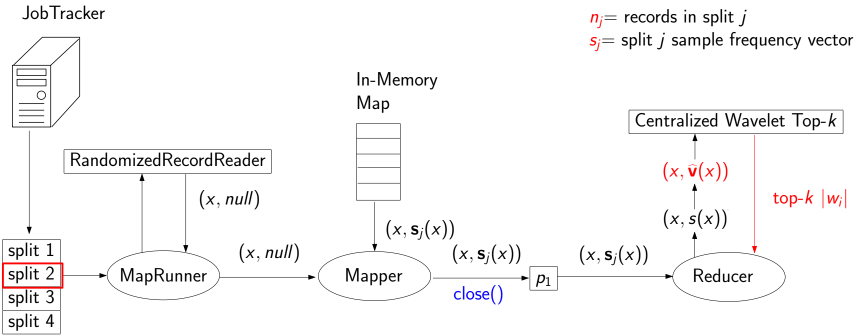

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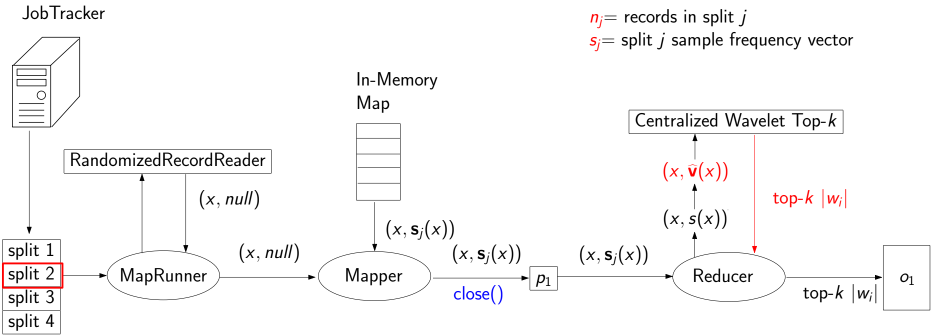

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