R-tree: Indexing Structure for Data in Multi-dimensional Space

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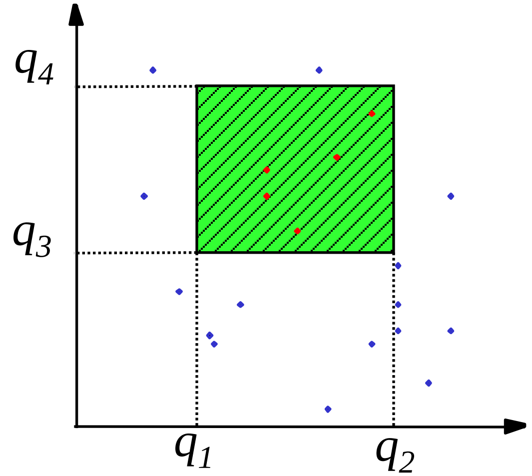
(Many slides made available by Ke Yi)

Until now: Data Structures



- General planer range searching (in 2-dimensional space):
  - kdB-tree: $O(\sqrt{N/B} + T/B)$ query, $O(\frac{N}{B})$ space

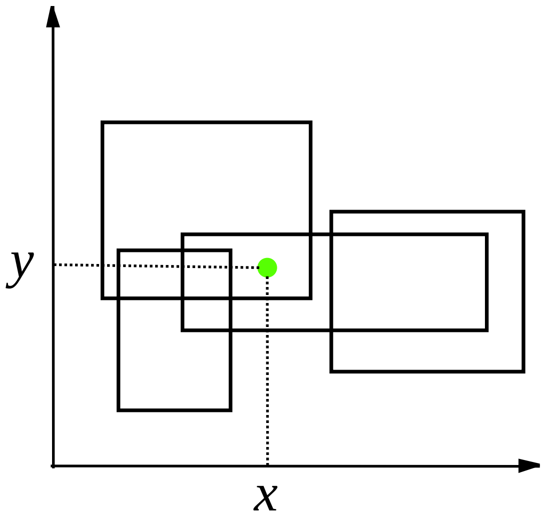
Other results

- Many other results for e.g.
  - Higher dimensional range searching
  - Range counting, range/stabbing max, and stabbing queries
  - Halfspace (and other special cases) of range searching
  - Queries on moving objects
  - Proximity queries (closest pair, nearest neighbor, point location)
  - Structures for objects other than points (bounding rectangles)

- Many heuristic structures in database community

Point Enclosure Queries

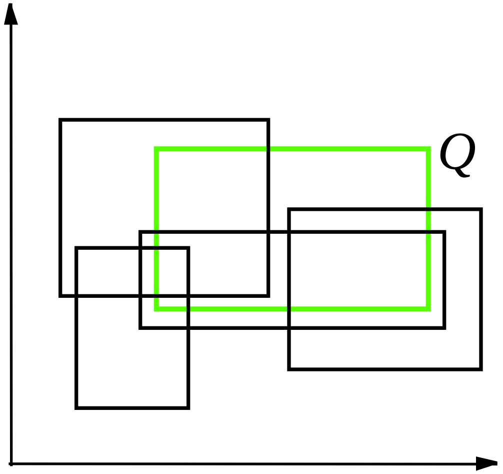
- Dual of planar range searching problem
  - Report all rectangles containing query point (x,y)



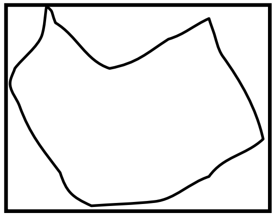
- Internal memory:
  - Can be solved in O(N) space and $O(\log N + T)$ time
  - Persistent interval tree

Rectangle Range Searching

- Report all rectangles intersecting query rectangle Q

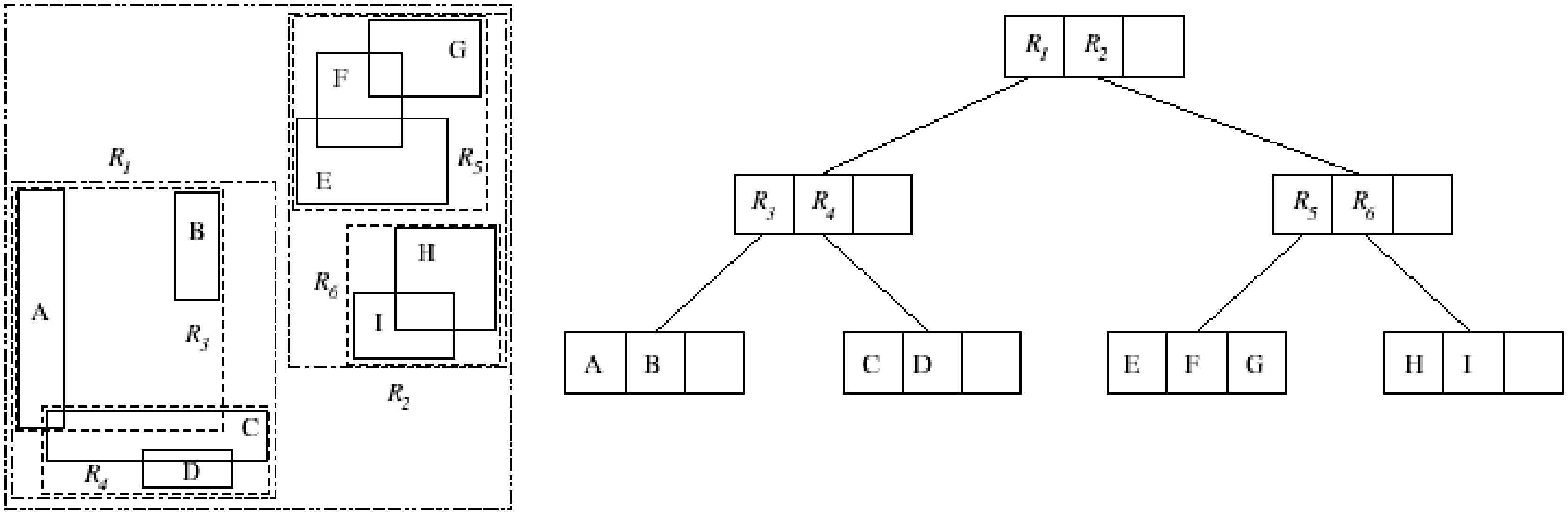


- Often used in practice when handling complex geometric objects
  - Store minimal bounding rectangles (MBR)



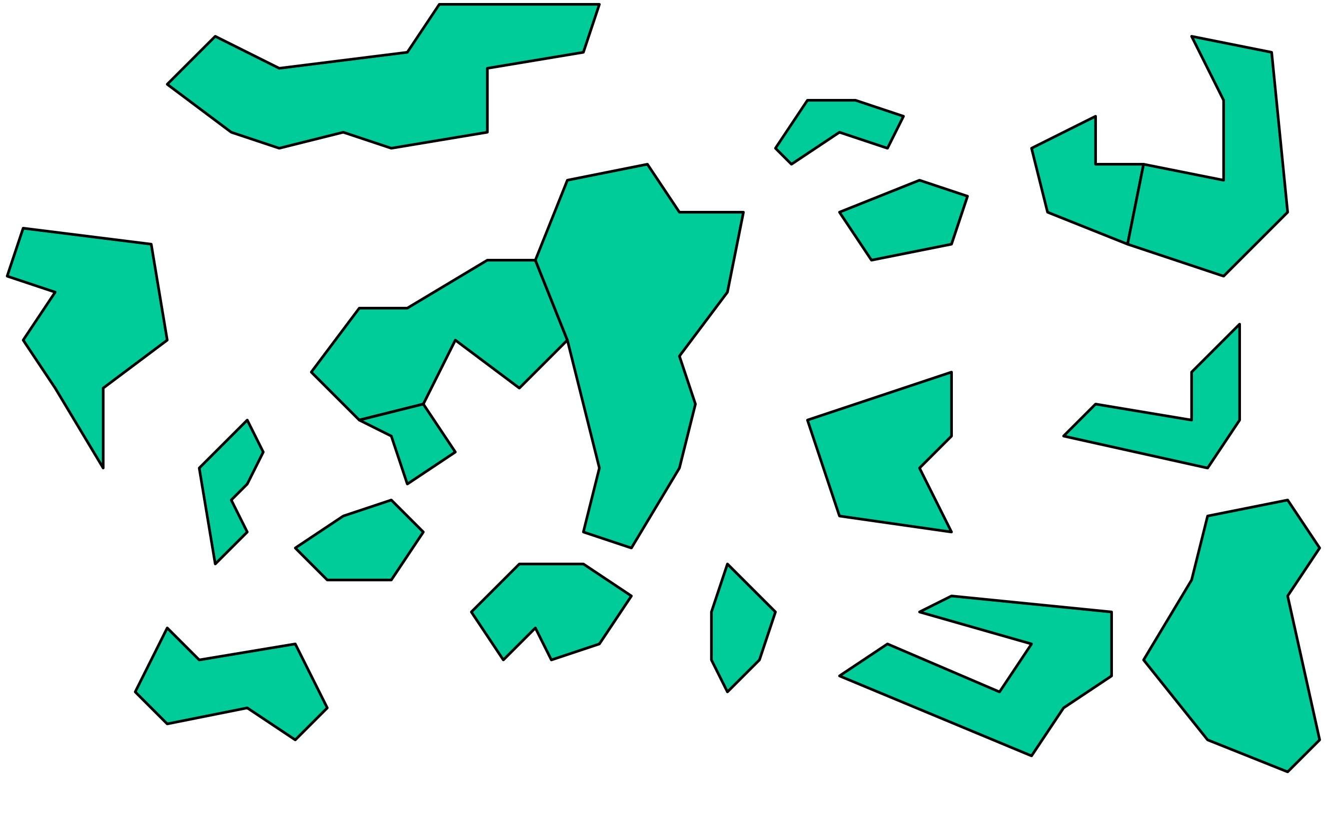
Rectangle Data Structures: R-Tree [Guttman, SIGMOD84]

- Most common practically used rectangle range searching structure
- Similar to B-tree
  - Rectangles in leaves (on same level)
  - Internal nodes contain MBR of rectangles below each child

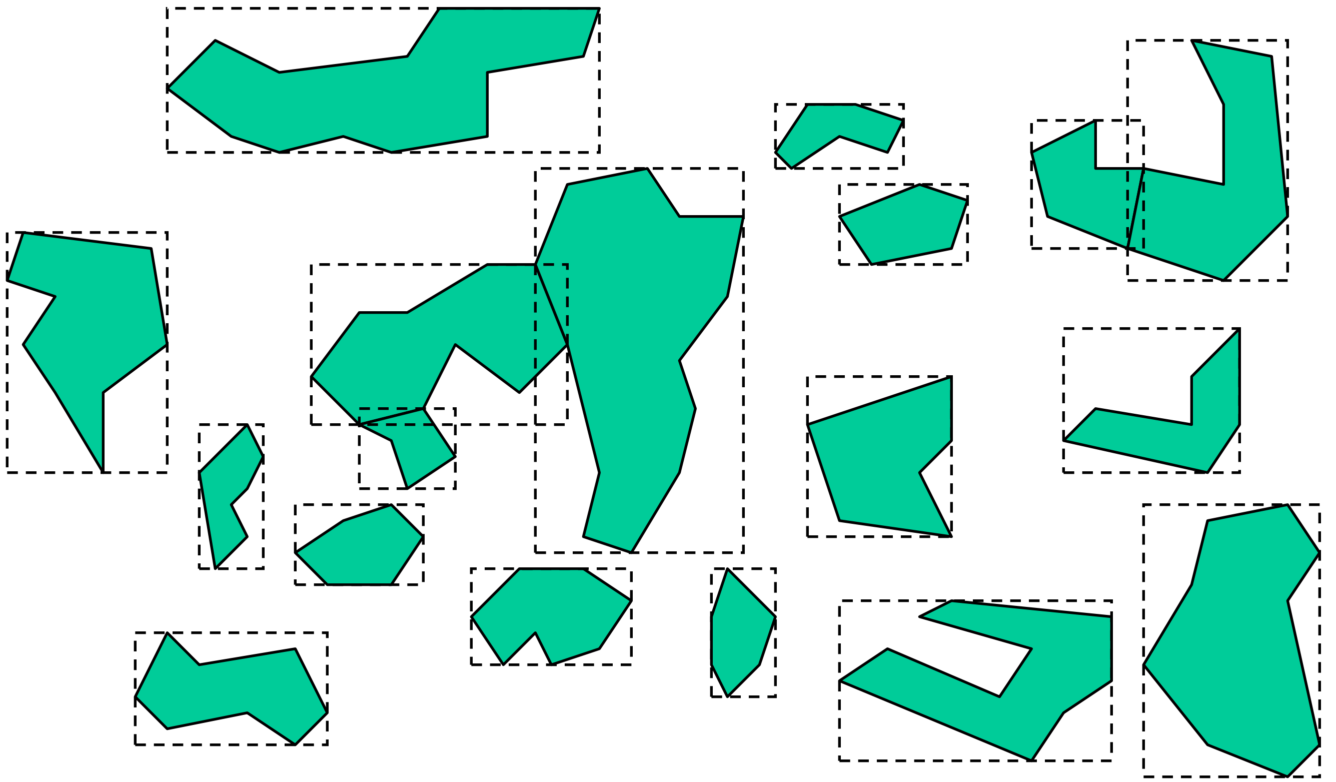


- Note: Arbitrary order in leaves/grouping order

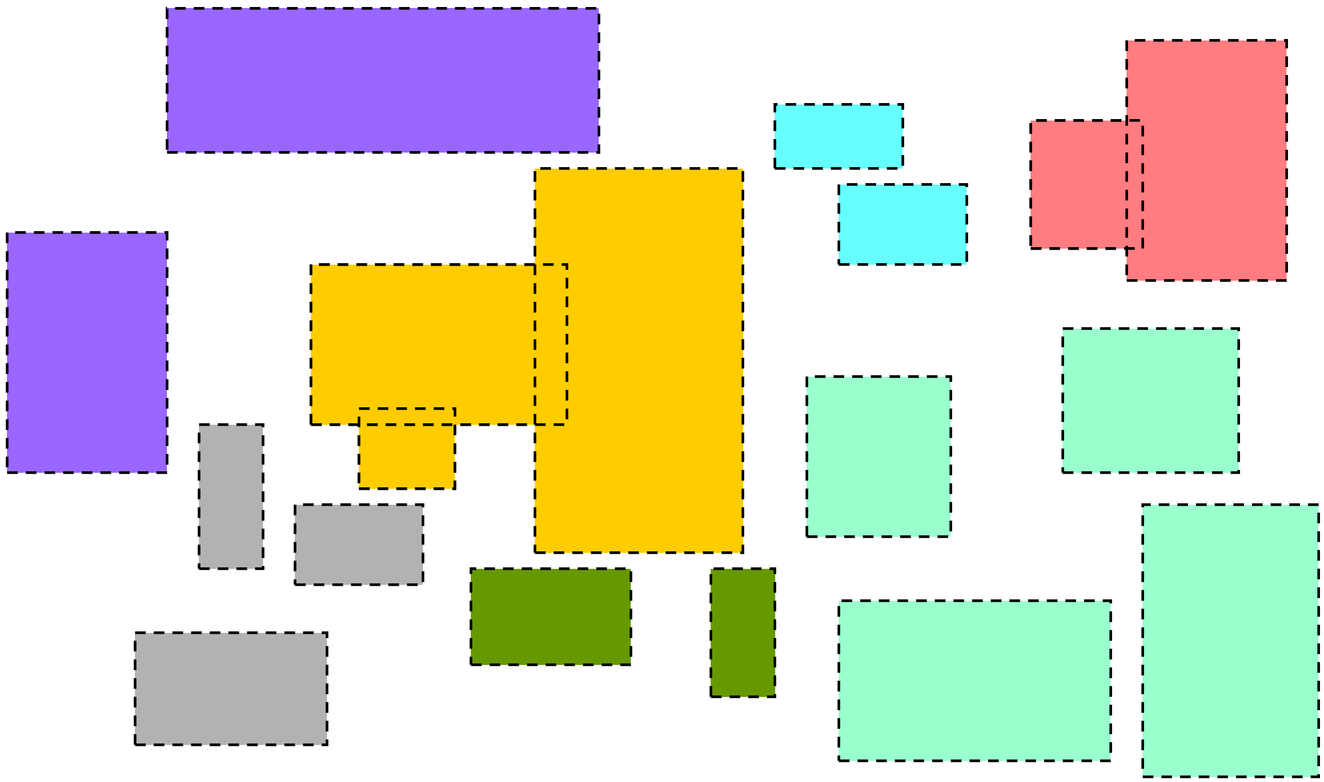
Example



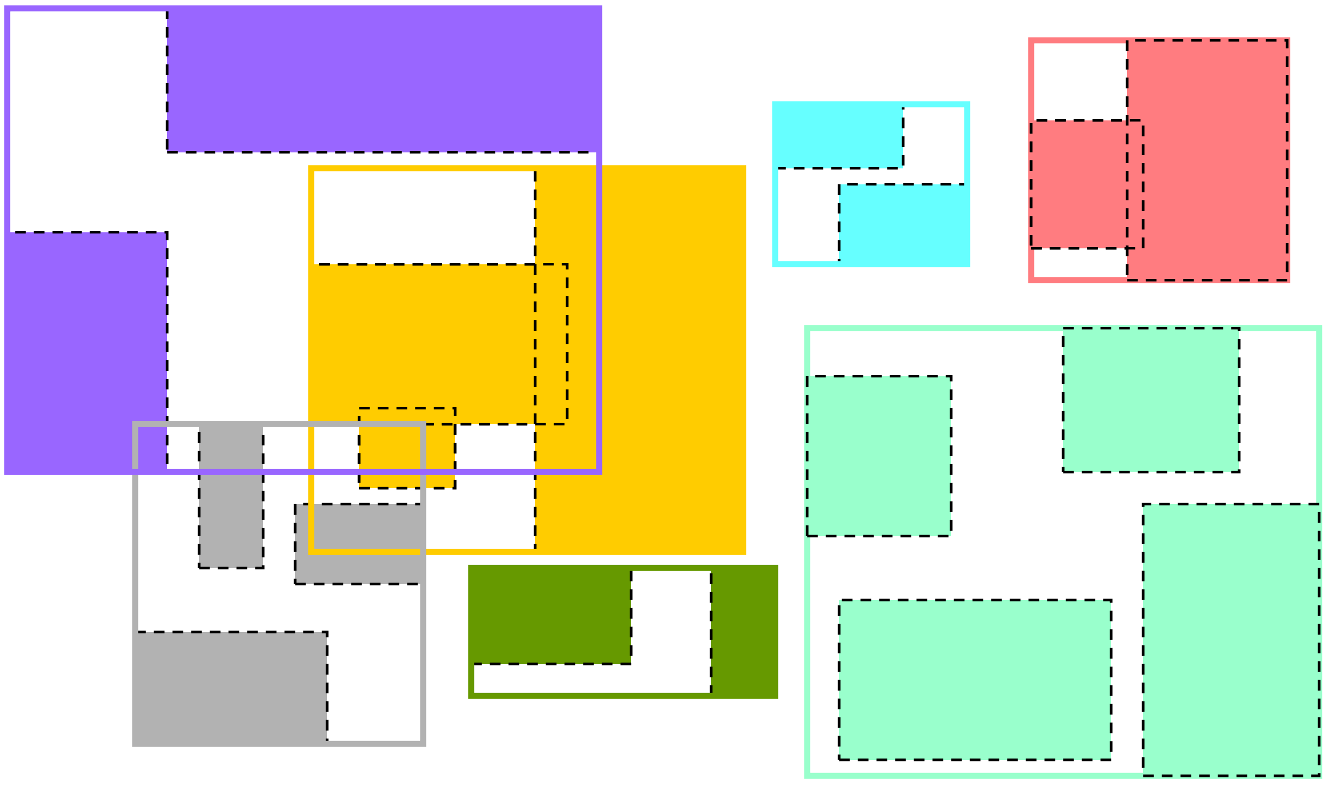
Example



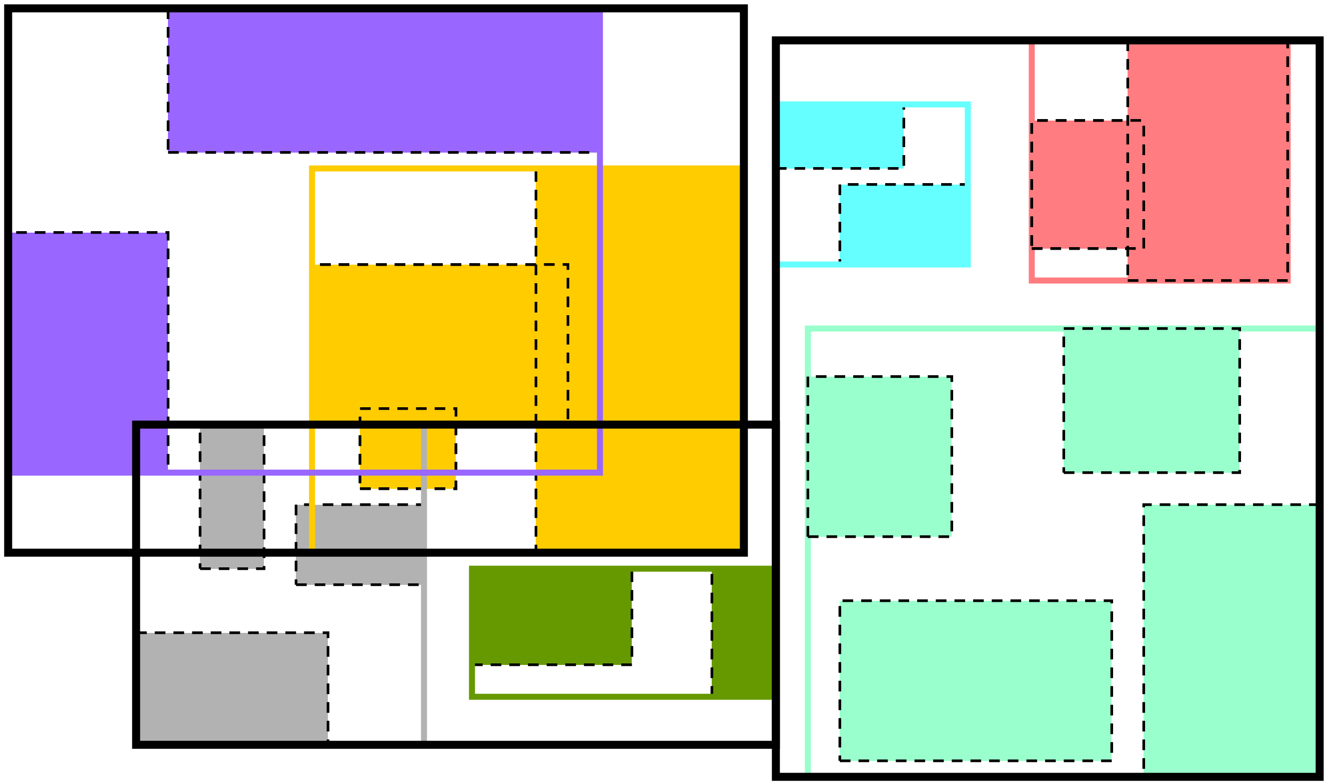
Example



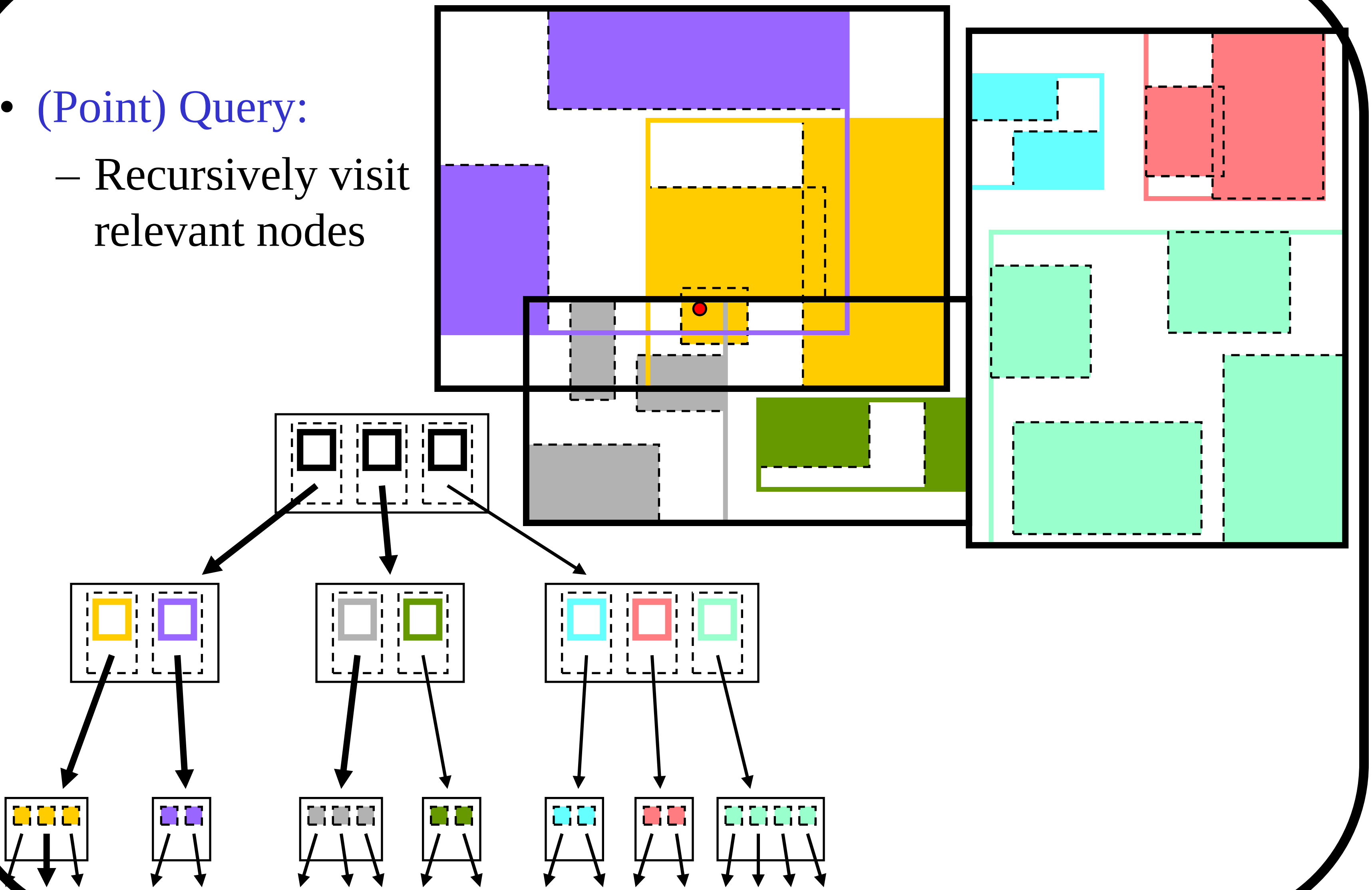
Example



Example

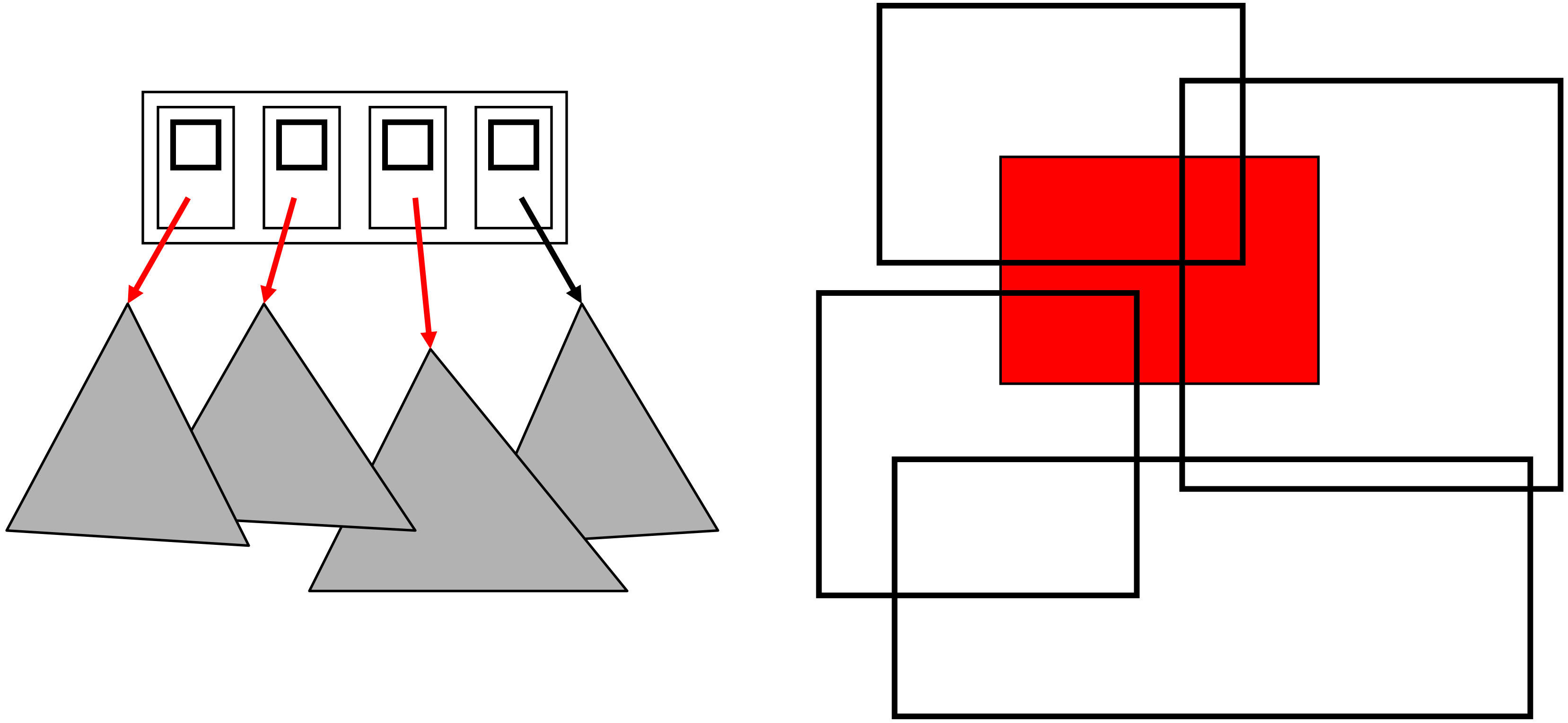


- (Point) Query:
  - Recursively visit relevant nodes



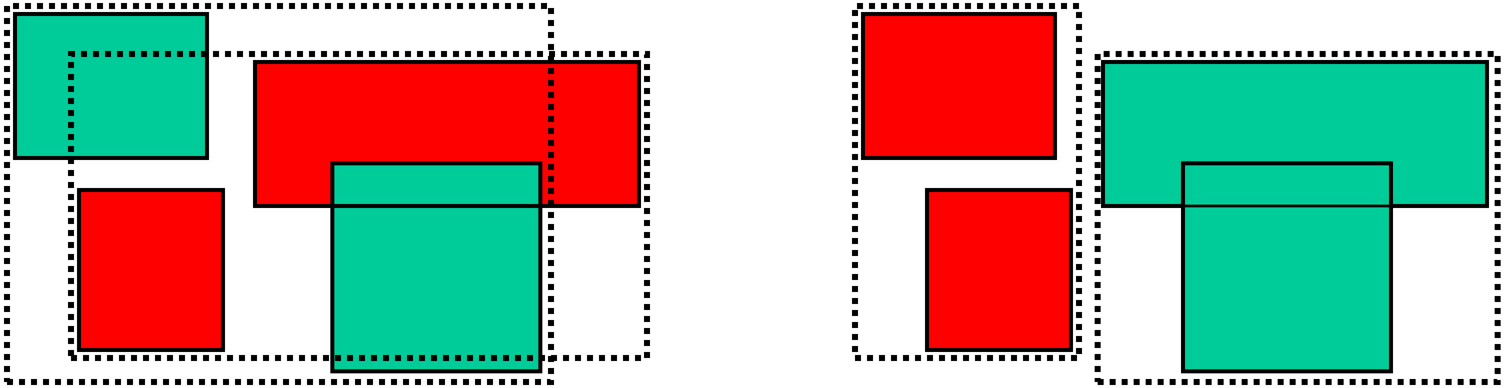
Query Efficiency

- The fewer rectangles intersected the better



Rectangle Order

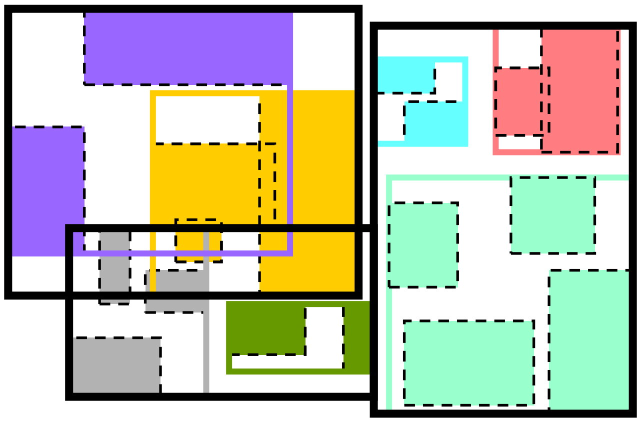
- Intuitively
  - Objects close together in same leaves
    $\Rightarrow$ small rectangles $\Rightarrow$ queries descend in few subtrees

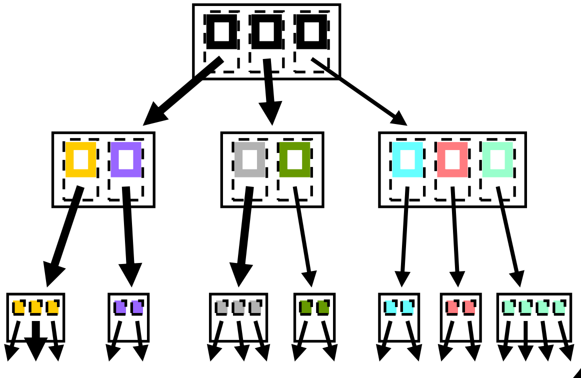


- Grouping in internal nodes?
  - Small area of MBRs
  - Small perimeter of MBRs
  - Little overlap among MBRs

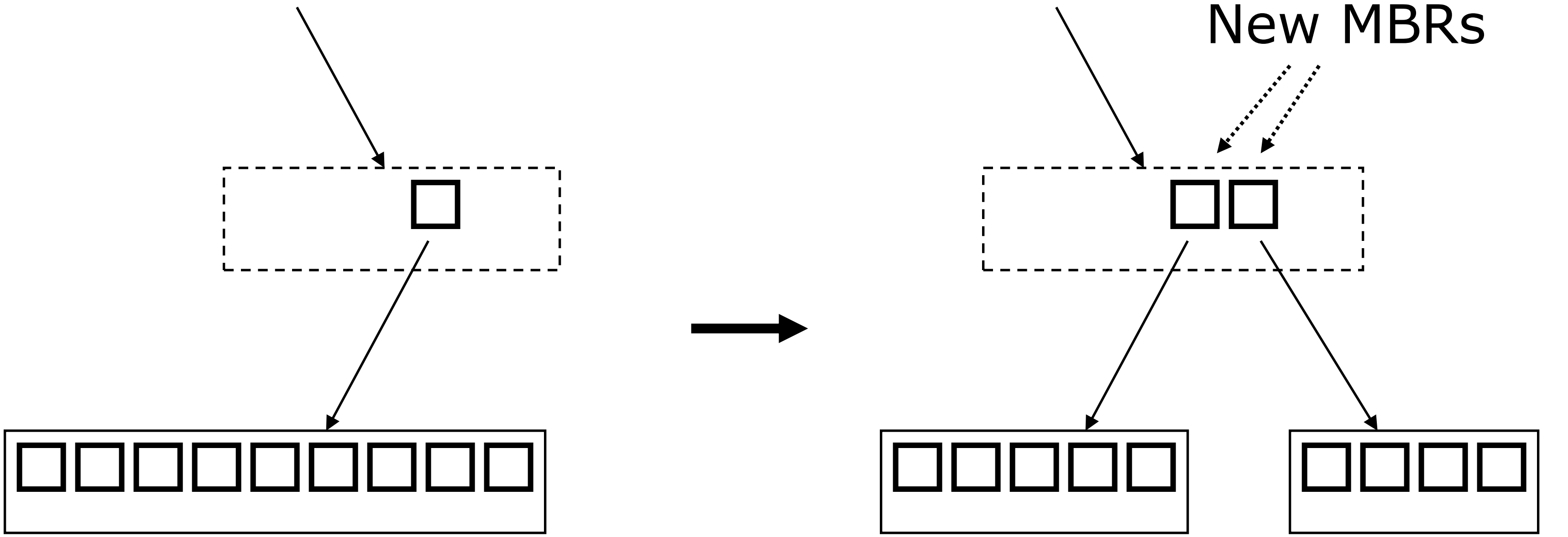
R-tree Insertion Algorithm

- When not yet at a leaf (*choose subtree*):
  - Determine rectangle whose area increment after insertion is smallest (small area heuristic)
  - Increase this rectangle if necessary and recurse
- At a leaf:
  - Insert if room, otherwise *Split Node* (while trying to minimize area)



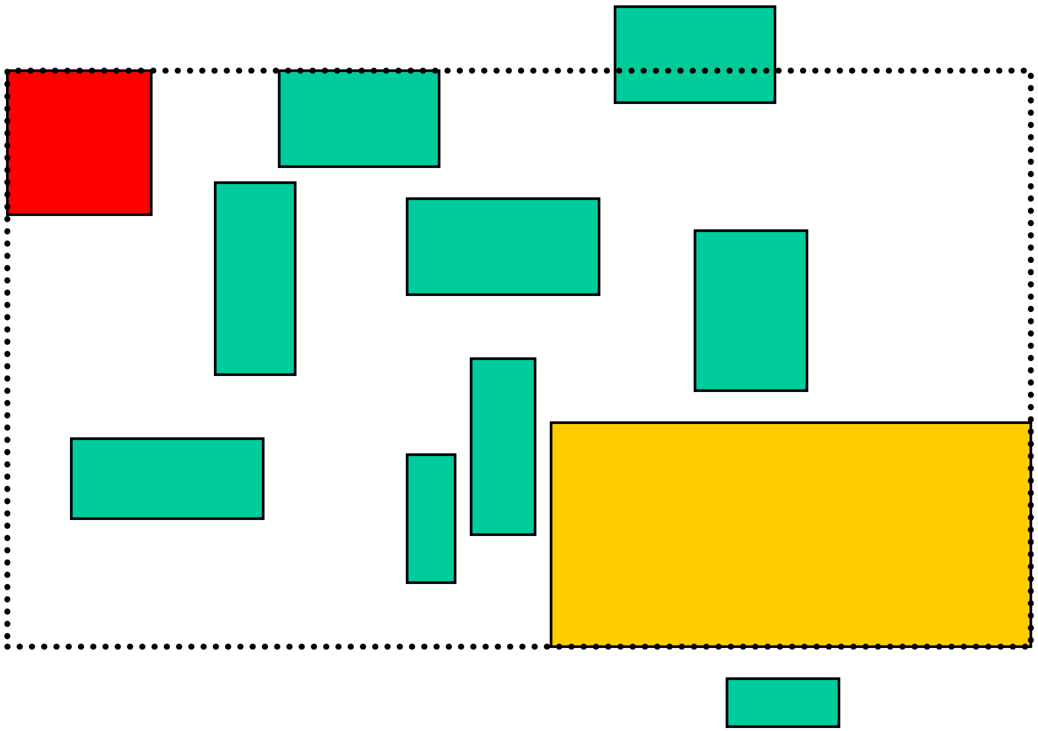


Node Split



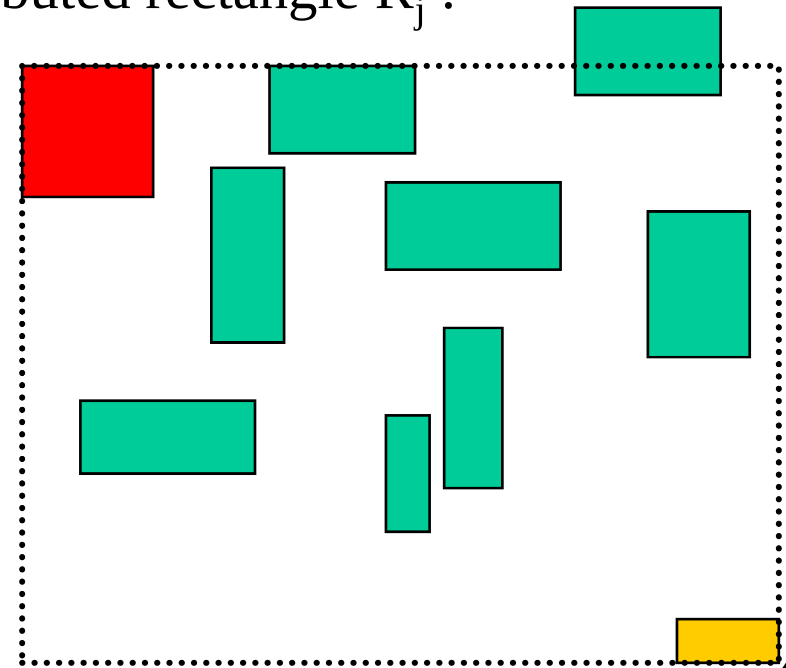
Linear Split Heuristic

- Determine the furthest pair R_1 and R_2 : the *seeds* for sets S_1 and S_2
- While not all MBRs distributed
  - Add next MBR to the set whose MBR increases the least



Quadratic Split Heuristic

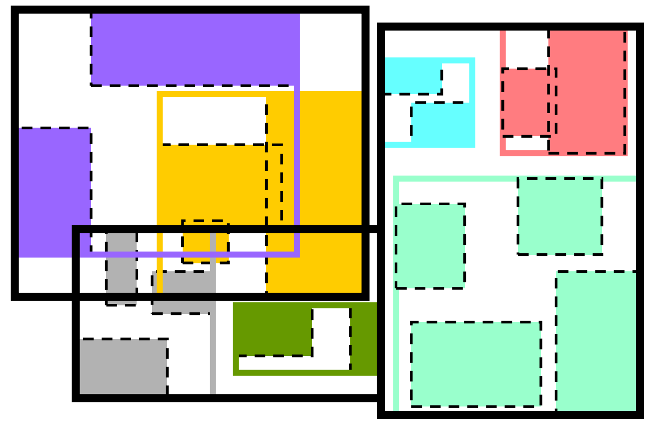
- Determine R1 and R2 with largest *area*(MBR of R1 and R2)-*area*(R1) - *area*(R2): the *seeds* for sets S1 and S2
- While not all MBRs distributed
  - Determine of every not yet distributed rectangle R_j :
    d_1 = area increment of $S_1 \cup R_j$
    d_2 = area increment of $S_2 \cup R_j$
  - Choose Ri with maximal $|d_1 - d_2|$ and add to the set with smallest area increment



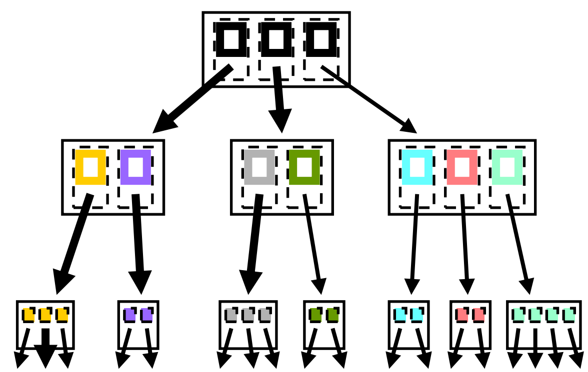
R-tree Deletion Algorithm

- Find the leaf (node) and delete object; determine new (possibly smaller) MBR
- If the node is too empty:
  - Delete the node recursively at its parent
  - Insert all entries of the deleted node into the R-tree

R*-trees [Beckmann et al. SIGMOD90]



- Why try to minimize area?
  - Why not overlap, perimeter,…



- R*-tree:
  - Better heuristics for *Choose Subtree* and *Split Node*

R-Tree Variants

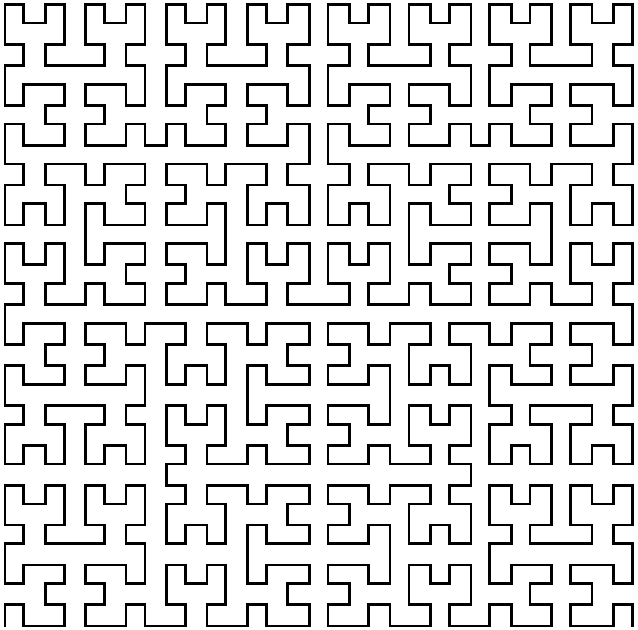
- Many, many R-tree variants (heuristics) have been proposed

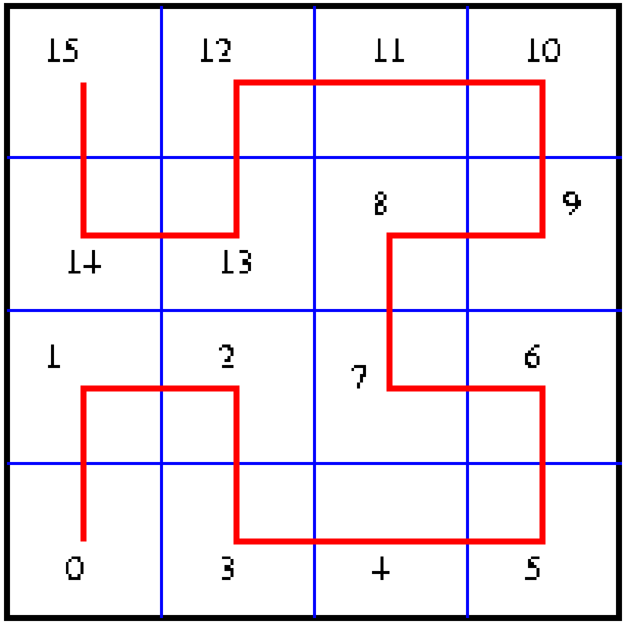
- Often bulk-loaded R-trees are used
  - Much faster than repeated insertions
  - Better space utilization
  - Can optimize more “globally”
  - Can be updated using previous update algorithms

How to Build an R-Tree

- Repeated insertions
  - [Guttman84]
  - R^+-tree [Sellis et al. 87]
  - R*-tree [Beckmann et al. 90]
- Bulkloading
  - Hilbert R-Tree [Kamel and Faloutos 94]
  - Top-down Greedy Split [Garcia et al. 98]
  - Advantages:
    - Much faster than repeated insertions
    - Better space utilization
    - Usually produce R-trees with higher quality

R-Tree Variant: Hilbert R-Tree





Hilbert Curve

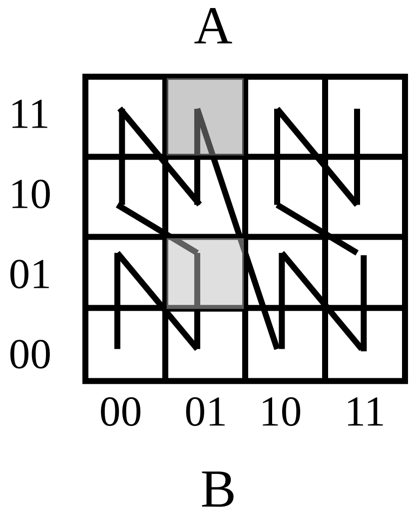
- To build a Hilbert R-Tree (cost: $O(N/B \log_{M/B} N)$ I/Os)
  - Sort the rectangles by the Hilbert values of their centers
  - Build a B-tree on top

Z-ordering

- Basic assumption: Finite precision in the representation of each co-ordinate, K bits (2^K values)
- The address space is a square (image) and represented as a $2^K \times 2^K$ array
- Each element is called a pixel

Z-ordering

- Impose a linear ordering on the pixels of the image → 1 dimensional problem



$Z_A = \text{shuffle}(x_A, y_A) = \text{shuffle}(\text{“01”}, \text{“11”})$

$= 0111 = (7)_{10}$

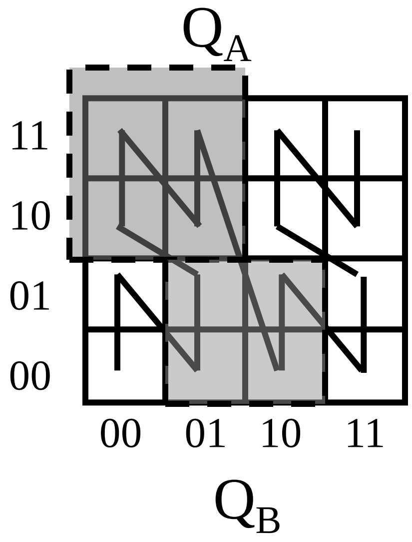
$Z_B = \text{shuffle}(\text{“01”}, \text{“01”}) = 0011$

Z-ordering

- Given a point (x, y) and the precision K find the pixel for the point and then compute the z-value
- Given a set of points, use a B+-tree to index the z-values
- A range (rectangular) query in 2-d is mapped to a set of ranges in 1-d

Queries

- Find the z-values that contained in the query and then the ranges



$Q_A \rightarrow$ range [4, 7]

$Q_B \rightarrow$ ranges [2,3] and [8,9]

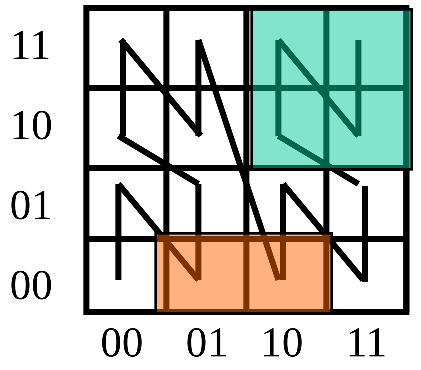
Handling Regions

- A region breaks into one or more pieces, each one with different z-value
- We try to minimize the number of pieces in the representation: precision/space overhead trade-off

Z_{R1} = 0010 = (2)
Z_{R2} = 1000 = (8)

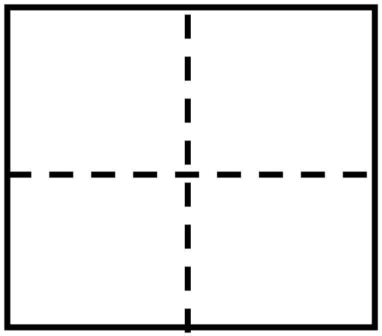
Z_G = 11

(“11” is the common prefix)



Z-ordering for Regions

- Break the space into 4 equal quadrants: <u>level-1</u> blocks
- Level-i block: one of the four equal quadrants of a level-(i-1) block
- Pixel: level-K blocks, image level-0 block
- For a level-i block: all its pixels have the same prefix up to 2i bits; the z-value of the block

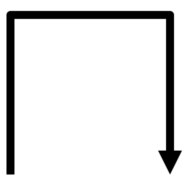


Hilbert Curve

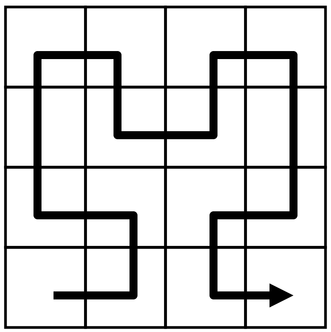
- We want points that are close in 2d to be close in the 1d
- Note that in 2d there are 4 neighbors for each point where in 1d only 2.
- Z-curve has some “jumps” that we would like to avoid
- Hilbert curve avoids the jumps : recursive definition

Hilbert Curve- example

- It has been shown that in general Hilbert is better than the other space filling curves for retrieval [Jag90]
- Hi (order-i) Hilbert curve for $2^i x 2^i$ array

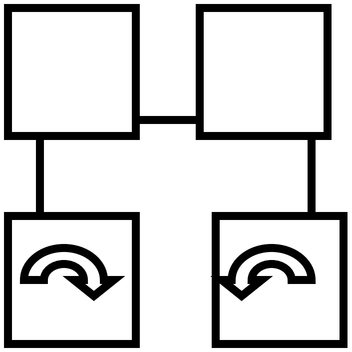


H1



H2

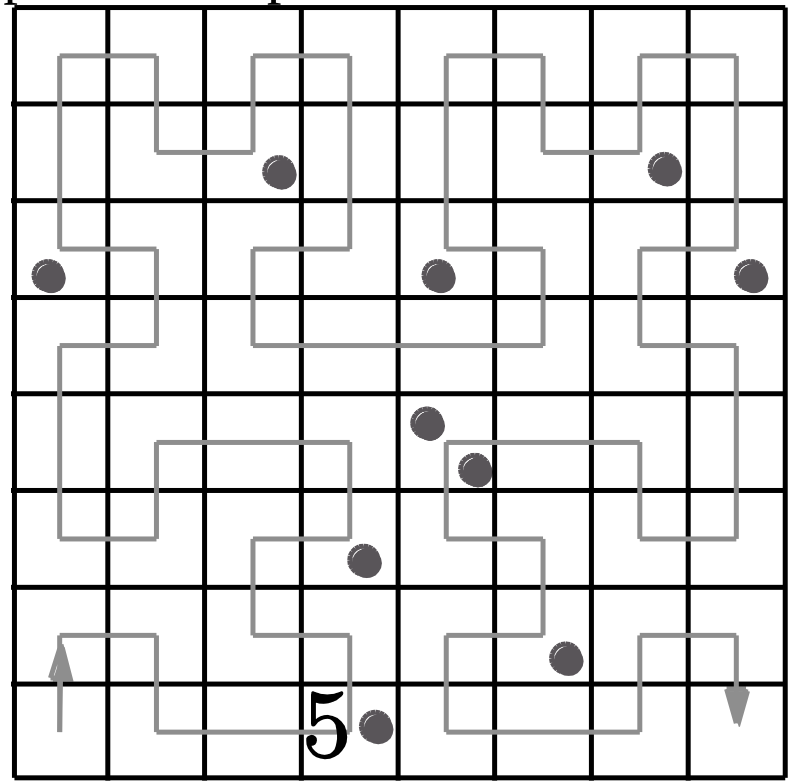
...

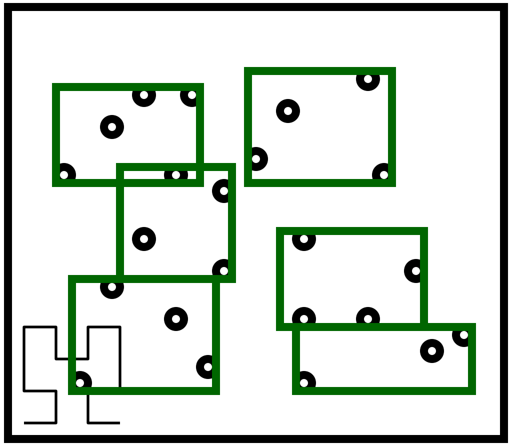


H(n+1)

R-trees - variations

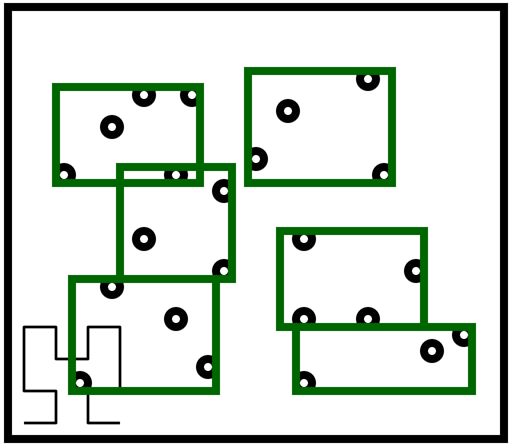
- A: plane-sweep on HILBERT curve!





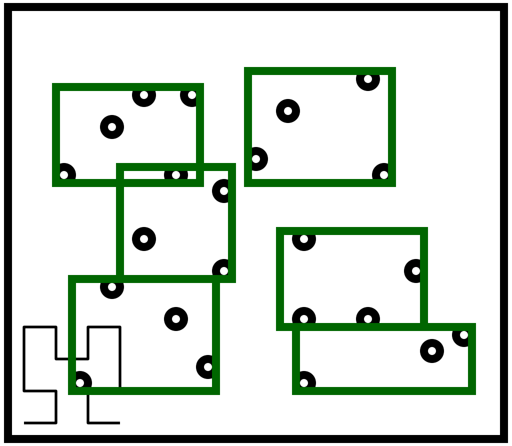
R-trees - variations

- A: plane-sweep on HILBERT curve!
- In fact, it can be made dynamic (how?), as well as to handle regions (how?)



R-trees - variations

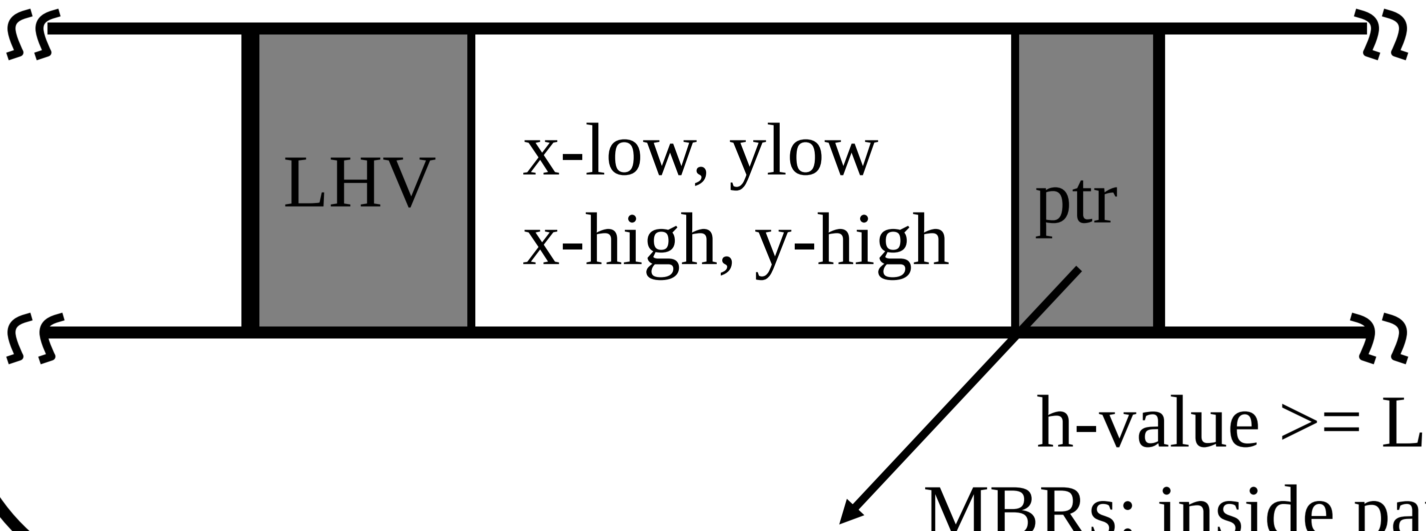
- Dynamic (‘Hilbert R-tree):
  - each point has an ‘h’-value (hilbert value)
  - insertions: like a B-tree on the h-value
  - but also store MBR, for searches



R-trees - variations

- Data structure of a node?

~B-tree

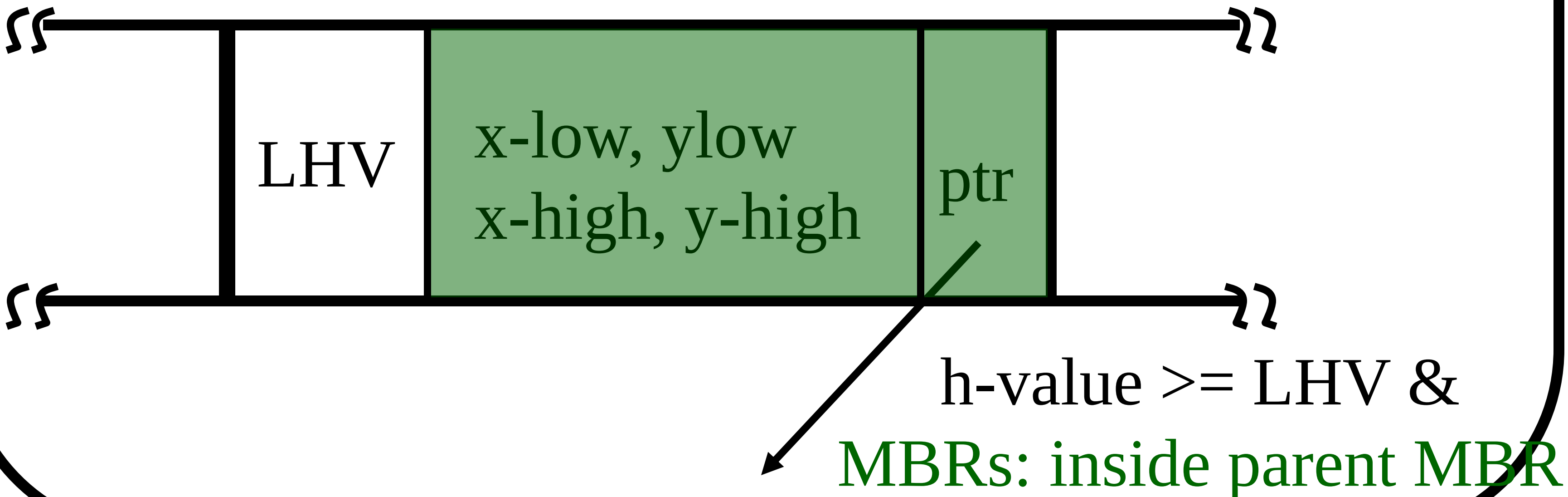


h-value >= LHV &
MBRs: inside parent MBR

R-trees - variations

- Data structure of a node?

~ R-tree



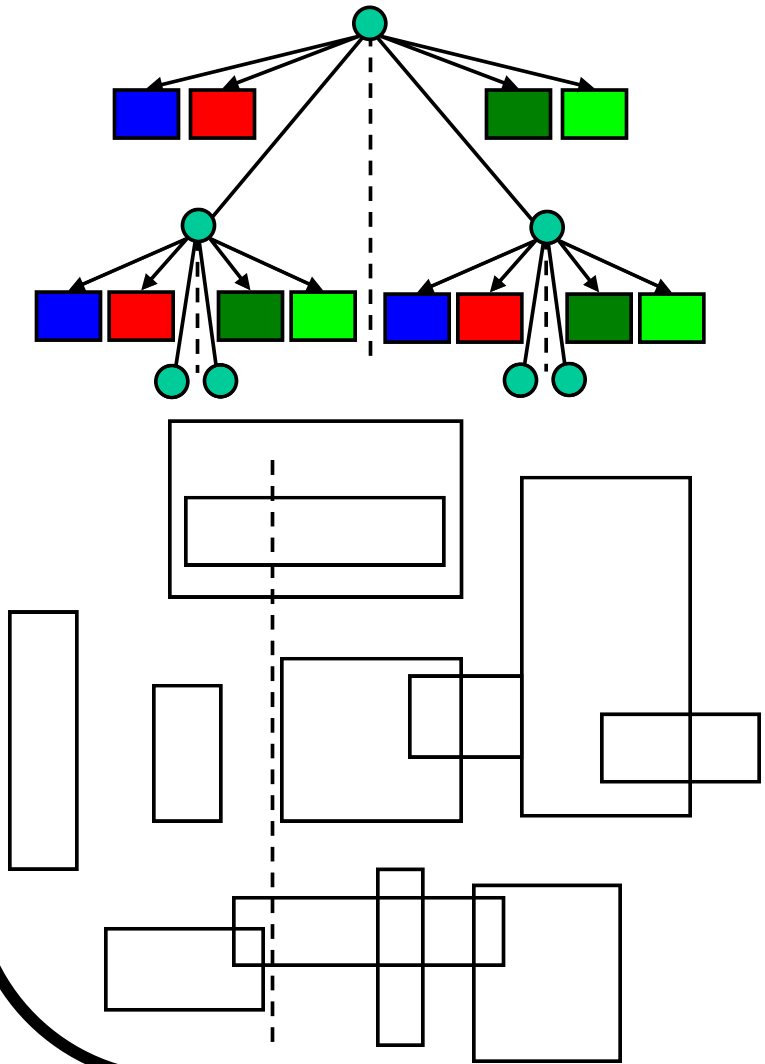
Theoretical Musings

- **None of existing R-tree variants has worst-case query performance guarantee!**
  - In the worst-case, a query can visit all nodes in the tree even when the output size is zero
- R-tree is a generalized kdB-tree, so can we achieve $O(\sqrt{N/B}+T/B)$?
- **Priority R-Tree** **[Arge, de Berg, Haverkort, and Yi, SIGMOD04]**
  - **The first R-tree variant that answers a query by visiting $O(\sqrt{N/B}+T/B)$ nodes in the worst case**
    - T: Output size
  - **It is optimal!**
    - Follows from the kdB-tree lower bound.

Roadmap

- Pseudo-PR-Tree
  - Has the desired $O(\sqrt{N/B} + T/B)$ worst-case guarantee
  - Not a real R-tree
- Transform a pseudo-PR-Tree into a PR-tree
  - A real R-tree
  - Maintain the worst-case guarantee
- Experiments
  - PR-tree
  - Hilbert R-tree (2D and 4D)
  - TGS-R-tree

Pseudo-PR-Tree



1. Place B extreme rectangles from each direction in priority leaves
2. Split remaining rectangles by x_{min} coordinates (round-robin using x_{min}, y_{min}, x_{max}, y_{max} – like a 4d kd-tree)
3. Recursively build sub-trees

Query in $O(\sqrt{N/B} + T/B)$ I/Os

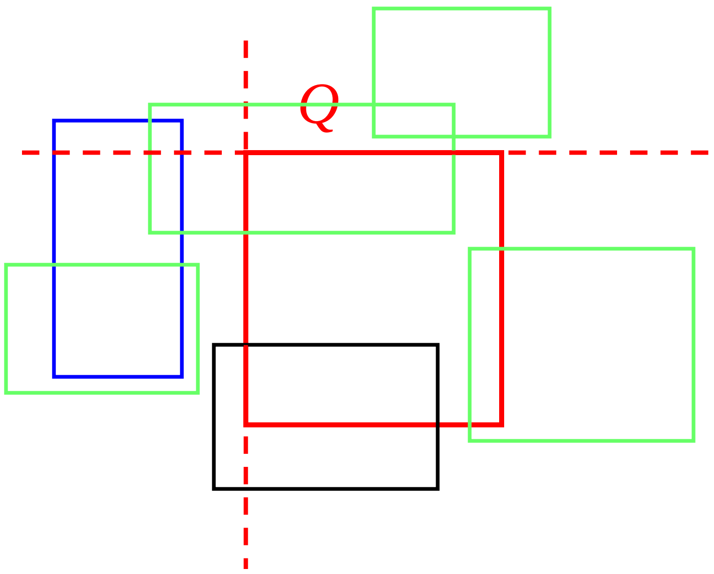
– $O(T/B)$ nodes with priority leaf completely reported

– $O(\sqrt{N/B})$ nodes with no priority leaf completely reported

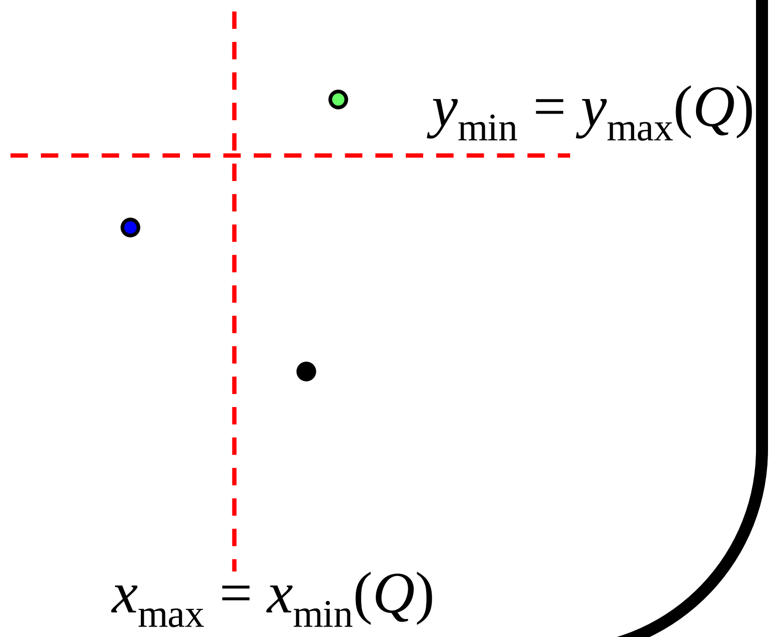
Pseudo-PR-Tree: Query Complexity

- Nodes *v* visited where all rectangles in at least one of the priority leaves of *v*'s parent are reported: O(*T*/*B*)
- Let *v* be a node visited but none of the priority leaves at its parent are reported completely, consider *v*'s parent *u*

2d

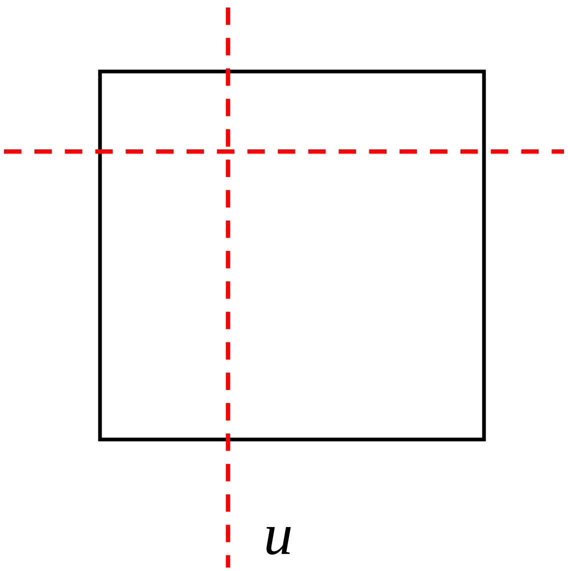


4d

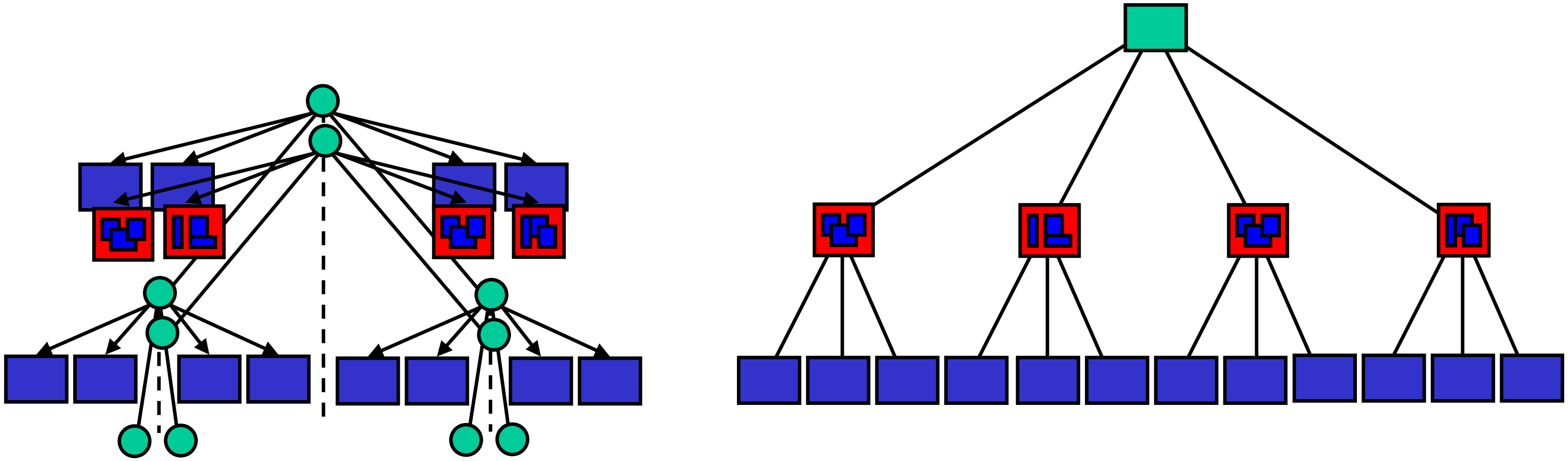


Pseudo-PR-Tree: Query Complexity

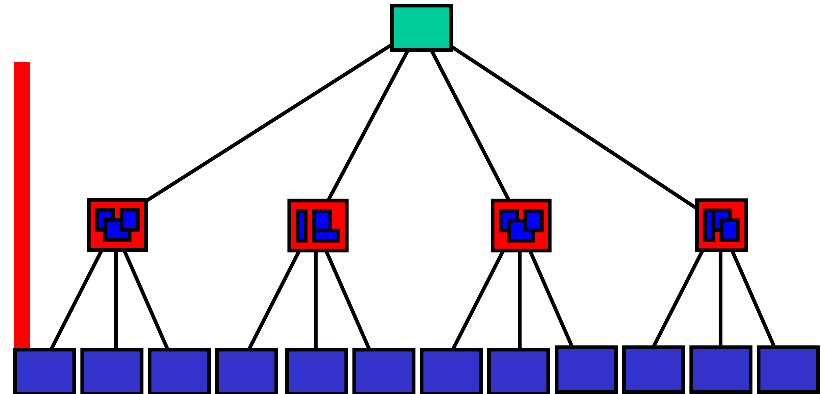
- The cell in the 4d kd-tree of *u* is intersected by two different 3-dimensional hyper-planes defined by sides of query *Q*
- The intersection of each pair of such 3-dimensional hyper-planes is a 2-dimensional hyper-plane
- Lemma: # of cells in a *d*-dimensional kd-tree that intersect an axis-parallel *f*-dimensional hyper-plane is $O((N/B)^{f/d})$
- So, # such cells in a 4d kd-tree: $O(\sqrt{N/B})$
- Total # nodes visited: $O(\sqrt{N/B} + T/B)$



PR-tree from Pseudo-PR-Tree



Query Complexity Remains Unchanged



$$\sqrt{N/B^3} + \sqrt{N/B^2}/B + \sqrt{N/B}/B^2 + T/B^3$$

Next level: $\sqrt{N/B^2} + \sqrt{N/B}/B + T/B^2$

nodes visited on leaf level $\sqrt{N/B} + T/B$

PR-Tree

- PR-tree construction in $O(\frac{N}{B}\log_{M/B}\frac{N}{B})$ I/Os
  - Pseudo-PR-tree in $O(\frac{N}{B}\log_{M/B}\frac{N}{B})$ I/Os
  - Cost dominated by leaf level
- Updates
  - $O(\log_B N)$ I/Os using known heuristics
    * Loss of worst-case query guarantee
  - $O(\log_B^2 N)$ I/Os using logarithmic method
    * Worst-case query efficiency maintained
- Extending to *d*-dimensions
  - Optimal $O((N/B)^{1-1/d} + T/B)$ query