ProbStats L07a

Continuous Random Variables

February 14, 2023

Random Variable : $X : \Omega \to \mathbb{R}$

Discrete Ω

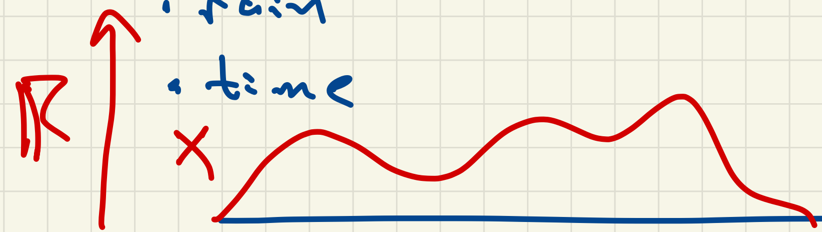
Ω	$X(\omega)$
$\omega = 1$	0.7

pdf $f_X(a) = Pr(X = a) \in [0, 1]$

cdf $\boxed{F_X(a) = Pr(X \le a)}$

$\Omega \subseteq \mathbb{R}$

- snow
- rain
- time



Ω

pdf $Pr(X = a) = 0$

$f_X(a) =$ ___

cdf

$$\boxed{F_X(a) = Pr(X \le a)} = \int_{x=-\infty}^{a} f(x)\,dx$$

Contin R.V. $X \sim f_X$

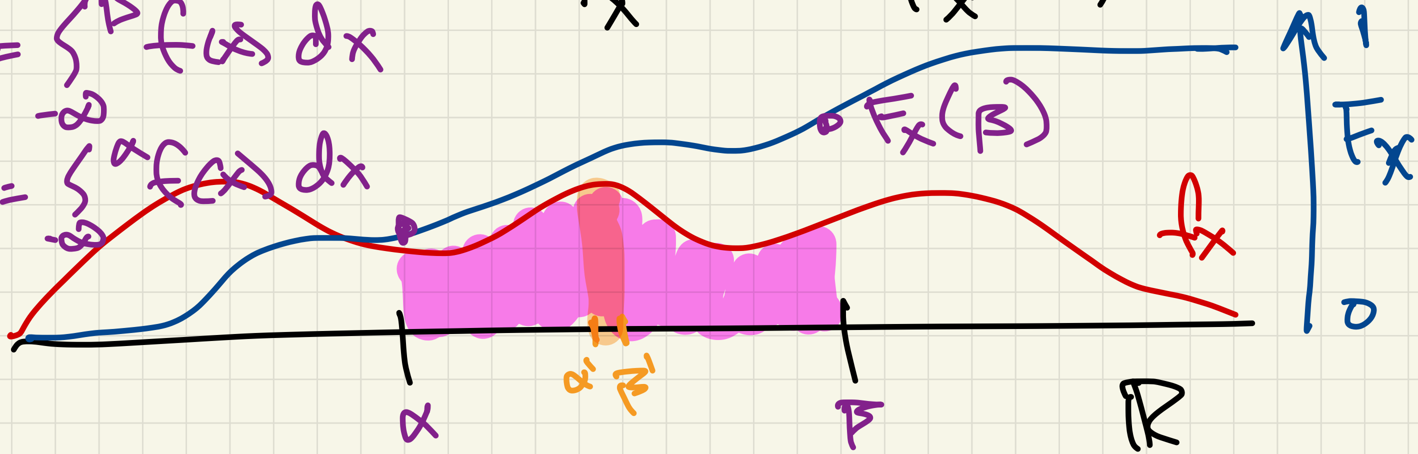
$$\boxed{F_X(a) = \Pr(X \le a)}$$

$$\Pr(\alpha \le X \le \beta) = \int_{x=\alpha}^{\beta} f_X(x)\,dx$$

$$= F_X(\beta) - F_X(\alpha)$$

$$F_X(\beta) = \int_{-\infty}^{\beta} f(x)\,dx$$

$$F_X(\alpha) = \int_{-\infty}^{\alpha} f(x)\,dx$$



continuous pdf

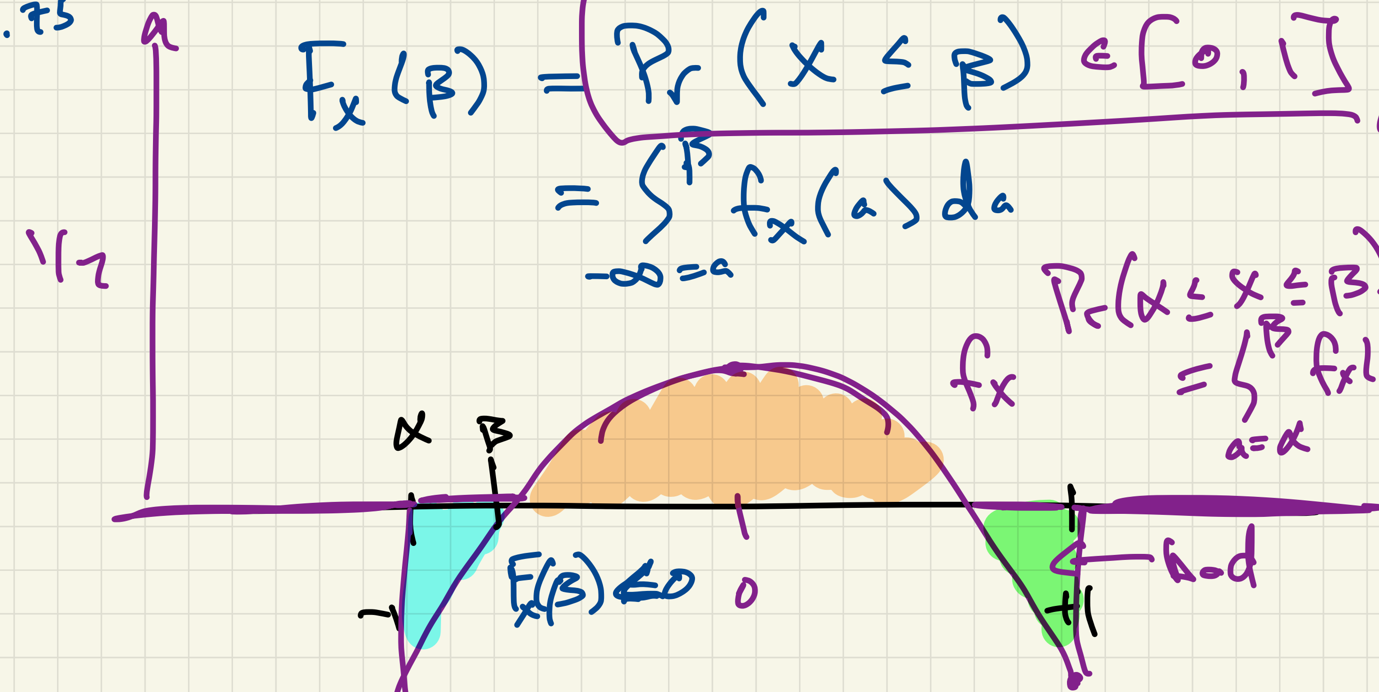
$$f(x) = \begin{cases} \frac{1}{2} - 2x^2 & \text{if } x\,[-1, +1] \\ 0 & \text{o.w.} \end{cases}$$

$\beta = -0.75$

$$\boxed{F_X(\beta) = \Pr(X \le \beta) \in [0, 1]}$$

$$= \int_{-\infty = a}^{\beta} f_X(a)\, da$$

$$\Pr(\alpha \le X \le \beta) = \int_{a=\alpha}^{\beta} f_X(a)\, da$$



Rules of continuous pdf

f_X

① $f_X(a) \geq 0 \quad \in [0, \infty)$

② $\int_{a=-\infty}^{\infty} f_X(a)\, \partial a = 1$

Rules for cdf

① $F_X(a) \in [0, 1]$

② $F_X(-\infty) = 0 \quad // \quad F_X(\infty) = 1$

③ $= a > a'$

$F_X(a) \geq F_X(a')$

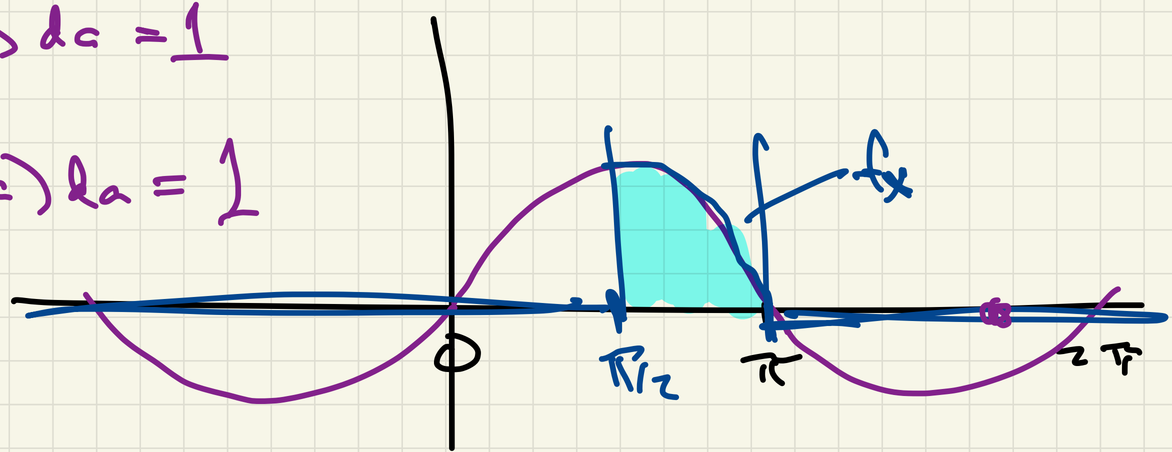
$$f_x = \begin{cases} \sin(x) & \text{if } x \in [\pi/2, \pi] \\ 0 & \text{o.w.} \end{cases}$$

Rules ① check $f_x(a) \geq 0 \quad \forall a$

$\sin(\pi/2) = 1 \qquad \sin(\pi) = 0$

② $\int_{-\infty}^{\infty} f_x(a)\, da = 1$

$= \int_{\pi/2}^{\pi} f_x(a)\, da = 1$

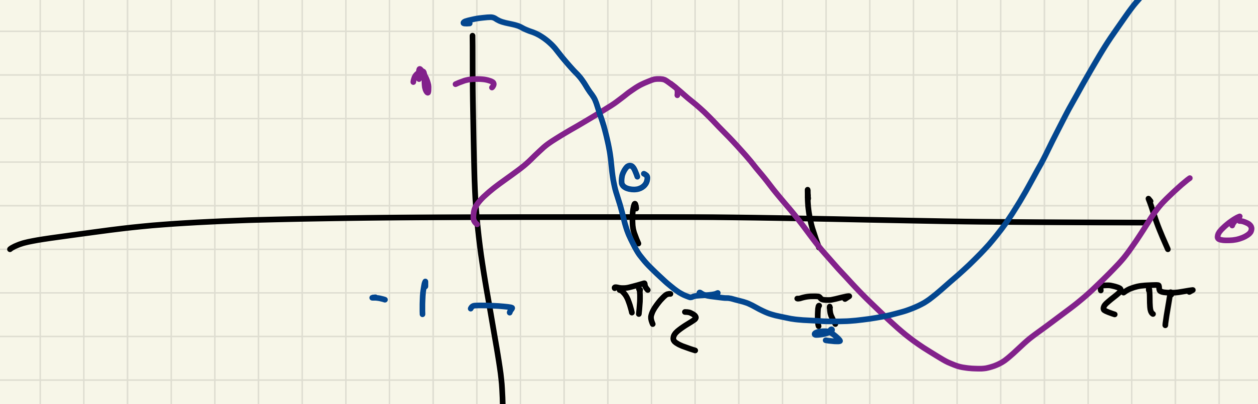


$$\int_{a=\pi/2}^{\pi} \sin(a)\,da = -\cos(a)\Big|_{a=\pi/2}^{a=\pi}$$

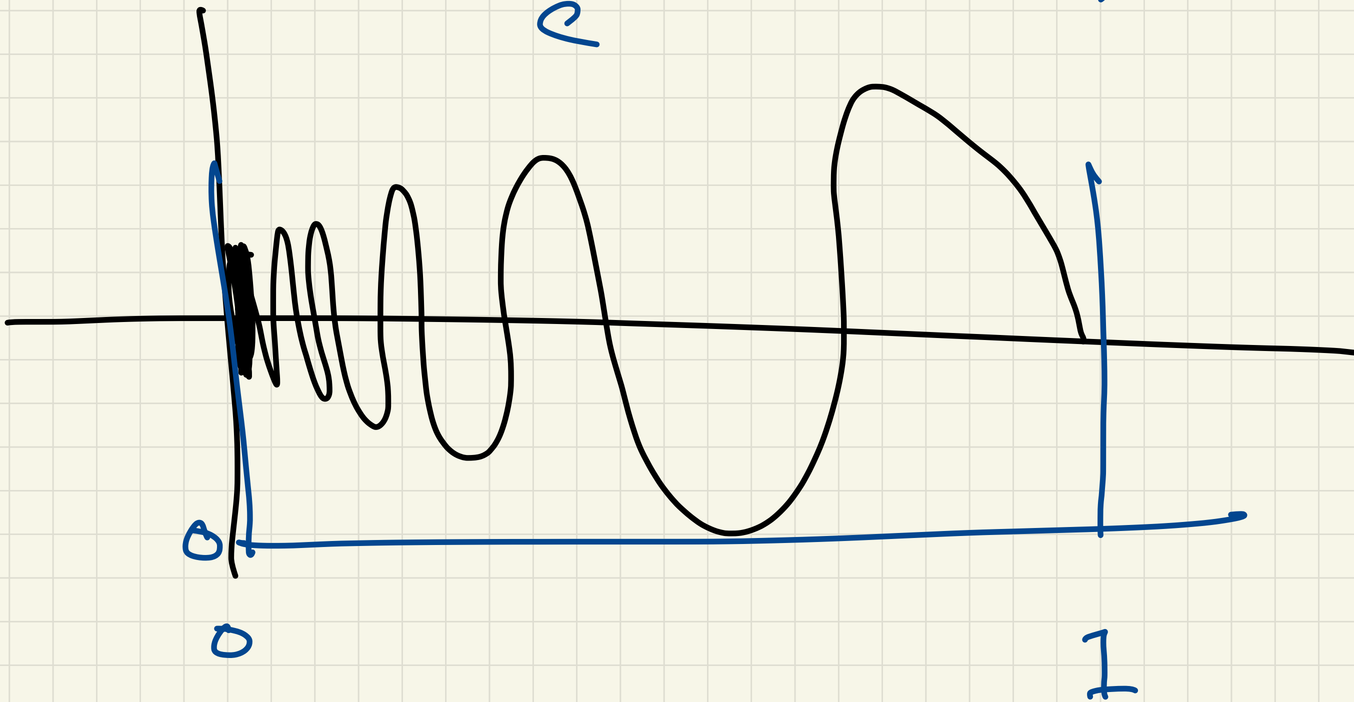
$$= -\cos(\pi) - \left(-\cos(\pi/2)\right)$$

$$\cancel{0} \quad \cancel{- (1)) = 1}$$

$$-(-1) - (-0) = 1$$



$$\frac{\sin(1/x) + 1}{c}$$?



Unif Distribution $\quad X \sim U(\alpha, \beta)$

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \quad x \; [\alpha, \beta] \\ 0 & \text{o.w.} \end{cases}$$

$$\int_{x=\alpha}^{\beta} f(x)\,dx = 1$$

$$= \cancel{(\beta-\alpha)}\left(\frac{1}{\cancel{\beta-\alpha}}\right) = 1$$

$X \sim U(0, 1/2)$

$$f(1/4) = 2 = \frac{1}{1/2 - 0} = \frac{1}{1/2} = 2$$