


Prob Stats LO6a

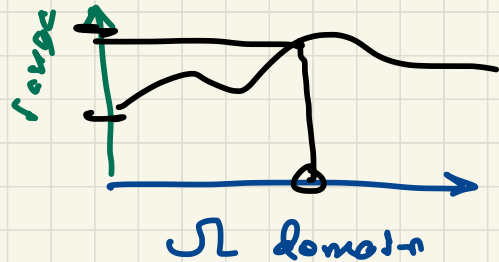
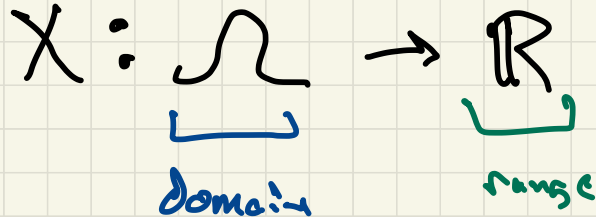
Discrete Random Variables

February 2, 2023



Random Variable X

X is function from
sample space Ω
to the real values \mathbb{R}



Example

Coin sample space

$$\Omega = \{H, T\}$$

Ω	X
H	0
T	1

X (input)

Flip coin twice

$$\Omega = \{HH, HT, TH, TT\}$$

Ω	X
HH	0
HT	0
TH	1
TT	1

$$P_c(X=1) = \frac{1}{4}$$

$$P_c(X=0) = \frac{1}{2}$$

$$P_c(\underline{X < 2}) = \frac{3}{4}$$

Roll 2 Dice $D_1 = \{1, 2, \dots, 6\}$ $D_2 = \{1, 2, \dots, 6\}$

$$\Omega = \{11, 12, \dots, 16, 21, 22, \dots, 66\}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

SZ

Random Variable

$S =$ sum of values of two dice

$$\{S=9\}$$

$$\{S=4\} = \{31, 22, 13\}$$

$$P_r(S=4) = \frac{|\{S=4\}|}{|\Omega|} = \frac{3}{36} \quad \{S=a\} = \{\omega \in \Omega : S(\omega) = a\}$$

Roll 2 Die $D_1, D_2 = \{1, \dots, 6\}$

R.V. $X =$ product of outcomes of D_1, D_2

• $P_r(X=a)$ for

$a = 3, 4, 12, 13$

$$P_r(X=3) = \frac{2}{36}$$

$$\{X=3\} = \{13, 31\}$$

$$P_r(X=4) = \frac{3}{36} = \frac{1}{12}$$

$$\{X=4\} = \{14, 22, 41\}$$

$$P_r(X=12) = \frac{1}{9}$$

$$\{X=12\} = \{34, 43, 26, 62\}$$

$$P_r(X=13) = 0$$

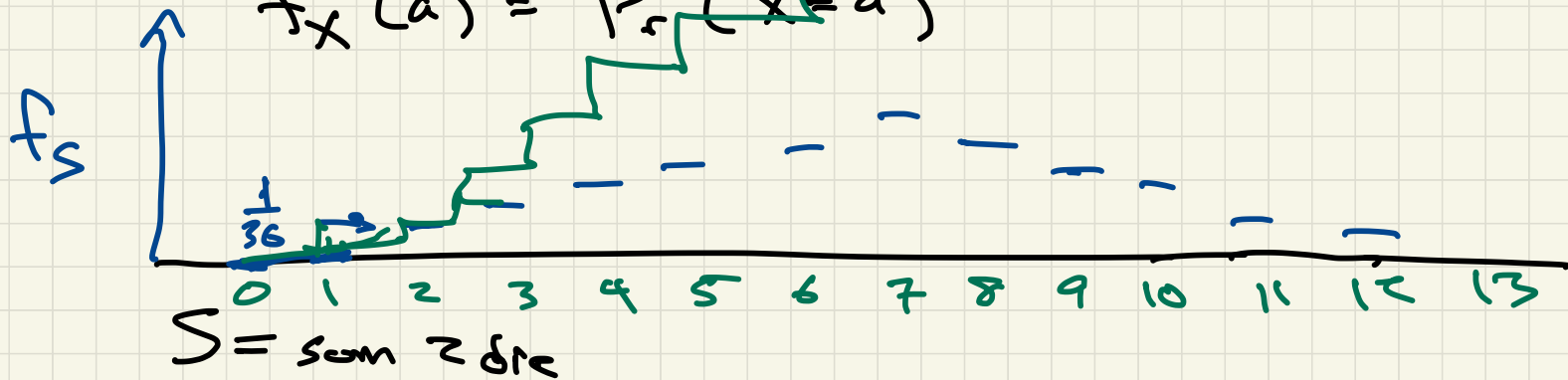
probability density function pdf

probability mass function pmf

$$f: \mathbb{R} \rightarrow [0, 1]$$

f_x is the pdf for R.V. x

$$f_x(a) = P_r(x=a)$$



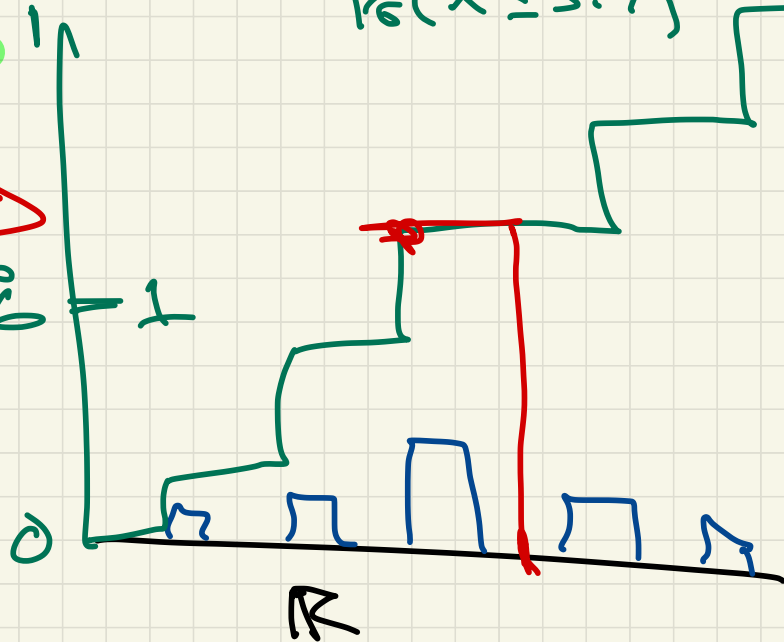
$S = \text{Sum of } 2 \text{ Dice}$

a	$f_S(a)$	$F_S(a)$
-1	0	0
2	1/36	1/36
3	2/36	3/36
4	3/36	6/36
5	4/36	10/36
6	5/36	15/36
7	6/36	21/36
8	5/36	26/36
9	4/36	30/36
10	3/36	33/36
11	2/36	35/36
12	1/36	36/36
13	0	1

$$F_S(1.5) = 0$$

$$F_S(5.7) = \frac{10}{36}$$

$$P_S(X \leq 5.7)$$



Cumulative Density Function (cdf)

$$F : \mathbb{R} \rightarrow [0, 1]$$

F_x : cdf for R.V. X

$$F_x(a) = P_r(X \leq a)$$