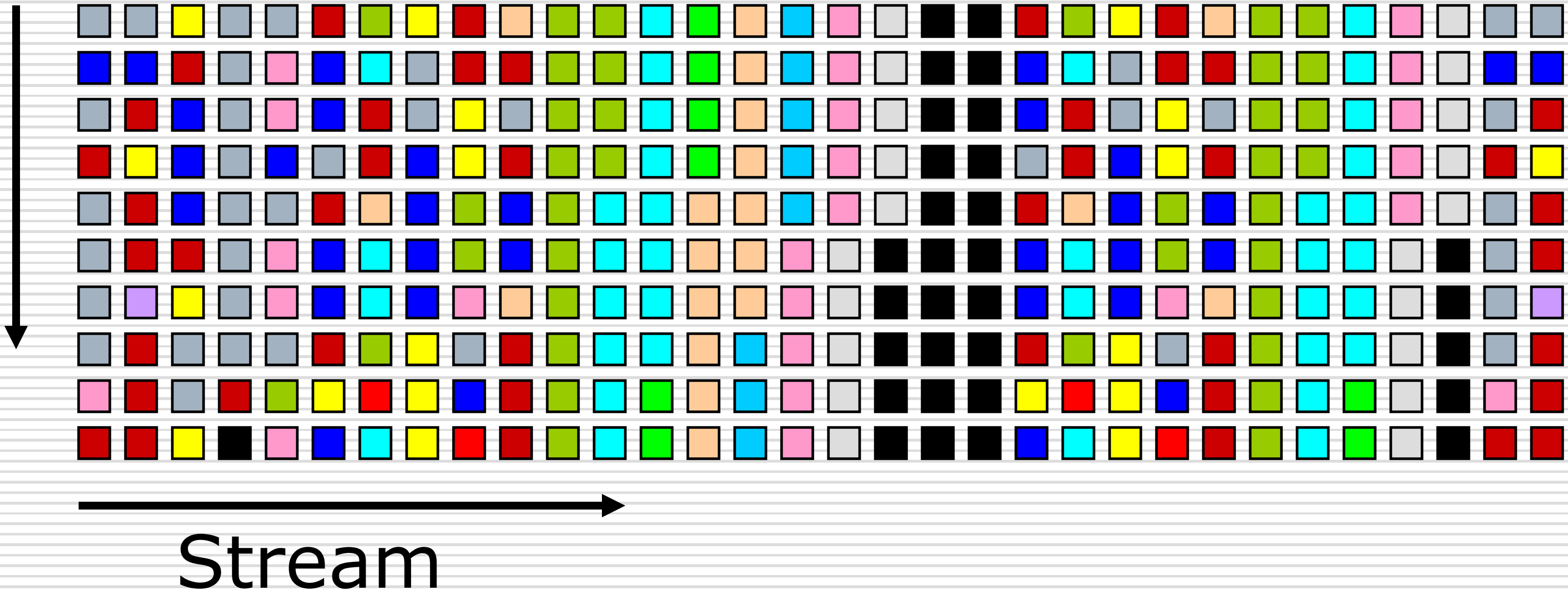
Frequency Counts over Data Streams

The Problem ...

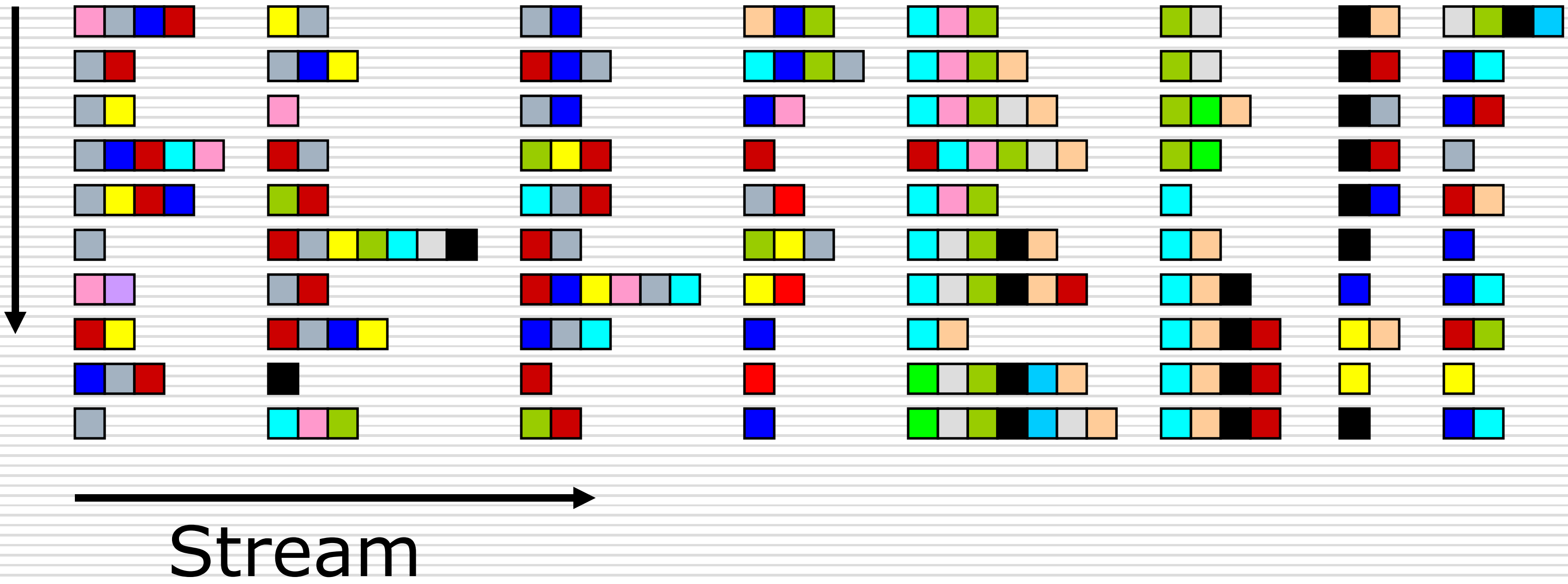


- Identify all elements whose current frequency exceeds support threshold $s = 0.1\%$.

Formal Definition

- All item whose true frequency exceeds sN are output. There are no false negatives.
- No item whose true frequency is less than (s- ε)N is output.
- Estimated frequencies are less than the true frequencies by at most εN!

A Related Problem ...



- Identify all subsets of items whose current frequency exceeds s = 0.1%.

Frequent Itemsets / Association Rules

Applications

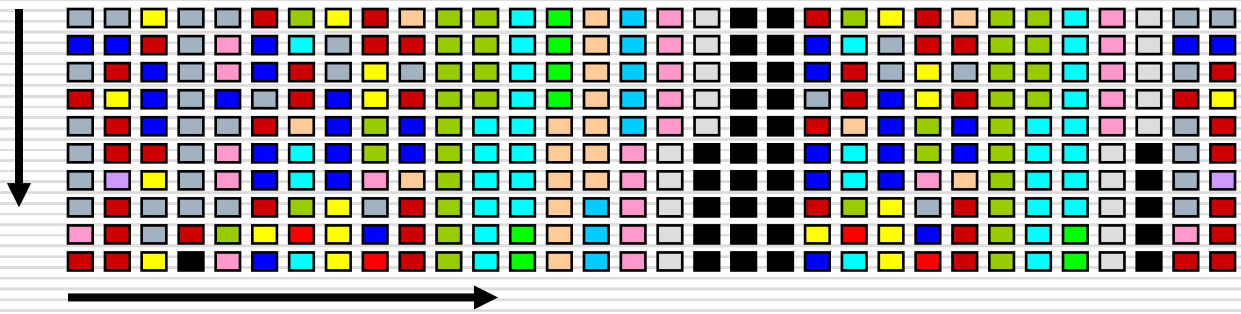
Flow Identification at IP Router [EV01]

Iceberg Queries [FSGM+98]

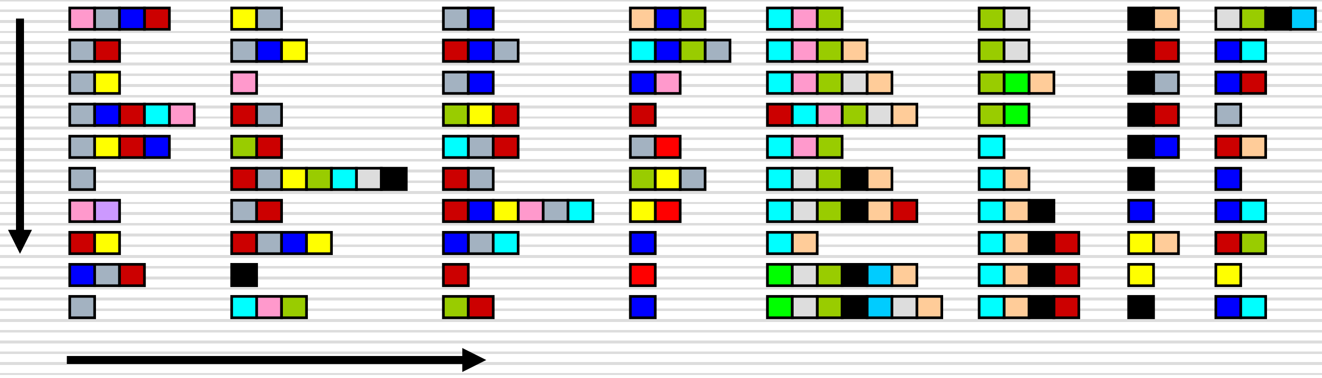
Iceberg Datacubes [BR99 HPDW01]

Association Rules & Frequent Itemsets [AS94 SON95 Toi96 Hid99 HPY00 …]

Presentation Outline ...



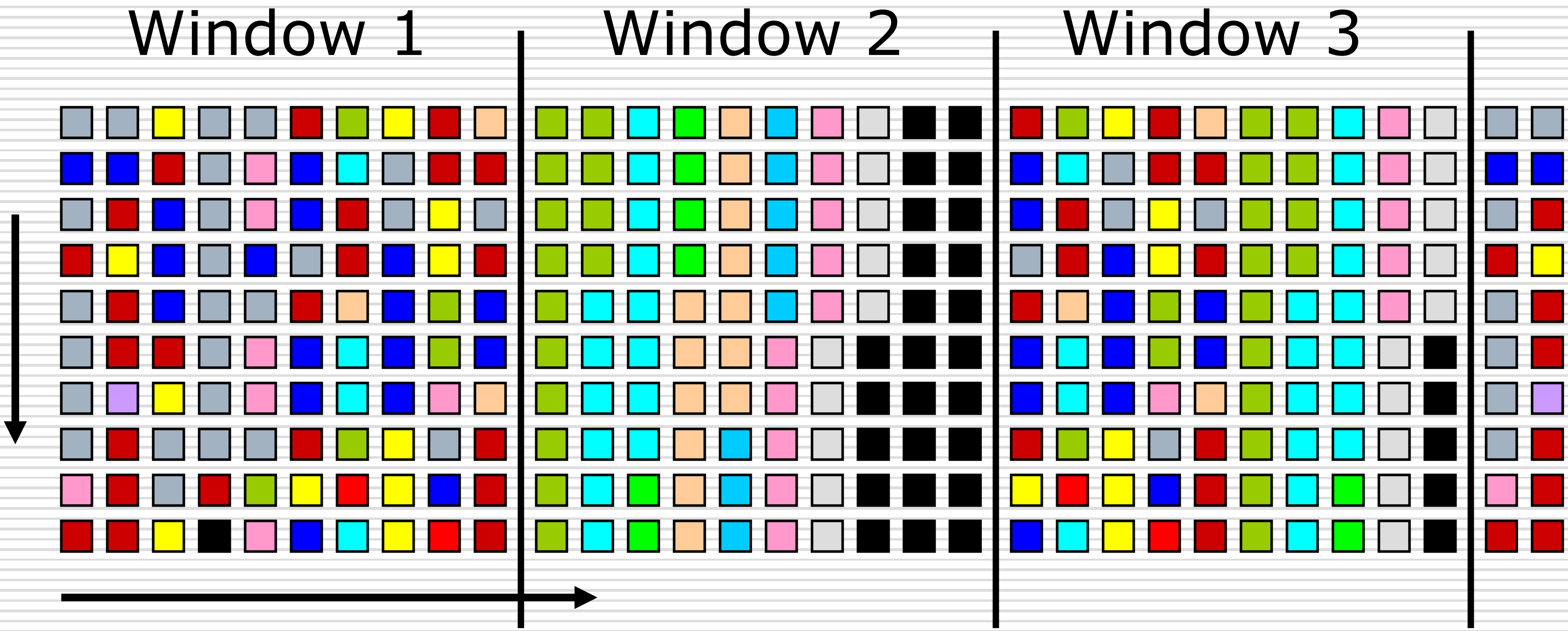
1. Lossy Counting 2. Sticky Sampling



3. Algorithm for Frequent Itemsets

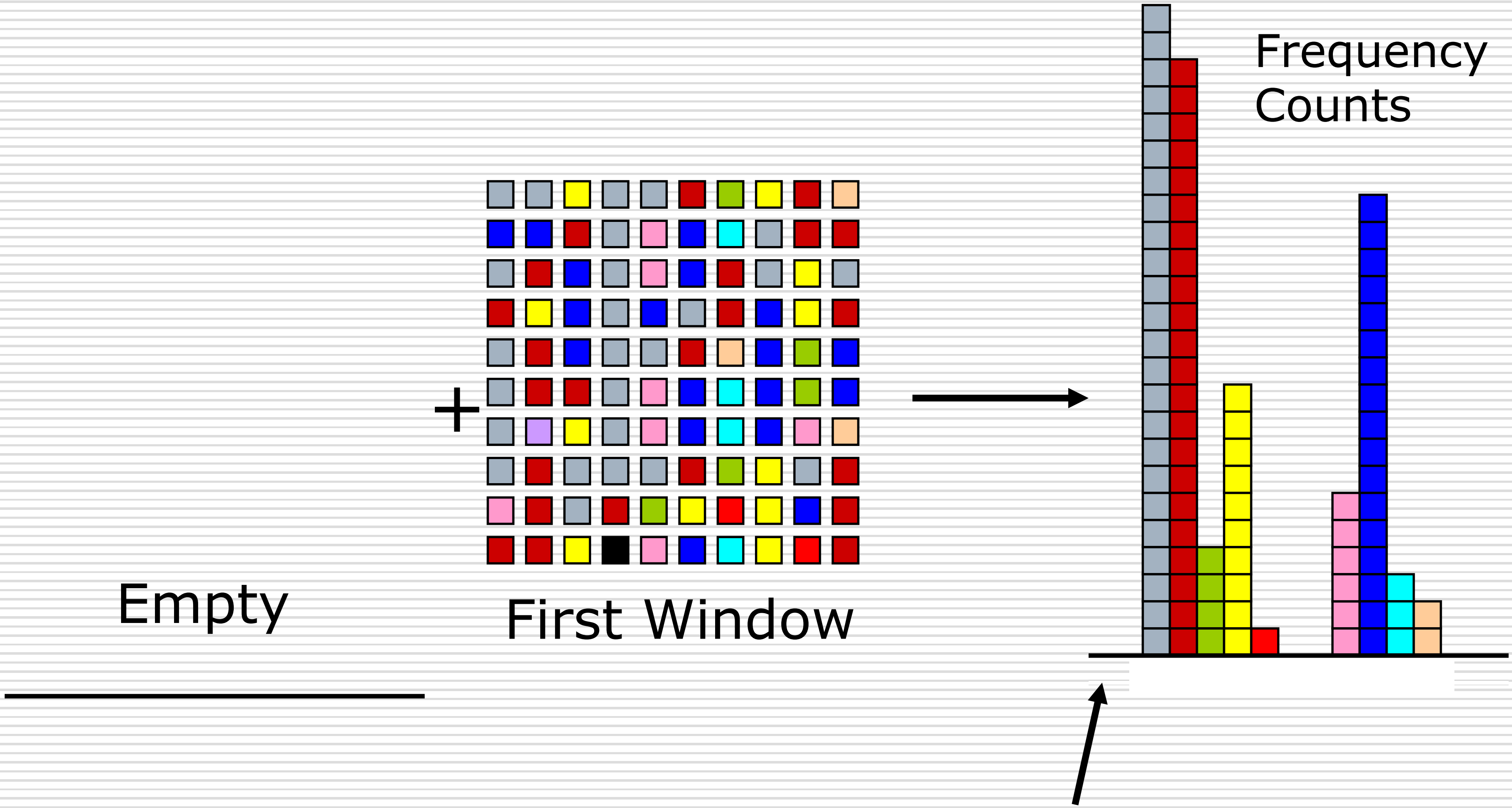
Algorithm 1: Lossy Counting

Step 1: Divide the stream into 'windows'



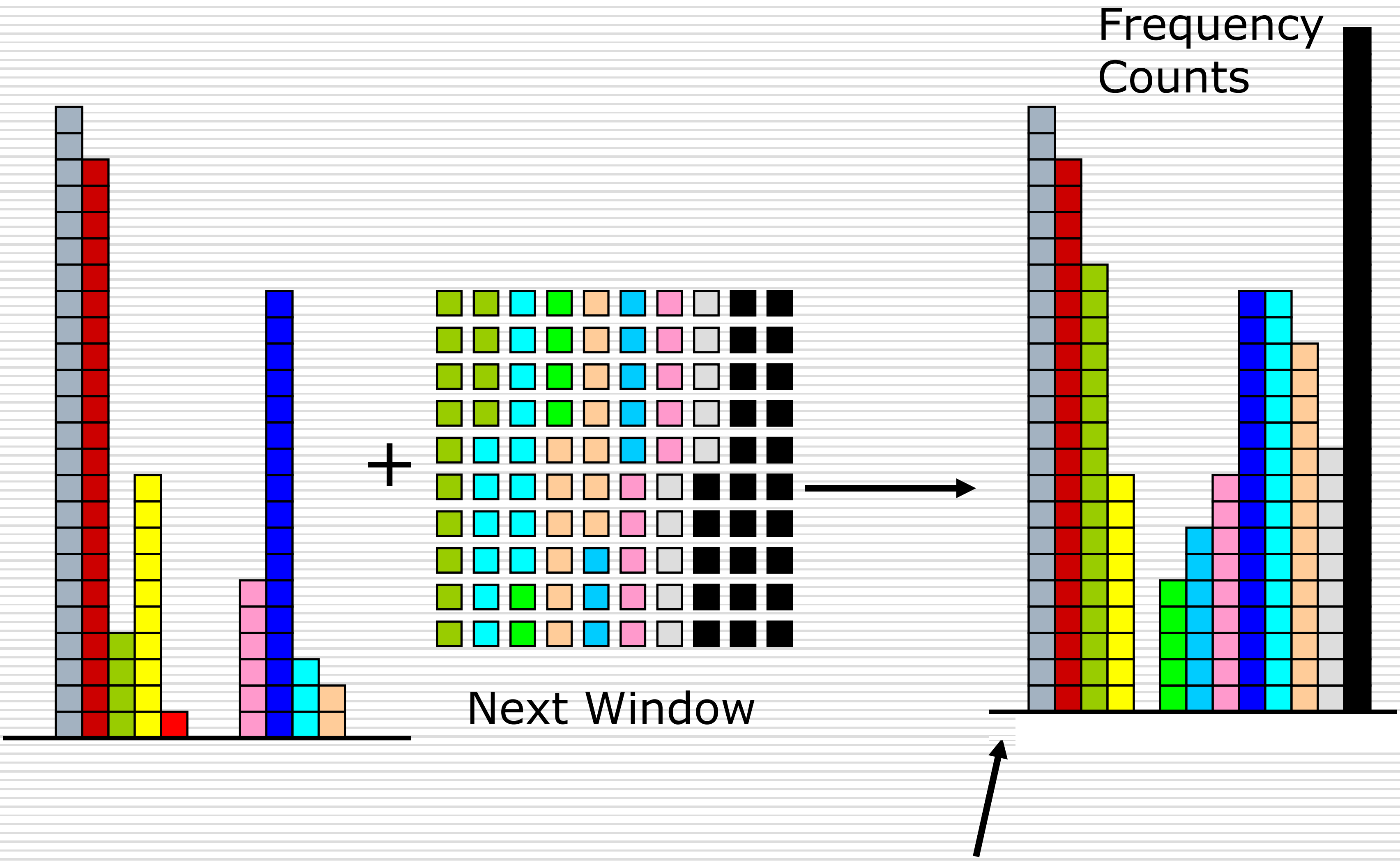
Is window size a function of support s? Will fix later…

Lossy Counting in Action ...



At window boundary, decrement all counters by 1

Lossy Counting continued ...



At window boundary, decrement all counters by 1

Error Analysis

How much do we undercount?

If	current size of stream	$= N$
and	window-size	$= 1/\varepsilon$

then frequency error $\leq$ #windows $= \varepsilon N$

Rule of thumb:

Set ε = 10% of support s

Example:

Given support frequency $s = 1\%$,

set error frequency $\varepsilon = 0.1\%$

Output:

Elements with counter values exceeding $sN - \varepsilon N$

Approximation guarantees

- Frequencies underestimated by at most εN
- No false negatives
- False positives have true frequency at least $sN - \varepsilon N$

How many counters do we need?

Worst case: $1/\varepsilon \log(\varepsilon N)$ counters (see proof later)

Enhancements ...

Frequency Errors

For counter (X, c), true frequency in [c, c+εN]

Trick: Remember window-id's

For counter (X, c, w), true frequency in [c, c+w-1]

If (w = 1), no error!

At the time of deletion, the true frequency of X is at most w which is less than εN_w! (N_w is the number of elements seen so far in the stream at the end of window w)

Batch Processing

Decrements after k windows

The Enhanced Algorithm

0. The data structure we keep is D, which has entries in the form of (X, c, Δ);

1. In window b, an element X appears, if X is in D, increase its count c by 1; otherwise, insert (X, 1, b-1) into D;

2. At the end of window b, delete all entries if their $c+ \Delta \leq b$.

Worse Case Bound

How many counters do we need?

Worst case: 1/ε log (ε N) counters

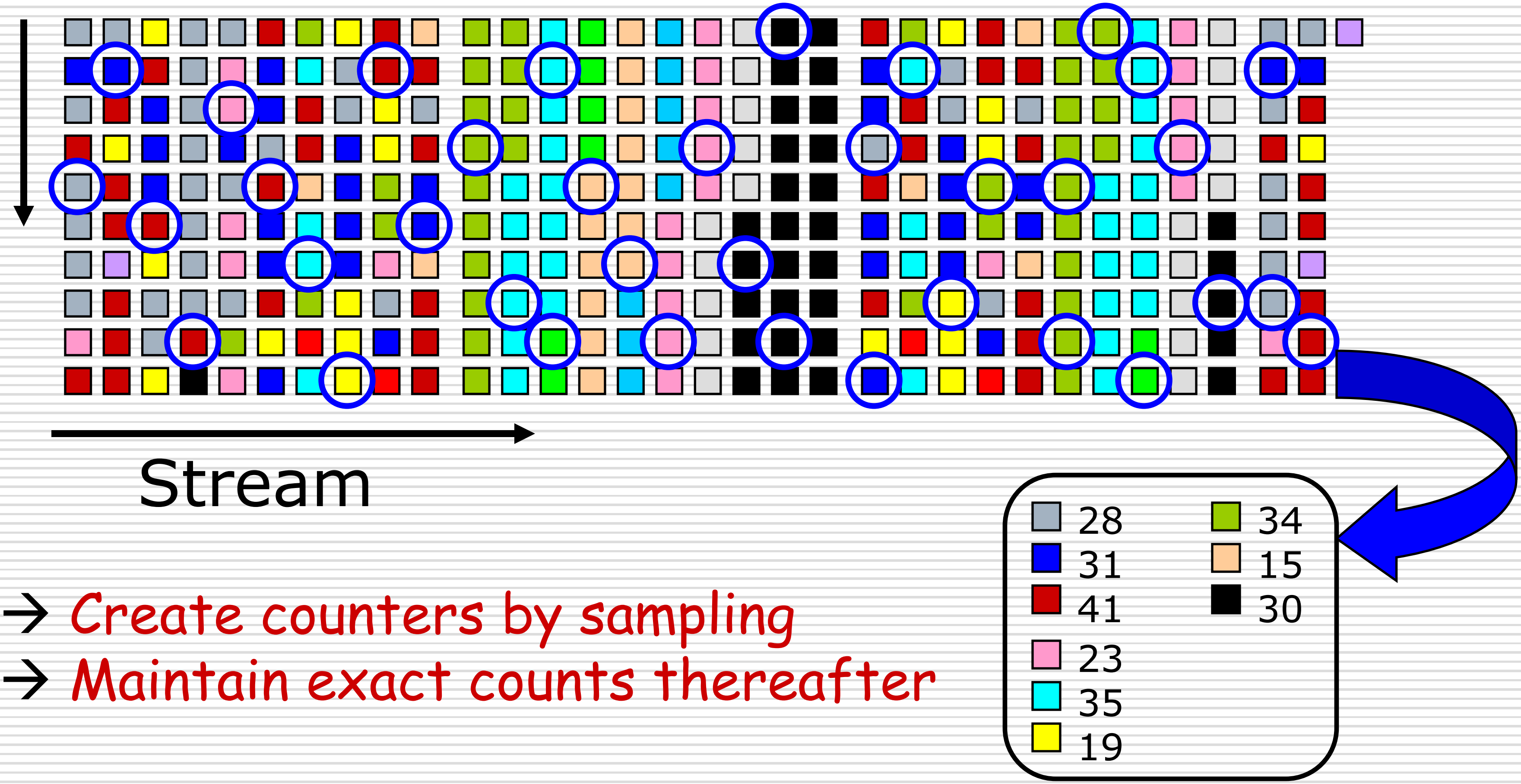
Let B be the current bucket id, d_i denote the number of entries we kept whose bucket id is B-i+1 for i\in [1, B]: The item corresponding to such entry must occur at least i times in bucket B-i+1 through B;

$$\sum_{i=1}^{j} i \times d_i \le jw, \text{ for } j = 1,2,\ldots B.$$

$$\text{We claim}: \sum_{i=1}^{j} d_i \le \sum_{i=1}^{j} w/i, \text{ for } j = 1,2,\ldots B$$

Prove By Induction.

Algorithm 2: Sticky Sampling



→ *Create counters by sampling*

→ *Maintain exact counts thereafter*

What rate should we sample?

Sticky Sampling contd...

For finite stream of length N

Sampling rate = $2/N\varepsilon \log 1/(s\delta)$

δ = probability of failure

Output:

Elements with counter values exceeding $sN - \varepsilon N$

Approximation guarantees (probabilistic)

- Frequencies underestimated by at most εN
- No false negatives
- False positives have true frequency at least $sN - \varepsilon N$

Same error guarantees as Lossy Counting but probabilistic

Same Rule of thumb:

Set ε = 10% of support s

Example:

Given support threshold s = 1%,
set error threshold ε = 0.1%
set failure probability δ = 0.01%

Sampling rate?

Finite stream of length N

Sampling rate: $2/N\varepsilon \ \log 1/(s\delta)$

Infinite stream with unknown N

Gradually adjust sampling rate (discussed later)

In either case,

Expected number of counters = $2/\varepsilon \log 1/s\delta$

Independent of N!

Infinite Stream

Let $t=1/\varepsilon \log(1/s\delta)$
First 2t, sample rate =1
Next 2t, sample rate =1/2
Next 4t, sample rate=1/4
Next 8t, sample rate=1/8

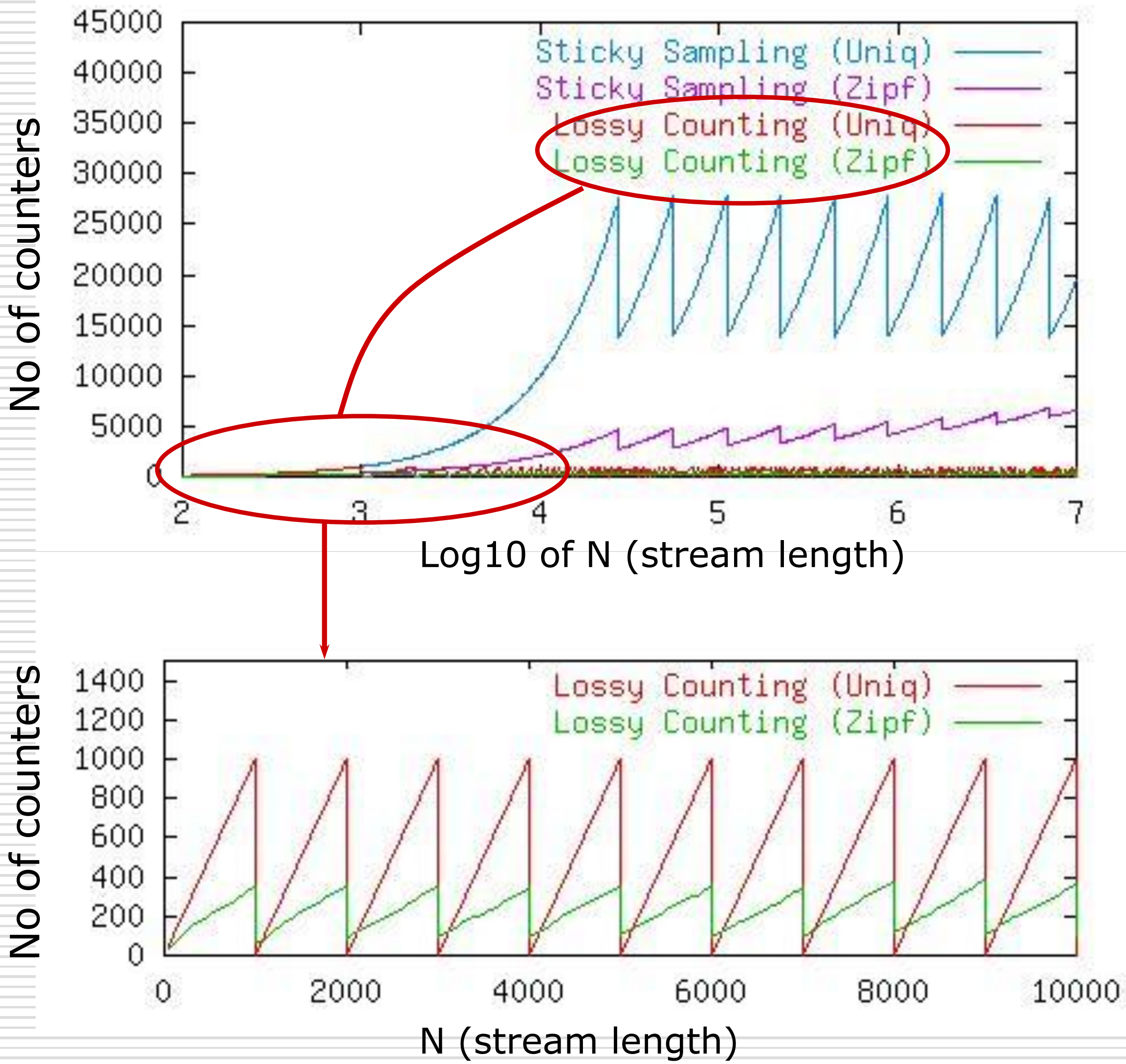
And, whenever sample rate changes, do:
For each entry kept, flip coin (with p=1/2) continuously,
Until a head appears, for each tail, decrease the count
Of the entry by 1; remove the entry when its count is 0.

Result (proof in class)

Sticky sampling satisfies our requirement with Probability at least 1- δ using at most $2/\varepsilon \log(1/s\delta)$ expected number of entries.

Proof: In class, Please take notes.

Sticky Sampling Expected: 2/ε log 1/sδ
Lossy Counting Worst Case: 1/ε log εN



Support s = 1%
Error ε = 0.1%

From elements

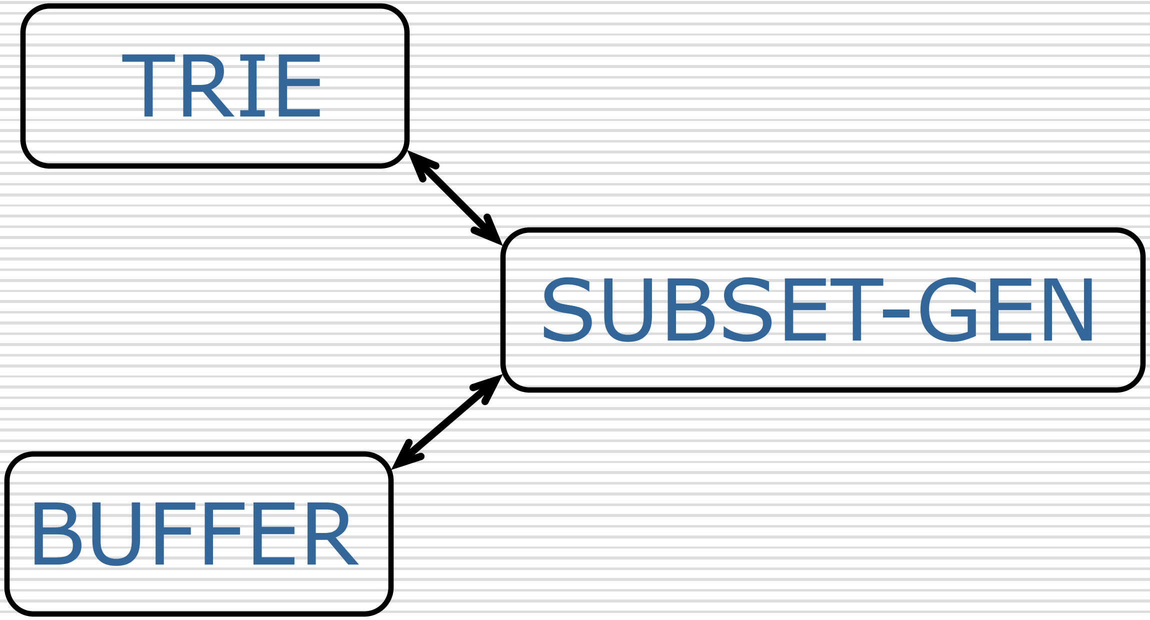
to *sets* of elements …

Frequent Itemsets Problem ...

- Identify all subsets of items whose current frequency exceeds $s = 0.1\%$.

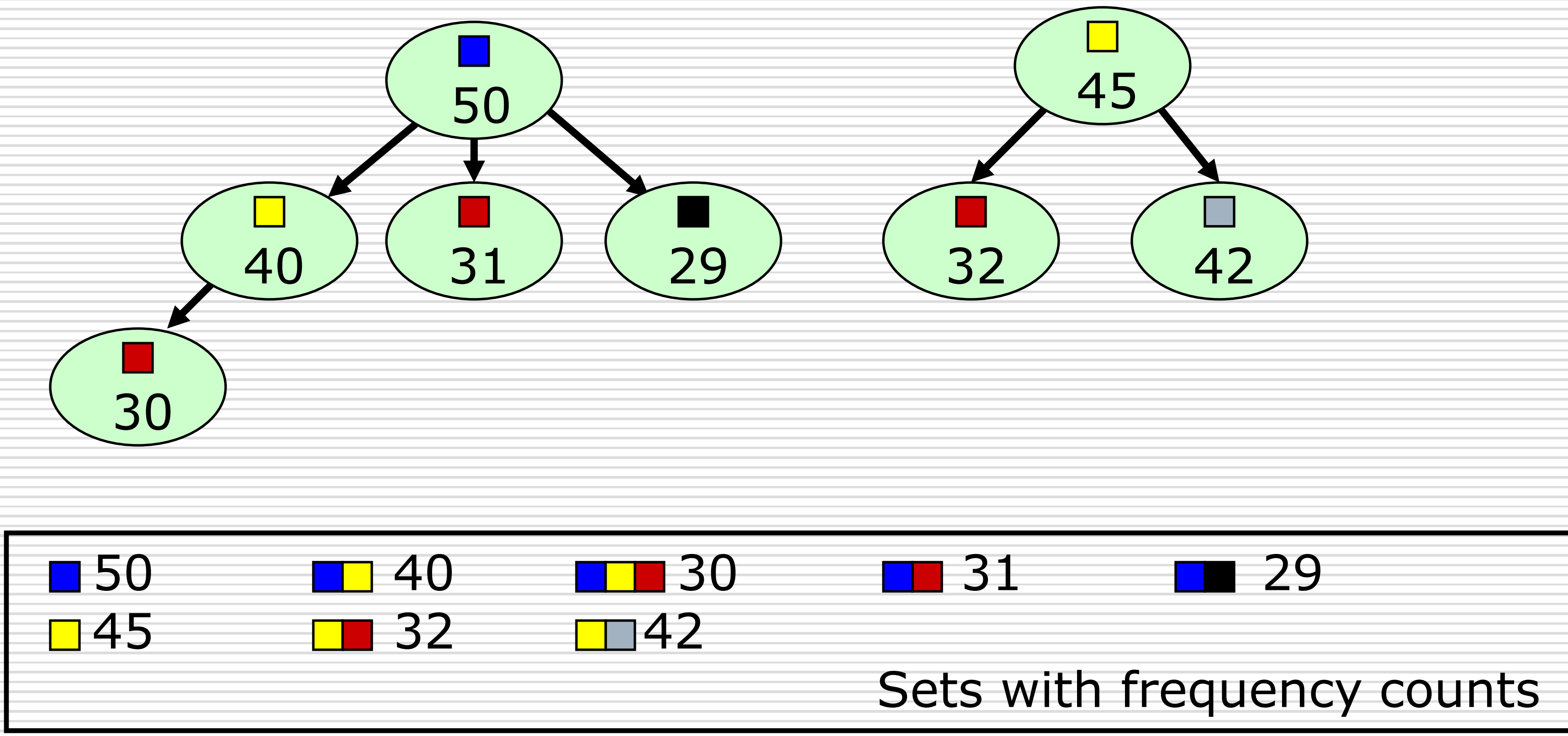
Frequent Itemsets => Association Rules

Three Modules

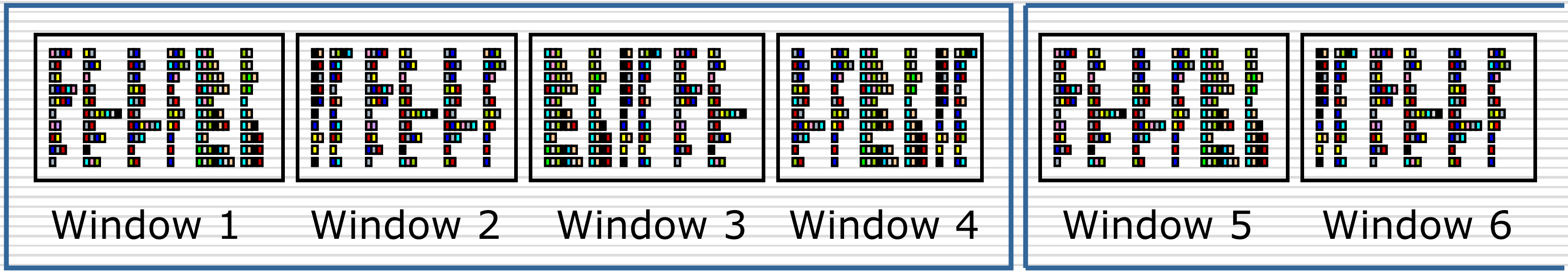


Module 1: TRIE

Compact representation of frequent itemsets in lexicographic order.



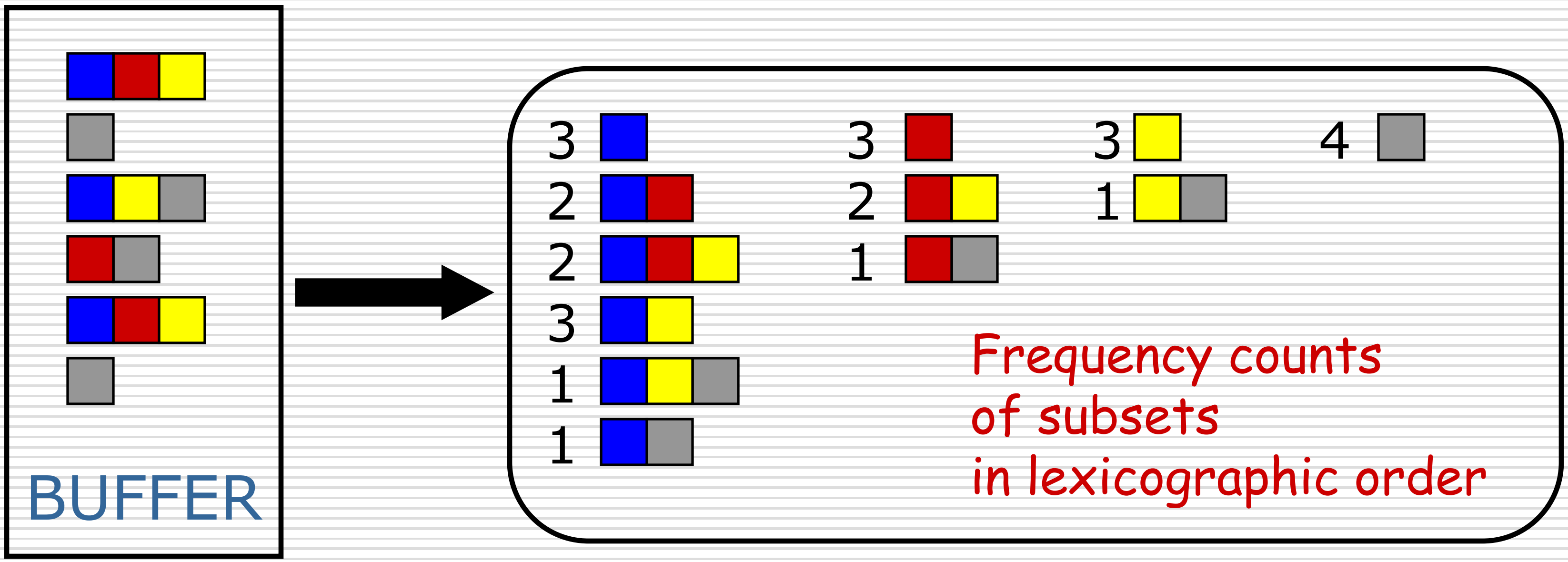
Module 2: BUFFER



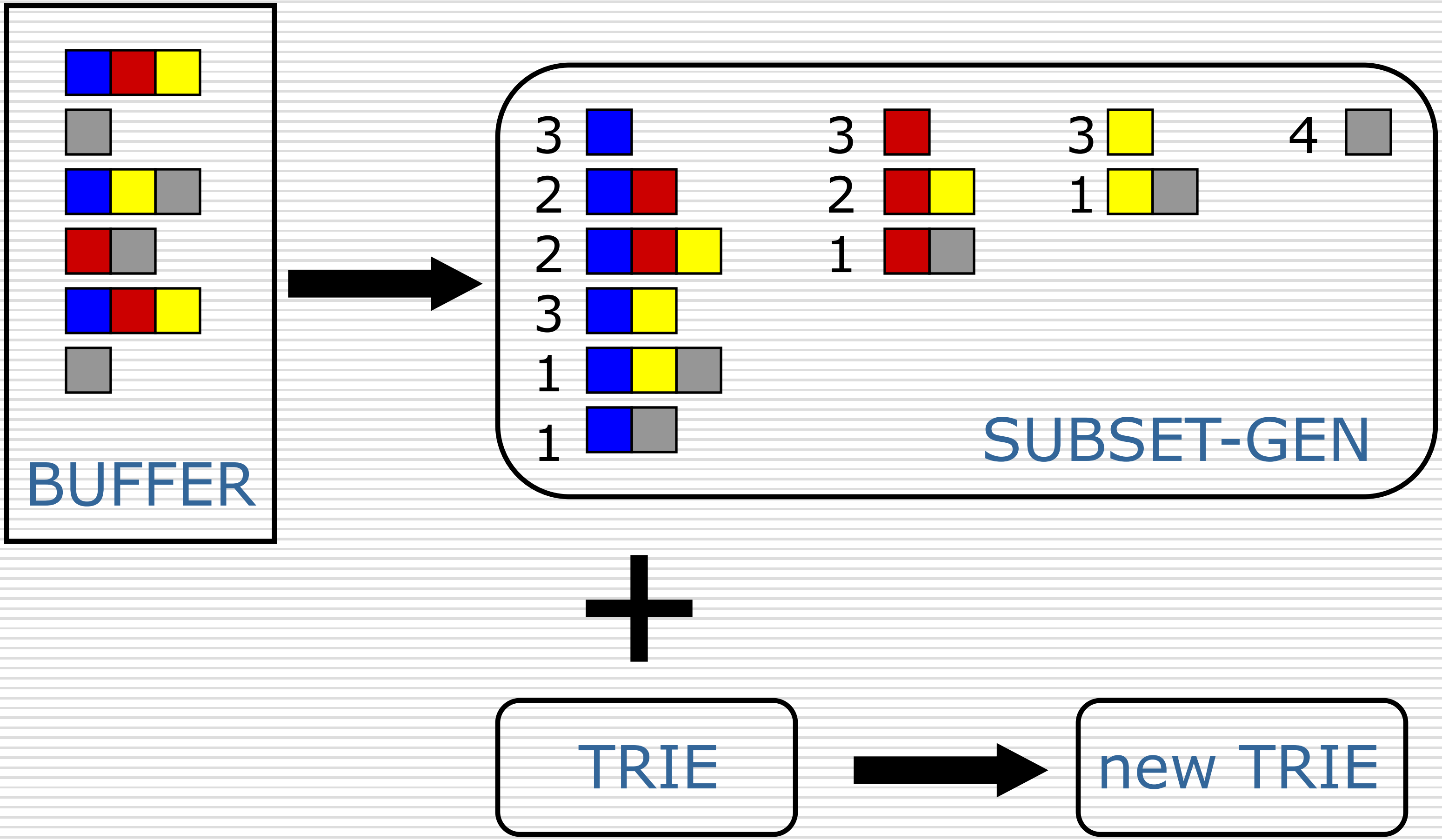
In Main Memory

Compact representation as sequence of ints
Transactions sorted by item-id
Bitmap for transaction boundaries

Module 3: SUBSET-GEN



Overall Algorithm …



Problem: Number of subsets is exponential!

SUBSET-GEN Pruning Rules

A-priori Pruning Rule

If set S is infrequent, every superset of S is infrequent.

Lossy Counting Pruning Rule

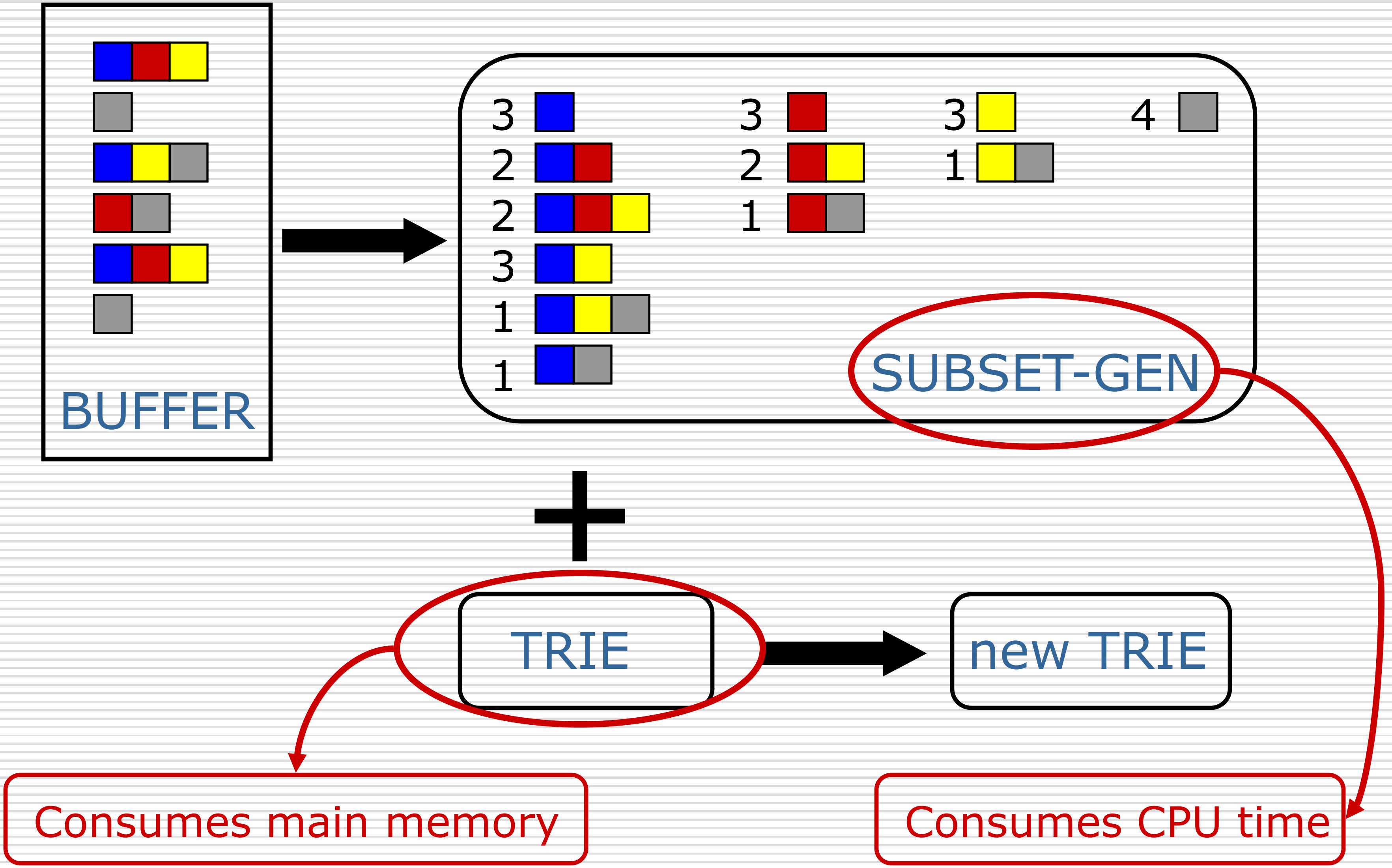
At each 'window boundary' decrement TRIE counters by 1.

Actually, 'Batch Deletion':

At each 'main memory buffer' boundary,
decrement all TRIE counters by b.

See paper for details ...

Bottlenecks …



Design Decisions for Performance

TRIE — Main memory bottleneck

- Compact linear array
  - → (element, counter, level) in preorder traversal
  - → No pointers!
- Tries are on disk
  - → All of main memory devoted to BUFFER
- Pair of tries
  - → old and new (in chunks)
- mmap() and madvise()

SUBSET-GEN — CPU bottleneck

- Very fast implementation
  - → See paper for details

Experiments …

IBM synthetic dataset T10.I4.1000K

N = 1Million   Avg Tran Size = 10   Input Size = 49MB

IBM synthetic dataset T15.I6.1000K

N = 1Million   Avg Tran Size = 15   Input Size = 69MB

Frequent word pairs in 100K web documents

N = 100K   Avg Tran Size = 134   Input Size = 54MB

Frequent word pairs in 806K Reuters newsreports

N = 806K   Avg Tran Size = 61   Input Size = 210MB

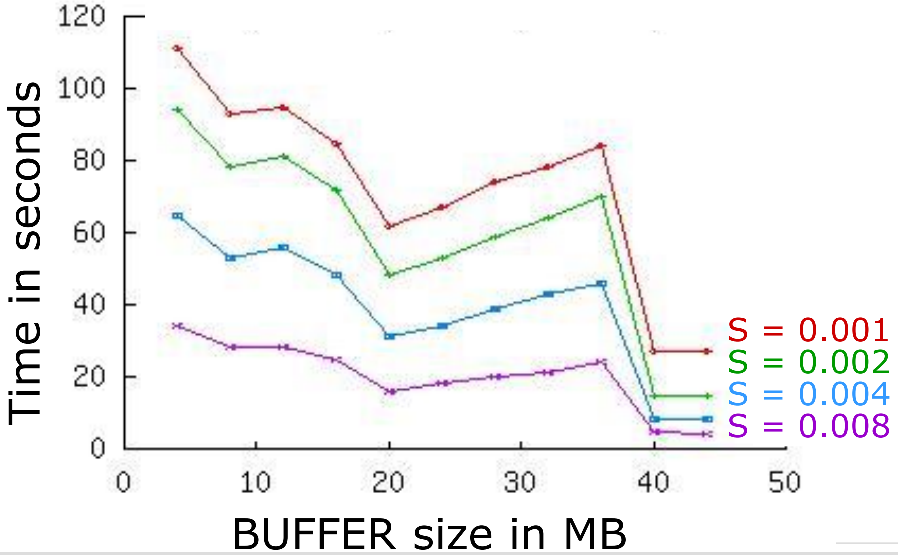
What do we study?

For each dataset

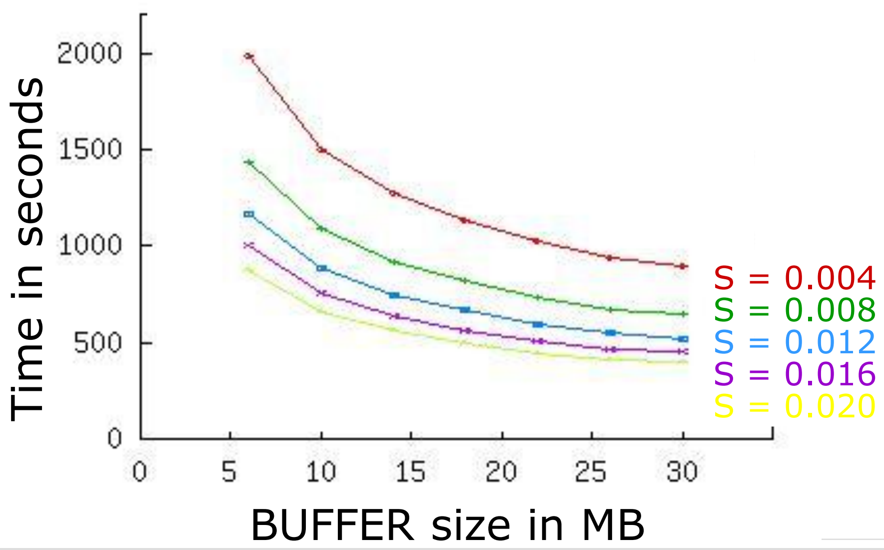
Support threshold	s		Three independent variables Fix one and vary two
Length of stream	N		
BUFFER size	B		
Time taken	t		Measure time taken

$\boxed{\text{Set } \varepsilon = 10\% \text{ of support } s}$

Varying support s and BUFFER B



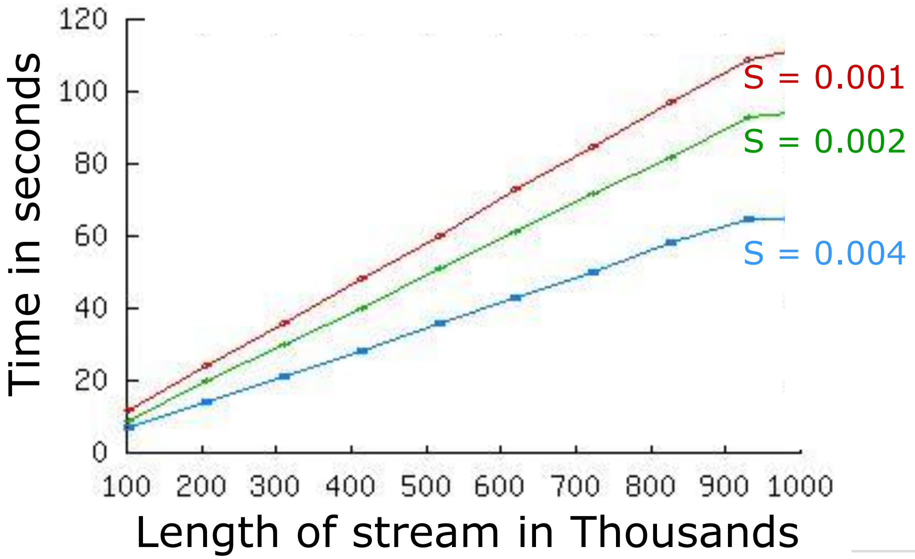
IBM 1M transactions



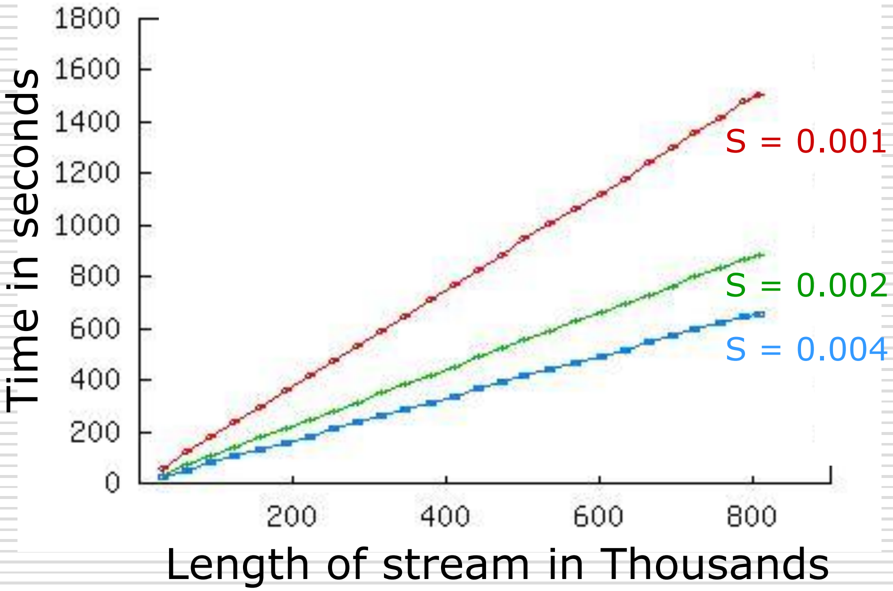
Reuters 806K docs

Fixed:	Stream length	N
Varying:	BUFFER size	B
	Support threshold	s

Varying length N and support s



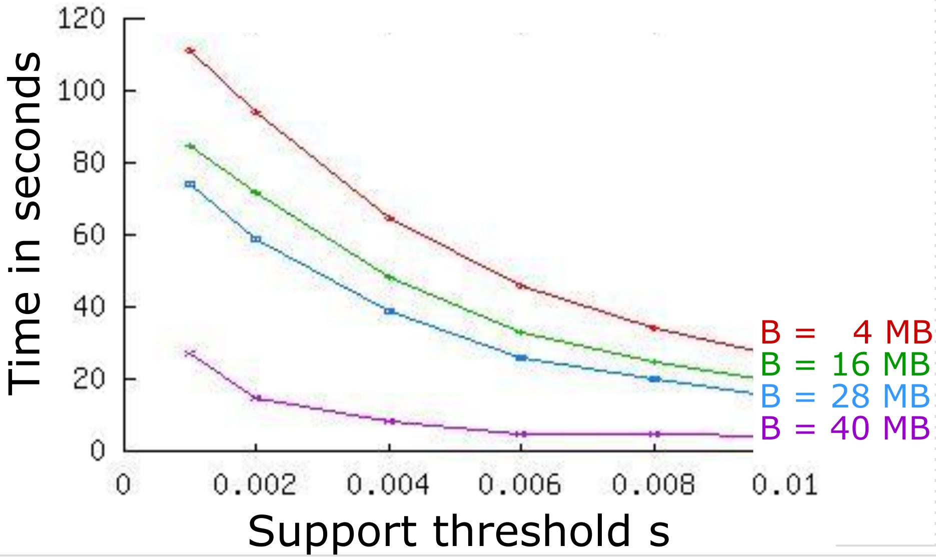
IBM 1M transactions



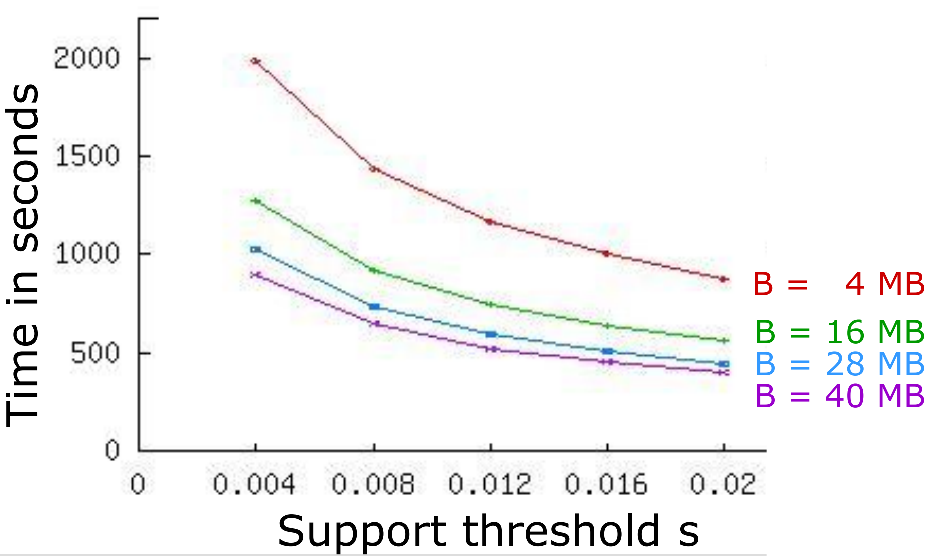
Reuters 806K docs

Fixed:	BUFFER size	B
Varying:	Stream length	N
	Support threshold	s

Varying BUFFER B and support s



IBM 1M transactions



Reuters 806K docs

Fixed:	Stream length	N
Varying:	BUFFER size	B
	Support threshold	s

Comparison with fast A-priori

	APriori		Our Algorithm with 4MB Buffer		Our Algorithm with 44MB Buffer	
Support	Time	Memory	Time	Memory	Time	Memory
0.001	99 s	82 MB	111 s	12 MB	27 s	45 MB
0.002	25 s	53 MB	94 s	10 MB	15 s	45 MB
0.004	14 s	48 MB	65 s	7MB	8 s	45 MB
0.006	13 s	48 MB	46 s	6 MB	6 s	45 MB
0.008	13 s	48 MB	34 s	5 MB	4 s	45 MB
0.010	14 s	48 MB	26 s	5 MB	4 s	45 MB

Dataset: IBM T10.I4.1000K with 1M transactions, average size 10.

A-priori by Christian Borgelt http://fuzzy.cs.uni-magdeburg.de/~borgelt/software.html

Comparison with Iceberg Queries

Query: Identify all word pairs in 100K web documents
which co-occur in at least 0.5% of the documents.

[FSGM+98] multiple pass algorithm:
7000 seconds with 30 MB memory

Our single-pass algorithm:
4500 seconds with 26 MB memory

Our algorithm would be much faster if allowed multiple passes!

Lessons Learnt ...

Optimizing for #passes is wrong!

Small support s $\Rightarrow$ Too many frequent itemsets!

Time to redefine the problem itself?

Interesting combination of Theory and Systems.

Other Interesting Work

Frequency Counts over Sliding Windows

Multiple pass Algorithm for Frequent Itemsets

Iceberg Datacubes

Summary

Lossy Counting: A Practical algorithm for online frequency counting.

First ever single pass algorithm for Association Rules with user specified error guarantees.

Basic algorithm applicable to several problems.

Sticky Sampling Expected: $2/\varepsilon \log 1/s\delta$
Lossy Counting Worst Case: $1/\varepsilon \log \varepsilon N$

But ...

ε	s	SS worst	LC worst	SS Zipf	LC Zipf	SS Uniq	LC Uniq
0.1%	1.0%	27K	9K	6K	419	27K	1K
0.05%	0.5%	58K	17K	11K	709	58K	2K
0.01%	0.1%	322K	69K	37K	2K	322K	10K
0.005%	0.05%	672K	124K	62K	4K	672K	20K

LC: Lossy Counting
SS: Sticky Sampling
Zipf: Zipfian distribution
Uniq: Unique elements