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# Screw Axis Measurement for Kinematic Calibration of the Sarcos Dextrous Arm 

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#### Abstract

A reecntly developed kinematic calibration technique for 6 degree-offreetom (DOF) arms, the Jacobian measurement method, is generalized to inchude underconstruined and overconslrained arms. Using joint torque sensing and a urist 6 -axis force/toriue sensor to measure the Jacobian, experimental resulls are presented for the first 5 DOFs of the Sarcos Dextrous Arm. Results are compared to two standard calibration methods, op then loop calibration and circle point analysis. Measurements of endpoint position and orientation were obtained with the Optotrak and the Bird sensor, two commercial motion tracking systems.


## 1 Introduction

Recently, an alternative kinematic calibration approach was introduced that involves direct measurement of the Jacobian matrix [4]. The Jacobian matrix can be measured either by sensing joint torques and the corresponding endpoint forces and torgues, or by sensing joint angle velocities and the corresponding endpoint linear and angular velocities. The kinematic parameters can then be extracted directly from the elements of the Jacobian, exactly and without iteration. Simulations were presented for a 6-DOF manipulator with a nominal PUMA 560 geometry. The Denavit-Hartenberg parameterization [5] was used along with modified Hayati parameters [7].
In this paper, we extend the Jacobian measurement method, originally developed for 6-DOF manipulators, to overconstrained manipulators ( $\leq 5$ DOFs) and to redundant manipulators ( $\geq$ i DOFs). We then experimentally implement the method on the first 5 DOFs of the Sarcos Dextrous Arm [13, 12], by using joint torque sensing intrinsic to the arm's design and a six-axis force/torque sensor (Assurance Technology, Garner, North Carolina) at the endpoint. We compare the results to those obtained by applying two standard calibration techniques, open-loop kinematic calibration [1, 11] and circle point analysis ( CPA ) [14], using endpoint measurements obtained respectively with the Optotrak, a commercial optoelectronic motion tracking system (Northern Digital Inc., Waterloo, Ontario), and the Bird sensor, a magnetic motion tracking system (Ascension Technology, Burlington, Vermont).

In open-loop kinematic calibration, the manipulator is placed in a number of poses and the endpoint position and orientation are measured. Nonlinear optimization, usually the Levenberg-Marquardt algorithm, is performed to extract the kinematic parameters. The phrase open-loop refers to an endpoint which is positioned freely in space, in contrast to closed-loop methods where the endpoint is attached to the enviromment [2,3].

Circle point analysis (CPA) is also a kind of open-loop method, but proceeds quite differently from the open-loop method just described. Each joint is moved alone; a linear optimization then fits a least squares plane to the data. followed by a nonlinear optimization which fits a circle in this plane. The plane gives the orientation of the joint axis, while the circle yields a point on this axis; thus the line in space for each joint axis is known. Different procedures have been proposed to extract the kinematic parameters from a knowledge of
the joint axes [14. 17].
The Jacobjan measurement method may be either open or closed loop. depending on the sensing. When endpoint force/torque sensing and joint torgue sensing are used, the manipulator forms a closed loop. When endpoint linear and angular velocity and joint angle velocity are sensed, the manipulator is in an open loop.

There is a strong relation between C.PA and the Jacobian measurement method, because both involve measurement of the screw coordinates of the joint axes. In CPA, the screw axes are obtained one at a time by individual joint motion. In the Jacobian measurement method, the screw axes are deduced together because the Jacobian as a whole is calculated from the measurements by matrix inversion. Once the screw axes have been obtained, whether from CPA or the Jacobian measurement method, the same methods can be applied to extract the kinematic parameters from the screw axes. The method that we proposed in [.f] is most similar to Sklar's method [14, 16].

Since CPA and the Jacobian measurement method have this strong relation. it may be worth coining a new phrase to describe a more general class of calibration methods that includes them both. We suggest screw axis measurement. Other screw axis measurement methods besides these two are conceivable, and so this phrase may have additional utility.

In the following. we first extend the Jacobian measurement method to overconstramed 5 -DOF arms, then to redundant 7 -DOF arms. The experimental setup involving the Sarcos Dextrous Arm is then described, and calibration results are presented.

## 2 Jacobian Measurement Method

For brevity, we will describe only the approach using joint torque sensing and wrist force/torque sensing. Extensions to velocity sensing are straightforward, and for the 6-DOF case are described in [4]. The basis for the approach is the well-known static relation:

$$
\begin{equation*}
\tau^{i}=\mathbf{J}^{T} \mathbf{w}^{i}+\mathbf{g} \tag{1}
\end{equation*}
$$

where $\tau^{i}$ is the vector of joint torgues. $\mathrm{w}^{i}$ is the wrench, g is the gravity torguc vector, and the superscript $i$ refers to the $i$ th measurement point.

The Jacobian J is not superscripted because the manipulator position is always the same in this method, and is related to the screw Jacobian matrix $\mathrm{J}_{s}$ by the relation:

$$
\mathrm{J}=\left[\begin{array}{ll}
\mathbf{0}_{3} & \mathrm{I}_{3}  \tag{2}\\
\mathrm{I}_{3} & \mathbf{0}_{3}
\end{array}\right] \mathrm{J}_{s}=\mathrm{AJ}_{s}
$$

where $0_{3}$ is the 3 -by-3 zero matrix, $I_{3}$ is the 3 -by- 3 identity matrix. For an $n$-joint manipulator, $\mathrm{J}_{s}$ has the form [2]:

$$
\mathbf{J}_{s}=\left[\begin{array}{ccc}
z_{0} & \cdots & z_{n-1}  \tag{3}\\
z_{0} \times b_{1} & \cdots & z_{n-1} \times \mathbf{b}_{n}
\end{array}\right]
$$

where $\mathbf{z}_{j-1}$ is the rotation axis for joint $j$, and $\mathbf{b}_{j}$ is any vector from axis $\boldsymbol{z}_{j-1}$ to the endpoint.

### 2.1 Fully Constrained Manipulators

For 6 -DOF manipulators, we estimate J by exerting six joint torques $\tau^{i}, i=1, \ldots, 6$, that yield independent endpoint wrenches wi. The bias due to gravity torques is eliminated by exerting both positive $\tau^{i}$ and negative $-\tau^{i}$ torques for each torque vector application, then subtracting the corresponding static equations (1):

$$
\begin{align*}
\tau^{i} & =\mathrm{J}^{T} \mathrm{w}_{+}^{i}+\mathrm{g}  \tag{4}\\
-\tau^{i} & =\mathrm{J}^{T} \mathrm{w}_{-}^{i}+\mathrm{g} \tag{5}
\end{align*}
$$

where $\mathrm{w}_{+}^{i}$ and $\mathrm{w}_{-}^{i}$ are the wrenches resulting from the positive and negative torque applications respectively. Subtracting these equations climinates gravity:

$$
\begin{equation*}
2 \tau^{i}=\mathrm{J}^{T} \Delta \mathrm{w}^{i} \tag{6}
\end{equation*}
$$

where $\Delta w^{i}=w_{+}^{i}-w_{-}^{i}$. From the six differential measurements ( 6 ). form the matrices $T=\left(2 r^{1}, \ldots, 2 r^{6}\right)$ and $W=\left(\Delta w^{1}, \ldots . J w^{6}\right)$. Then

$$
\begin{align*}
\mathrm{T} & =\mathrm{J}^{T} \mathrm{~W}  \tag{1}\\
\mathrm{~J}^{T} & =\mathrm{T} \mathrm{~W}^{-1} \tag{8}
\end{align*}
$$

where the measured Jacobian ( 8 ) is obtained from (7) by inverting W.

To generate an independent wrench set, the manipulator must not be in a singular position. A singularity may be detected by checking the rank of the generated wrenches $W$. If singular, a new position and attachment point to the environment would have to be sought that is not at a singularity. When the position is not singular, an independent wrench set may be obtained by generating torque at one joint at a time. Let $\tau^{i}=\mathrm{e}^{i}$. where the $j$ th component of $\epsilon_{j}^{i}$ is 1 if $i=j$ and 0 otherwise. Then from (6)

$$
\begin{equation*}
\Delta \mathrm{w}^{i}=\mathrm{J}^{-T}\left(2 \mathrm{e}^{i}\right) \tag{9}
\end{equation*}
$$

As long as $J$ is invertible, the resulting wrenches are independent but not usually orthogonal.

### 2.2 Overconstrained Manipulators

When there are 5 or fewer DOFs, the endpoint wrenches generated by the joint torques do not span the full six-dimensional space. Hence it is not possible to use the exact procedure above. which requires an invertible W . Because experiments will be presented later for a manipulator with 5 DOFs, we restrict the following discussion to 5 -DOF arms. From (6), the pseudoinverse solution for $\Delta w^{i}$ is:

$$
\begin{equation*}
\Delta \mathrm{w}^{i}=\left(\mathrm{J}^{T}\right)^{\cdot} \cdot 2 \tau^{i}+\left(\mathrm{I}_{6}-\left(\mathbf{J}^{T}\right)^{\times} \mathrm{J}\right) \mathbf{v} \tag{10}
\end{equation*}
$$

where $\left(\mathrm{J}^{T}\right)^{x}=\mathrm{J}\left(\mathrm{J}^{T} \mathrm{~J}\right)^{-1}$ is the pseudoinverse of $\mathrm{J}^{T}, \mathbf{I}_{6}$ is the 6 -by-6 identity matrix, and $v$ is an arbitrary wrench. The second term on the right side represents a vector in the null space of $J^{T}$, i.e., a wrench which generates no joint torques. The first term on the right represents the endpoint wrenches which can actively be generated by the manipulator. Since in our scenario the manipulator is the active source of wrench generation (once gravity has been accounted for), the second term on the right side is zero. Thus the measured wrenches will correspond only to those actively generated by the joint torques.

Five independent measurements need to be generated, for example. by the single-joint actuation method described above. Combining these five independent measurements. (10) becomes

$$
\begin{equation*}
\mathrm{W}=\left(\mathbf{J}^{T}\right)^{*} \mathrm{~T} \tag{11}
\end{equation*}
$$

where $W$ consists of the 5 independent wrenches that are actually produced from the joint torgues.

The solution for $\mathbf{J}$ call now be obtained by taking the transpose of ( $i$ ) and using the pseudoinverse of $W^{T}$;

$$
\begin{equation*}
\mathrm{J}=\left(\mathrm{W}^{T}\right) \times \mathrm{T}^{T}=\mathrm{W}\left(\mathrm{~W}^{T} \mathrm{~W}\right)^{-1} \mathrm{~T}^{T} \tag{12}
\end{equation*}
$$

That (12) yield the correct solution may be verified by substituting (11).

### 2.3 Underconstrained Manipulators

With rigid attachment of the endpoint to the environment, a redundant manipulator with $i$ DOFs will in general form a mobile closed kinematic chain with 1 DOF. The main difficulty in extending the Jacobian measurement method to redundant manipulators is to generate a set of joint torques which do not result in an internal motion of the mobile closed chain. Borrowing a technique developed in [6] for redundancy resolution, we note that six components of joint torque can be considered independent while the seventh is dependent. Without loss of generality, assume $\tau$ - is the dependent torçue. If joint $\bar{\gamma}$ is immobile during the self motion, we may rearrange the Jacobian to place a mobile joint in position 7 . The mobility of joints may be discovered experimentally, by finding a joint that drives the chain when other joint torques are servoed to 0 .

First we need to counteract gravity for an arm to hold an arbitrary posture. This can be done. for example, by servo action on joint $\overline{7}$ to discover the required joint torque. Let $\tau_{0}$ be this applied torque vector and $w_{0}$ the resulting endpoint wrench. We can again eliminate the effect of gravity by a difference method, this time subtracting the following bias equation from each reading:

$$
\begin{equation*}
\tau_{0}=\mathrm{J}^{T} \mathrm{w}_{0}+\mathrm{g} \tag{13}
\end{equation*}
$$

Next define a reduced torque vector $\bar{\tau}^{i}$ from the first six joint torque components of $\tau^{i}$. Apply the method in Section 2.1 for fully constrained manipulators to generate six joint torques $\tilde{\tau}^{i}$. For each torque application $\bar{\tau}^{i}$, servo joint $i$ to remain stationary and read the resulting joint torgue $r_{i}^{i}$. Then read the resulting wrench $\mathbf{w}_{i}$. The static force-torque relation may be writtell as:

$$
\tau^{i}=\left[\begin{array}{c}
\bar{\tau}^{i}  \tag{1.1}\\
\tau_{i}^{i}
\end{array}\right]=\mathrm{J}^{T} \mathrm{w}^{i}+\mathrm{g}
$$

Subtracting the bias torque (13) to eliminate gravity,

$$
\begin{equation*}
\tau^{i}-\tau_{0}=\mathrm{J}^{T}\left(\mathrm{w}^{i}-\mathrm{w}_{0}\right) \tag{15}
\end{equation*}
$$

The 6-by- - Jacobian is then found straightforwardly using the procedures of Section 2.1.

## 3 Methods

At the time of the experiments; our Sarcos Dextrons Arm was configured with the first 5 arm joints: a roll-pitch-coll spherical shoulder joint. a rotary elbow joint, and a roll forearm joint. Hence for the Jacobian measurement method, the overconstrained method discussed in Section 2.2 was implemented.

Each joint of our Sarcos Dextrous Arm is sensed by two different joint angle sensors, 100,000 count optical encolers and Rotary Variable Differential Transducers (RVI)Ts). In this preliminary experimental study, readings from the Ams standard RVDTs were employed for both the Open-loop and CPA method. Their analog signals are converted by 12 -hit ADCs. The manufacturer specifics a linearity of $2 \%$ of full scale of a best fit straight line. In addition we observe a noise level of 2 bits.

The D-H parameters are employed and derived, namely for joint $j$ :


Figure 1: Diagram of Sarcos Dextrous Arm for CPA calibration.
$a_{j}$ is the distance along axis $\mathrm{x}_{j}$ from rotation axis $z_{j-1}$ to rotation axis $\mathbf{z}_{j}$.
$s_{j}$ is the distance along axis $z_{j-1}$ from axes $\mathrm{X}_{j-1}$ to $\mathrm{x}_{j}$.
$\alpha_{j}$ is the skew angle about $\mathbf{x}_{j}$ from $z_{j-1}$ to $z_{j}$.
$\theta_{j}^{o j f}$ is the joint angle offset.
The D-Il parameters derived from the manufacturer's specifications are listed in Table 1, and are used as a point of reference for the experimental results.

### 3.1 Open-Loop Method

The procedure presented in [11] was implemented using the OPTOTRAK 3020 , which has a stated accuracy of $0.1 \cdot 0.15 \mathrm{~mm}$ in a. 1 $\mathrm{m}^{3}$ workspace. A calibration frame was attached to the endpoint of the Sarcos Dextrous Arm, and six IREDs (infrared light emitting diodes) were attached in the shape of a rectangle.

The calibration frame was placed by the manipulator into 250 poses, carefully chosen to be in view of the camera system. At each pose. Optotrak IREDS and joint angle sensors were sampled and averaged for 200 times.

From the six IREDs, the position and orientation of the calibration frame were derived at each pose and compared to those predicted from the nominal parameters to yield the position and orientation errors. A standard iterative least squares method was employed to derive the D-H parameters, including extra parameters to locate the calibration frame in the last link and the base of the robot relative to the camera coordinate system.

### 3.2 Circle Point Analysis

The procedure presented in $[1.4,16]$ was implemented using an Ascension Technology Bird sensor, which has a stated accuracy of 2.5 mm in translation and $0.5^{\circ}$ rotation in a spherical workspace of 1.2 m diameter. Each joint of the arm was rotated by steps throughout its entire range. At each pose the position and orientation of the Bird sensor attached to the end-effector was sampled and averaged 25 times.
Theoretically, the resulting trajectory is a circle constrained to a plane. In practice, we first fit a plane to the experimental data. project the data points onto this plane, and then fit a circle to the projected data. The resulting normal to the plane through the center of the circle uniquely determines a line which is the screu aris of the rotating joint. With all the screw axes available, the arm's D-H parameters can be determined immediately exploiting simple geometric relationships.

### 3.3 Jacobian Measurement Method

In this method the robot arm's motion is completely constrained. To this end, a fixture was made which rigidly attached the arm's last link via the force torque sensor to the robot mounting table. Simulations were performed to find a pose for which the numerical calibration procedure is well conditioned. To cancel out the gravity terms, torques of opposite sign where generated at each joint. The resulting joint torques and wrenches can then be used to calculate the manipulator Jacobian.

For joint torque measurement, we employ the Sarcos Dextrous Arm's intrinsic joint torque sensors, which have a stated resolution of 1 part in 2000 . The torque sensors were calibrated by the manufacturer, and we assumed this calibration level to proceed. The measurement noise in the joint torque sensors is about 2 counts or $0.1 \%$ full scale.

An
Assur-
ance Technology FT 500/3000 model six-axis force/torque sensor was employed as the external wrench sensor. Its full scale of 500 lbs for force and $3000 \mathrm{in}-\mathrm{bs}$ for torque was selected to accommorlate the maximum wrenches exerted by the arm, and thus maximize the measurement accuracy: The manufacturer specifies an accuracy for the force/torque sensor of around $1 \%$.

## 4 Experimental Results

The results from the open-loop method, the CPA, and the Jacobian measurement method are presented in Tables 2, 3, and 4 respectively. The terms that are noted as $n / a$ are not considered as they are dependent on the fixtures of the base and of the end-effector and are not part of the arm's D-II parameter set.

In comparing the calibrated parameters to the manufacturer's parameters, we use the statistical root mean square (RMS) measure. For the length components, the RMS error $\psi$ is defined as:

$$
\begin{equation*}
\psi_{l}=\sqrt{1 / 2 N \sum_{j=1}^{N}\left(a_{j}-a_{j}^{m}\right)^{2}+\left(s_{j}-s_{j}^{m}\right)^{2}} \tag{16}
\end{equation*}
$$

where $a_{j}^{m}$ and $s_{j}^{m}$ are the manufacturer's parameters (Table 1), $a_{j}$ and $s_{j}$ are the parameters calibrated by one of the three methods (Tables 2-4), and $N$ is the number of joints. For the angle components, the RMS error $\psi^{\prime}$ is defined similarly:

$$
\begin{equation*}
\dot{\psi}_{a}=\sqrt{1 / 2 N \sum_{j=1}^{N}\left(\alpha_{j}-a_{j}^{m_{m}}\right)^{2}+\left(\theta_{j}^{o f f}-\theta_{j}^{o f f, m}\right)^{2}} \tag{17}
\end{equation*}
$$

The statistical errors are summarized in Table 5.

## 5 Discussion

Statistical analysis (Table 5) shows that the open-loop method using the Optotrak yielded the most accurate results, which was expected. The RMS error in the length parameters $t$ was 2 mm while that in the angle parameters $\ell_{a}$ was 0.3 degrees. These are reasonable results. considering the lack of precision in the RVDT angle sensors. In the future, we plan to redo the experiments using the substantially more accurate digital encoders.

The results from CPA using the Bird sensor were the least accurate, with a $\psi_{f}$ of 45 mm and $\xi_{n}$ of 1.2 degrees. The length parameter errors using the IBird sensor were about 20 times worse than for the Optotrak, which is roughly proportional to the price ratio of the two systems.

The Jacobjan measurement method using joint torque sensing and an Assurance Technology wrist force/torque sensor yielded a $\%$ of 17 mm and a $v_{a}$ of $0 . t$ degrees.

| $j$ | $a_{j}^{m}(\mathrm{~m})$ | $s_{j}^{m}(\mathrm{~m})$ | $\alpha_{j}^{m}(\mathrm{deg})$ | $\theta_{j}^{0 / \int, m}(\mathrm{deg})$ |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0.000 | $\mathrm{n} / \mathrm{a}$ | 90.0 | $\mathrm{n} / \mathrm{a}$ |
| 2 | 0.000 | 0.000 | -90.0 | 135.0 |
| 3 | 0.000 | 0.355 | 90.0 | -180.0 |
| 4 | 0.000 | 0.000 | -90.0 | 135.0 |
| 5 | $\mathrm{n} / \mathrm{a}$ | 0.235 | $\mathrm{n} / \mathrm{a}$ | 90.0 |

Table 1: D-H parameters according to manufacturer's specifications.

| $j$ | $a_{j}(\mathrm{~m})$ | $s_{j}(\mathrm{~m})$ | $\alpha_{j}(\mathrm{deg})$ | $\theta_{j}^{0 / f}(\mathrm{deg})$ |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0.001 | $\mathrm{n} / \mathrm{a}$ | 90.0 | -89.8 |
| 2 | 0.000 | 0.003 | -89.9 | 135.9 |
| 3 | 0.001 | 0.355 | 90.0 | -180.0 |
| 4 | 0.000 | 0.003 | -89.8 | 135.0 |
| 5 | $\mathrm{n} / \mathrm{a}$ | 0.232 | $\mathrm{n} / \mathrm{a}$ | 90.2 |

Table 2: D.H parameters derived from open-loop method.

| $j$ | $a_{j}(\mathrm{~m})$ | $s_{j}(\mathrm{~m})$ | $\alpha_{j}(\mathrm{deg})$ | $\theta_{j}^{0 j f}(\mathrm{deg})$ |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0.046 | 0.034 | 88.1 | -90.8 |
| 2 | -0.012 | 0.060 | -88.6 | 134.6 |
| 3 | -0.009 | 0.350 | 88.0 | -179.8 |
| 4 | 0.005 | 0.091 | -89.4 | 134.8 |
| 5 | 0.013 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ |

Table 3: D-H parameters derived from CPA.

| $j$ | $a_{j}(\mathrm{~m})$ | $s_{j}(\mathrm{~m})$ | $\alpha_{j}(\mathrm{deg})$ | $\theta_{j}^{o f f}(\mathrm{deg})$ |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0.005 | $\mathrm{n} / \mathrm{a}$ | 89.9 | $\mathrm{n} / \mathrm{a}$ |
| 2 | 0.030 | 0.003 | -89.6 | 134.9 |
| 3 | 0.028 | 0.343 | 90.0 | -179.9 |
| 4 | 0.017 | 0.003 | -90.0 | 135.3 |
| 5 | 0.001 | 0.220 | $\mathrm{n} / \mathrm{a}$ | 91.1 |

Table 4: D-H parameters derived from Jacobian measurement method.

| Method | $\psi_{1}(\mathrm{~m})$ | $\psi_{a}(\mathrm{deg})$ |
| ---: | :--- | :--- |
| Open-loop | 0.002 | 0.3 |
| CPA | 0.045 | 1.2 |
| JMM | 0.017 | 0.4 |
|  |  |  |

Table 5: RMS errors relative to the manufacturer's specifications for the D-H length parameters ( $\psi_{l}$ ) and for the D-H angle parameters (4)

The main limitation in this implementation of the Jacobian measurement method was the accuracy of the wrist force/torque sensor. While the stated accuracy is $1 \%$, the actual accuracy is likely to be worse because of nonlinear coupling among force directions. Hirose et al. (1990) have shown that such nonlinear coupling can degrade the accuracy for seusors initially rated at $0.2 \%$ down to $3 \%$, and have proposed a compensation method that reduces the error below $1 \%$. If only on-axis wrenches are generated, wrist force sensors are at their most accurate. For this purpose, it is possible to recast this method into a multi-step procedure, first generating a rough model with the method presented, then using this model to generate approximately on-axis wrench components, and finally refining these components through a small experimental search procedure to produce on-axis wrenches. Future force sensor developments should also result in more accurate sensors, such as the optically-based force sensors in $[8]$ or force sensors based on magnetic levitation, which have no coupling.

For CPA, one limitation in our experimental results was deflections in stationary joints during motion of the joint to be calibrated, up to 1 degree. This was due to inadequate stiffiness in the PD controller. We will implement a. PID controller to compensate for these deflection errors.

## 6 Conclusion

The primary purpose of this paper was an experimental implementation of the Jacobian measurement method, using joint torque sensing coupled with wrist force/torque sensing. This method was implemented on the Sarcos Dextrous Arm, which has joint torque sensing at every joint. Because our arm was configured for 5 DOFs, we extended our original approach [4], developed for 6-DOF arms, to overconstrained arms with less than 6 DOFs. The main issue in the extension was that 6 independent wrenches could not be generated, and the proposed solution involved the pseudoinverse of the wrench matrix $W$. Our arm is currently configured as a redundant $i$-DOF arm, and so the method was also extended to redundant arms. The main issue in this extension is to prevent internal motion of the manipulator when joint torques are exerted. and a solution was devised in which one of the joint torques is treated as a dependent variable.

We also showed how the Jacobian measurement method is related to another calibration method, circle point analysis (CPA). Both methods determine the screw axes of the joints, and we suggested the term screw axis measurement to encompass these two and other conceivable approaches.

The experimental results of Jacobian measurement method were compared to experimental results using the open-loop method and also CPA. For the open-loop method, the Optotrak system was employed to provide high-accuracy position measurements of the endpoint. For CPA, we decided to employ the much less expensive Bird sensor for interest to see how well calibration could be performed. For both open-loop and CPA method, the RVDT sensors were used to provide joint-angle measurements. Baseline results were chosen to be the manufacturer's specifications.

These results indicate that at the moment the Jacobian measurement method using force and torque sensing is suitable for coarse calibration, but not for high-accuracy parameter determination. One use for the method is that the frame of the wrist force/torque sensor with regard to the end link can be determined. Another use is to check, or determine an initial value for the joint angle offset. which in many manipulators has to be recalibrated frequently.

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