Prob/Stats L02

Conditional Probability

Jan 19, 2023

Sets

Sample Space Ω ← everything of interest

Subsets $A \subseteq \Omega$

$$Pr(A) = \frac{|A|}{|\Omega|}$$ if all $x \in \Omega$ equally likely

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intersection $A \cap B$ both A and B are true

union $A \cup B$ A or B are true (or both)

complement A^c A is not true.

Probability Rule

$A, B \subseteq \Omega$

Inclusion – Exclusion:

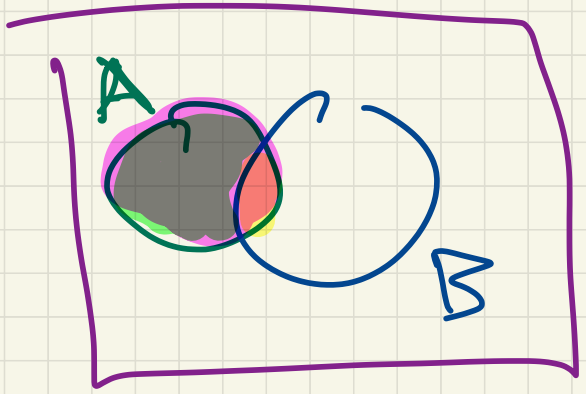
$$P_r(A \cup B) = P_r(A) + P_r(B) - P_r(A \cap B)$$

$0.2 + 0.7 - 0.1 = 0.8$ $\quad 0.2 \quad 0.7 \quad 0.1$ ~~0.3~~

Not Legal

$P_r(A) = 0.7$

$P_r(A \cap B) = 0.3$



Rule $P_r(C \cup D) = P_r(C) + P(D)$ if $C \cap D = \emptyset$

$0.2 \quad 0.3$

Complement Rule

$$P_r(A^c) = 1 - P_r(A)$$

Difference Rule

$$P_r(A - B) = P_r(A) - P_r(A \cap B)$$

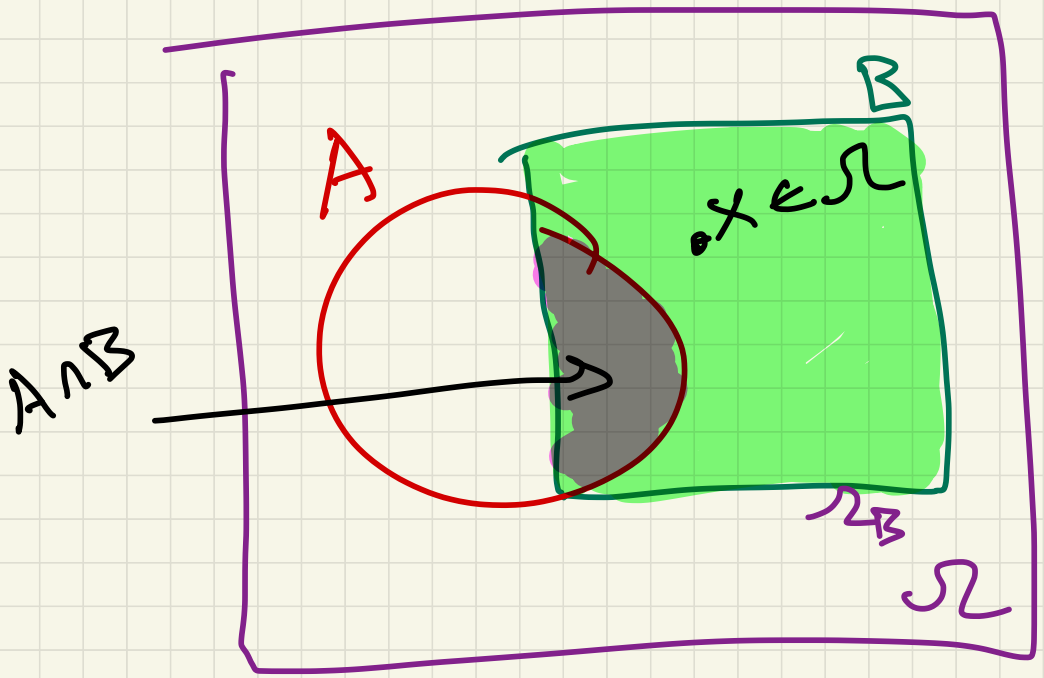
Conditional Probability

events A, B

$Pr(A \mid B)$

probability of A, if we know that B is true

$$Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)}$$



$$P_r(A \mid B) = \frac{P_r(A \cap B)}{P_r(B)}$$

$$= \frac{0.1}{0.5}$$

$$= \frac{1}{5} = \boxed{0.2}$$

$P_r(B) = 0.5$

$P_r(A) = 0.3$

$P_r(A \cap B) = 0.1$

Brain Teaser 2 coins, fair

B = at least one coin heads

$$\Omega_B = \{HT, TH, HH\}$$

A = second coin heads

$\{HH\}$

$$P(A|B) = \frac{|\{HH\}|}{|\{HT, TH, HH\}|} = \frac{1}{3}$$

Multiplication Rule

events A, B

$$P_r(A \cap B) = P_r(A \mid B) \cdot P_r(B)$$

$$P_r(B) \cdot P_r(A \mid B) = \frac{P_r(A \cap B)}{P_r(B)} \cdot P_r(B)$$

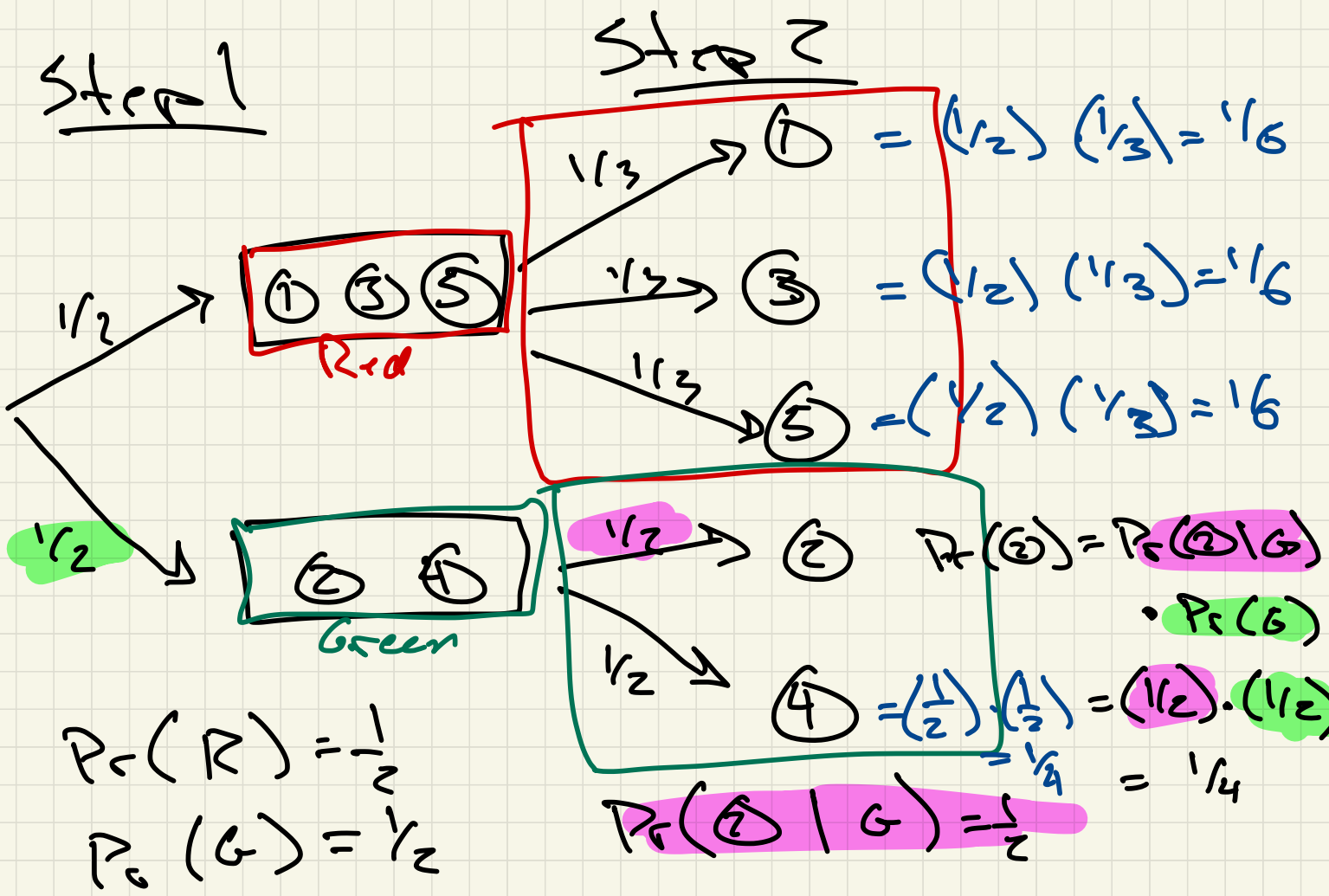
Tree Diagram : two-stage problems

2 Boxes contain 5 balls $\{1, 2, 3, 4, 5\}$

Step 1 choose 1 box (w/ equal prob)

Step 2 choose 1 ball from box.

? Prob 1 choose (2)?


Step 1
Step 2
1/2
1/2
1 3 5
Red
6 4
Green
1/3
1/3
1/3
1
3
5
= (1/2)(1/3) = 1/6
= (1/2)(1/3) = 1/6
= (1/2)(1/3) = 1/6
1/2
1/2
2
4
P(2) = P(2|G) · P(G)
= (1/2)·(1/2) = 1/4
P(R) = 1/2
P(G) = 1/2
P(2 | G) = 1/2

Sampling without Replacement

1 box 10 green balls
10 red balls
draw 2 balls (w/o replacement)

Probability of 2 red balls?

$R1$ = event that ball 1 red

$R2$ = event that ball 2 red

$Pr(R1) = \frac{10}{20} = \frac{1}{2}$

$Pr(R1 \cap R2) = Pr(R2 \mid R1) \cdot Pr(R1)$

$\frac{9}{19} \cdot \frac{1}{2} = \frac{9}{38}$ $=$ $Pr(R2 \mid R1) = \frac{9}{19}$

Pack of 3 red balls w/o replacement

10 red, 10 green

$R1, R2, R3$

$P_r(R1 \cap R_2) = \frac{9}{38}$

$P_r((R1 \cap R2) \cap R3) = P_r(R3 \mid R1 \cap R2) \cdot P_r(R1 \cap R2)$

$= \frac{9}{38} \cdot \frac{4}{9} = \frac{4}{38} = \frac{2}{19}$

$P_r(R3 \mid R1 \cap R2) = \frac{8}{18} = \frac{4}{9}$