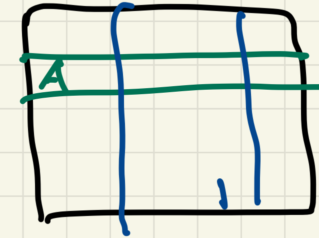
Prob Stats L05

Bayes' Rule

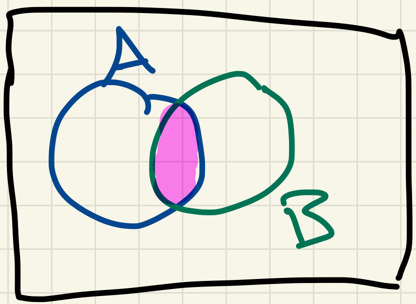
Jan 31, 2023

Two events A, B



$$\Pr(A \mid B) \overset{?}{\neq} \Pr(B \mid A)$$

$$= \frac{\Pr(A \cap B)}{\Pr(B)} \qquad\qquad = \frac{\Pr(A \cap B)}{\Pr(A)}$$



True if $\Pr(A) = \Pr(B)$

or $\Pr(A \cap B) = 0$

Bayes' Rule

$$\Pr(A \mid B) = \frac{\Pr(B \mid A) \cdot \Pr(A)}{\Pr(B)}$$

$$= \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$= \frac{\Pr(B \mid A) \cdot \Pr(A)}{\Pr(B)}$$

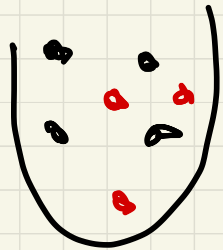
$$\Pr(B \mid A) = \frac{\Pr(B \cap A)}{\Pr(A)}$$

$$\Pr(B \mid A) \cdot \Pr(A) = \Pr(B \cap A)$$

$$= \Pr(A \cap B)$$

Example

urn 1



$Pr(B \mid U1) = \frac{4}{7}$

urn 2

$$Pr(R) = \left(\frac{1}{2}\right) \cdot \frac{3}{7} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{13}{28}$$

$$Pr(B) = \frac{15}{28} = 1 - Pr(R)$$

Q: $$Pr(U1 \mid B) = \frac{Pr(B \mid U1) \cdot P(U1)}{Pr(B)}$$

$$= \frac{(4/7) \cdot (1/2)}{(15/28)} = \frac{8}{15}$$

$Pr(M) = 0.1$

$Pr(M^c) = 1 - Pr(M)$

$Pr(B|M^c) = 0.1$

$Pr(B|M) = 0.15$

Law of Total Probability

$Pr(A) = Pr(A|E)Pr(E) + Pr(A|E^c)\cdot Pr(E^c)$

$\boxed{Pr(B)} = 0.105$

$= Pr(B|M)\cdot Pr(M) + Pr(B|M^c) \cdot P(M^c)$

$(0.15)(0.1) + (0.1)(0.9)$

$= 0.105$

$Pr(M|B) = ?$

$$= \frac{Pr(B|M)\cdot P(M)}{Pr(B)}$$

$$= \frac{(0.15)\cdot(0.1)}{(0.105)} = 0.143$$

Bayes' Rule in Machine Learning

M : my model describes the world

D : this is the data I collected.

$$\Pr(M \mid D) = \frac{\Pr(D \mid M) \cdot \Pr(M)}{\cancel{\Pr(D)}}$$

$$\approx \Pr(D \mid M) \cdot \underbrace{\Pr(M)}_{\text{prior}}$$

$D = \{x_1, x_2 \ldots x_n\}$ independent $\Pr(D) = P(x_1) \cdot P(x_2) \cdot P(x_3) \ldots$

$$\Pr(D) = \Pr(x_1) \cdot \Pr(x_2) \cdot \ldots$$

$$\Pr(x_i \mid M)$$