CS 4964 – Fall 2016 — Homework 2 Extra Questions

Q5.

a)

by CLT we can say $\bar{X}$ has a normal distribution with mean 3 and variance $\frac{10}{10} = 1$. Therefore:

$$E[\bar{X}] = 3$$

b)

$$Var[\bar{X}] = 1$$

c)

$$Std[\bar{X}] = \sqrt{Var[\bar{X}]} = 1$$

d)

We know that for both distributions the mean is equal to 3, however the variances are different. For $\bar{X}$ we have more concentration around the mean and less concentration in tails than X. Since the range $[4, \infty)$ is covering more tail than mean we can say $Pr[X > 4] > Pr[\bar{X} > 4]$.

e)

We can rewrite $Pr[X > 2] = 1 - Pr[X \leq 2]$ and $Pr[\bar{X} > 2] = 1 - Pr[\bar{X} \leq 2]$. By the same argument as before since range $(-\infty, 2]$ is only covering a tail we can say $Pr[X \leq 2] > Pr[\bar{X} \leq 2]$ and therefore $Pr[X > 2] < Pr[\bar{X} > 2]$.

Q6.

We define $Z = X + 1$, then we have $0 \leq Z \leq 3$ and

$$E[Z] = E[X + 1] = E[X] + 1 = 0 + 1 = 1$$

we want to find $Pr[X > 1.5]$. This is equal to

$$Pr[X > 1.5] = Pr[X + 1 > 1.5 + 1] = Pr[Z > 2.5]$$

now we can use Markov property on Z. We have:

$$Pr[Z > 2.5] \leq \frac{E[Z]}{2.5}$$

$$Pr[X > 1.5] = Pr[Z > 2.5] \leq \frac{1}{2.5}$$

Q7.

a)

For a matrix to be invertable it should be square and full rank. Therefore we should add a column that is not a linear combination of current columns. For instance,

$$V = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

b)

The same applies here, we should be careful that after removing the row the matrix should be full rank. For instance if we remove the last row, matrix would be square and full rank.

c)

We know $A^T A$ and AA^T are both square matrices, we just have to find the rank. If they are full rank they are invertable. AA^T is a 4×4 matrix with rank 3, therefore it is not invertable.

d)

$A^T A$ is a 3×3 matrix with rank 3, therefore it is invertable.