Prob Stats L15c

Hypothesis Testing

t-Test

and p-values

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Hypothesis Testing

Step 1: Formulate Hypothesis

θ_0 fixed known parameter

null hypothesis H_0: boring guess specific distribution $f(\theta_0)$

alternative hypothesis H_1: interesting ~~$f(\theta)$~~ with $\theta > \theta_0$

Step 2: Design Experiment

- Random Sample $X_1, X_2 \ldots X_n$ eg $\bar{X}_n$
- choose test statistic $T = T(X_1, \ldots X_n)$
- determine threshold critical value of α t_α

assume $X_i \sim H_0$

$\Pr(T \leq t_\alpha) = 1-\alpha$

Step 3 Run Experiment

realize sample $x_1, x_2 \dots x_n$ (lower case)

calculate $t = T(x_1 \dots x_n)$

actual calculation constant.

Compare t to t_α

if $t > t_\alpha \Rightarrow$ reject the null hypothesis

the probability, based on data $x_1 \dots x_n$, that H_0 is correct $\leq \alpha$.

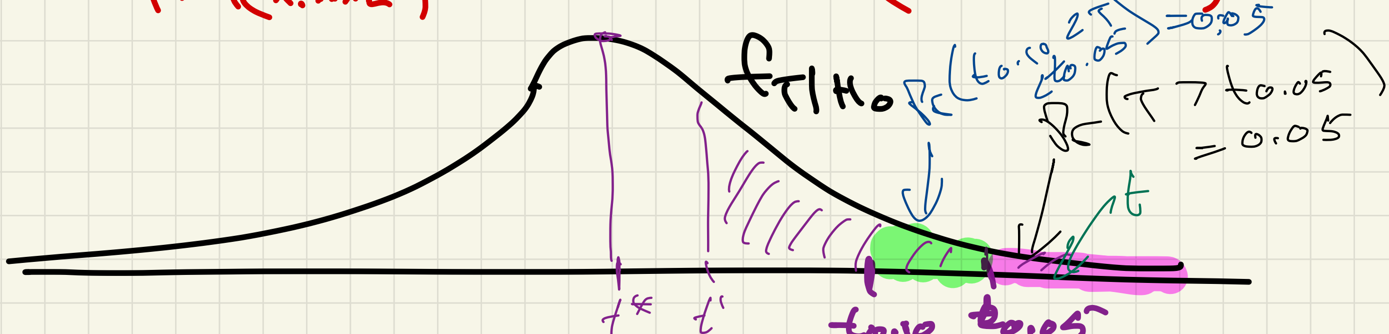
if $t \leq t_\alpha \Rightarrow$ do not reject null hypothesis

Consider $t_{0.1} \le t \le t_{0.05}$

What fraction of experiments would this happen, under H_0

$$\Pr_{X_i \underset{iid}{\sim} H_0}\left[t_{0.1} \le T \le t_{0.05}\right] = (0.1 - 0.05) = 0.05$$

$T = T(x_1 \ldots x_n)$



p-value : Probability, under H_0, that the realized test statistic t, or something more extreme (eg. larger) could occur.

$$Pr_{H_0}(T \le t) = 1 - p$$

$$\iff Pr_{H_0}(T > t) \doteq p$$

rejecting H_0, using critical value

if $p < 0.05 \iff t > t_{0.05}$

p-value < 0.05

Example: People of Utah are tall!! compared to dwarves.

H_0 : people in Utah same as USA

$N(\underset{\mu}{40}, \underset{\text{not known}}{\sigma^2})$

H_1 : $\mu_U > 40$

Random Sample Utah $X_1, \ldots, X_n$ $\quad n = 64$

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$$

$$T = \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}} = \frac{\bar{X}_n - 40}{S_n/8} \sim t(df = 63)$$

critical value at α

$$\alpha = 0.05 \implies t_\alpha = qt(1-\alpha, df = 63)$$

Draw real sample from Utah

$\bar{X}_n = 68$ inches $\qquad S_n^2 = 36 \text{ in}^2$

$$\frac{\bar{X}_n - 40}{S_n / 8} = \frac{28}{6/8} = \frac{4}{3} \cdot 28 = 36.33 = t$$

$$t_{0.05} \approx 1.97$$

$t > t_{0.05} \implies$ reject null hypothesis

p-value $\quad \Pr_{H_0}(T > t)$

R: 1-pt(t, df=63) $\qquad$ p-value $\leq 1 \cdot 10^{-100}$

① $H_0: X_i \sim N(\mu, \sigma^2)$

$H_1: \mu^* > \mu$ interesting

② Set experiment

$$T = \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}} \sim t(n-1)$$

critical value at α

t_α s.t.

$$\Pr(T \le t_\alpha) = 1-\alpha$$

$$t_\alpha = qt(1-\alpha, df=n-1)$$

③ Collect data

$t = T(x_1, \dots, x_n)$

if $t > t_\alpha$ → reject H_0