Independence
Two events $A, B$

$A$ independent of $B$ when

$$P_r(A|B) = P_r(A)$$

$$P_r(A|B) = \frac{P_r(A \cap B)}{P_r(B)}$$

if $P_r(A|B) \neq P_r(A)$

then $A$ dependent on $B$
\[ P_c(A | B) = P_c(A) \]

\[ \iff \frac{P_c(A \cap B)}{P_c(B)} = P_c(A) \]

\[ \iff P_c(A \cap B) = P_c(A) \cdot P_c(B) \]

\[ \iff \frac{P_c(A \cap B)}{P_c(A)} = P_c(B) \]

\[ \iff P_c(B | A) = P_c(B) \]
Fair Die, throw 2 times

A: sum of values is 5

\[ A = \{ (1,4), (2,3), (3,2), (4,1) \} \]

\[ B = \{ (2,1), 22, 23, 24, 25, 26, 12, 22, 32, 42, 52, 62 \} \]

\[ P(A \cap B) = \frac{2}{36} \quad P(A) = \frac{4}{36} \quad P(B) = \frac{11}{36} \]

\[ \frac{1}{18} = \frac{2}{36} \neq \frac{4}{36} \cdot \frac{11}{36} = \frac{1}{9} \cdot \frac{11}{36} = \frac{11}{324} \]

\[ \text{NOT independent} \quad P(A \cup B) = P(A) \cdot P(B) \]
2 urns, select 1 urn (equal prob), 1 ball.

\[ R = \frac{3}{14} \]

\[ P(R | U1) + P(R | U2) \]

\[ R(R) = \frac{1}{2} \cdot \frac{3}{7} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{14} + \frac{1}{4} = \frac{13}{28} \]

\[ R(U1) = \frac{1}{2} \]

\[ R(U2) = \frac{1}{2} \]

\[ R = \text{Not independent} \]

\[ U1 \text{ independent of } R \]

Choose urn 1

\[ \frac{1}{2} \]

\[ U2 \]

\[ \frac{1}{2} \]

\[ \frac{1}{2} \]

\[ \frac{1}{2} \]

\[ \frac{1}{2} \]

\[ \frac{1}{2} \]

\[ \frac{1}{2} \]