Homework 4: Expectation and Variance

Instructions: Your answers are due at the beginning of class on the due date. You can either turn in a paper copy, or a pdf version through canvas. We highly recommend using latex (`http://www.cs.utah.edu/~jeffp/teaching/latex/`) for producing the assignment answers. If the answers are too hard to read (e.g. **do not create pdf using your phone's camera!**) you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

1. Consider a continuous random variable X with the following pdf:

$$f(x) = \begin{cases} (1/2)x + (1/4) & \text{for } 1 \leq x \leq c, \\ 0 & \text{otherwise,} \end{cases}$$

   where c is an unknown constant.

   (a) What value of c makes f a valid pdf? (use this value c for remaining questions)

   (b) What is $E[X]$?

   (c) What is $\text{Var}(X)$? (Hint: first think about which variance formula is easiest to apply.)

   (d) What is $E[4X - 1]$?

   (e) What is $\text{Var}(4X - 1)$?

2. You roll a die until you have seen a 5 on 4 of the rolls (e.g. $\langle \mathbf{5}, 3, 2, \mathbf{5}, 4, 1, 6, \mathbf{5}, 2, \mathbf{5} \rangle$. What is the expected number of rolls this will take?

3. Let $X \sim \textit{Unif}(0, 2)$. What is $E[\exp(2X/3) - 3]$?

4. You are playing a game where you roll a die and win 0 jellybeans for rolling a 1, 2, or 3; 1 jellybean for rolling a 4, or 5; and 3 jellybeans for rolling a 6. Each time you play the game, you must pay 1 jellybean.

   (a) What is the expected number of jellybeans you win each round (including the one you pay)?

   (b) What is the variance of the number of jellybeans that you win?

   (c) If you like jellybeans, is this a game you want to play?