Prob Stats L16

Confidence Intervals

vs.

Hypothesis Tests

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$X_1, X_2 \ldots X_n \underset{iid}{\sim} f(\theta)$ (usually $N(\mu, \sigma^2)$)

statistic $T = T(X_1, \ldots X_n)$ (usually $\bar{X}_n = \frac{1}{n}\sum_{i=1} X_i$)

estimate θ $\quad \hat{\theta} = T$ $\quad (\hat{\mu} = \bar{X}_n)$

1. bias($\tilde{\theta}$) $\quad E[\tilde{\theta}] = \theta$ $\quad$ efficient $\mathrm{Var}(\tilde{\theta})$ small.

$E[\bar{X}_n] = \mu$

$\mathrm{Var}(\bar{X}_n) = \sigma^2/n$

2. Confidence Interval

$P_r[L_n \le \theta \le R_n] = 1-\alpha$

$L_n = \bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

close

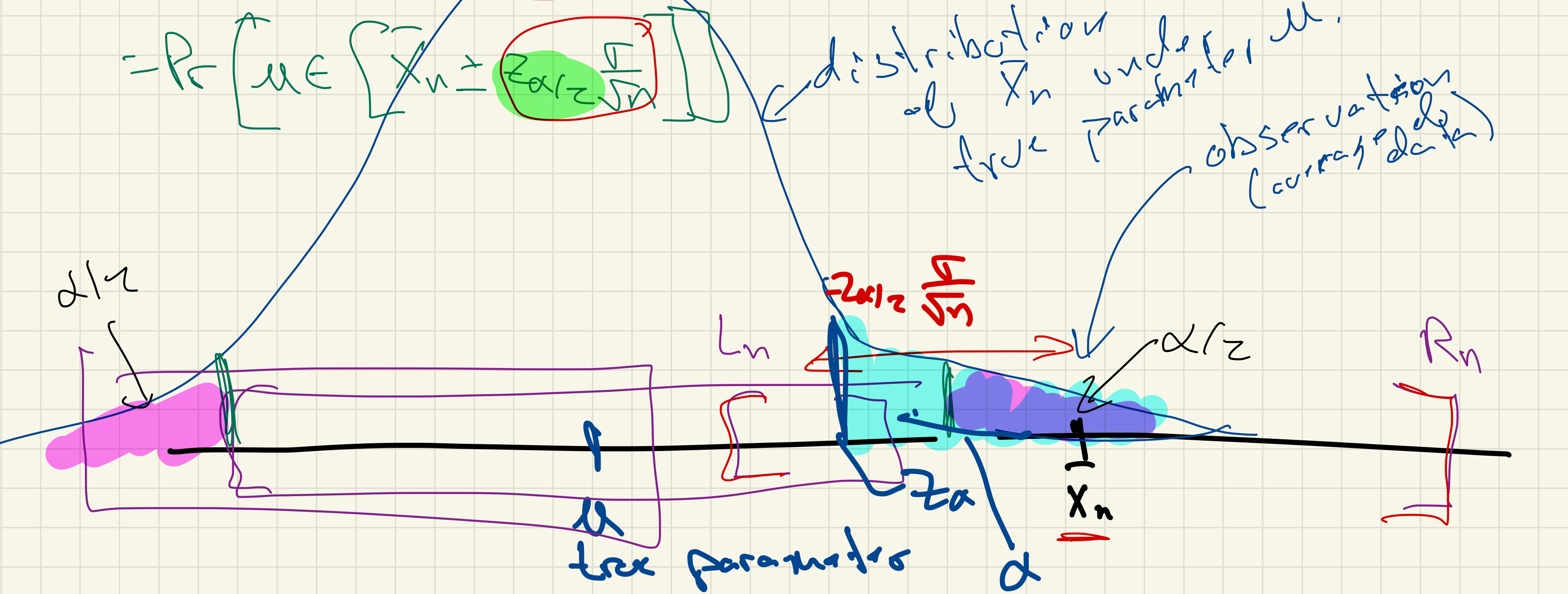
3. Hypothesis Test : Is $\bar{X}_n$ close to guess μ_0

$\bar{X}_n$ Random Variable

(lower case) $\bar{x}_n$ realization of data

$$Pr\left[\bar{X}_n \in \left[\mu \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]\right] = 1-\alpha$$

$$= Pr\left[\mu \in \left[\bar{X}_n \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]\right]$$



CI	HT
• Try find region likely to contain μ	• Hypothesis of choice of μ
• Center of CI informed by data	• Center of null hypothesis is from guess μ_0
• Range of how close is true value likely to be	• How likely is μ_0 to be true value.
• usually 2-sided	• usually μ_0 1-sided , $H_1: \mu_1 > \mu_0$