Total Probability

Jan 26, 2023
Review: Independent events $A, B$

\[
\begin{align*}
\Pr(A) &= \Pr(A \mid B) \\
\Pr(B) &= \Pr(B \mid A) \\
\Pr(A \cap B) &= \Pr(A) \cdot \Pr(B)
\end{align*}
\]

Also

\[
\begin{align*}
\Pr(A) &= \Pr(A \mid B^c) \\
\Pr(B^c) &= \Pr(B^c \mid A) \\
\Pr(A \cap B^c) &= \Pr(A) \cdot \Pr(B^c)
\end{align*}
\]
\[ P_c(A \cap B) = P_c(A) \cdot P_c(B) \]

\[ P_c(A - B^c) = P_c(A) \cdot P_c(B) \]

\[ P_c(A) - P_c(A \cap B^c) = P_c(A) - P_c(B) \]

\[ P_c(A) - P_c(A \cap B^c) = P_c(A)(1 - P_c(B^c)) \]

\[ P_c(A \cap B^c) = P_c(A) - P_c(B^c) \]
Total Probability

Partition the set $\Omega$ into

$\Omega = \bigcup_{i=1}^{k} B_i$

disjoint

$B_i \cap B_j = \emptyset$
Partition: $B_1, B_2, \ldots, B_k$ leads to

$$P_c(A) = P_c(A \mid B_1) P_c(B_1) + P_c(A \mid B_2) P_c(B_2) + \ldots + P_c(A \mid B_k) P_c(B_k)$$

$$= \sum_{j=1}^{k} P_c(A \mid B_j) P_c(B_j)$$

If $A = \Omega$

$$P_c(\Omega) = \frac{1}{\sum_{j=1}^{k} \frac{1}{P_c(B_j)}} = \frac{1}{\sum_{j=1}^{k} P_c(B_j)}$$
\[ Pr(A) = Pr(A \mid B) \cdot Pr(B) + Pr(A \mid B^c) \cdot Pr(B^c) \]

\[ A = (A \cap B) \cup (A \cap B^c) - \frac{A \Delta (B \cup B^c)}{\phi} \]
two urns: ① select urn ② select ball from urn

\[ A = \text{red} \]
\[ B = \text{urn 1} \]

\[ \Pr(\text{red ball}) = \Pr(B_{10}) \cdot \Pr(C_{10}) + \Pr(B_{112}) \cdot \Pr(C_{112}) \]

\[ \frac{3}{14} + \frac{1}{14} = \frac{13}{28} \]
Two Events A, B

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Probability</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

\[
P: 2^2 \rightarrow [0, 1]
\]

True: 0.1, 0.3, 0.4
False: 0.2, 0.4, 0.6