Sample Spaces, Events, Probability

CS 3130/ECE 3530:
Probability and Statistics for Engineers

Jan 12, 2023

Prof Phillips

pronouns he/him

Sets

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Not a valid set definition: $C = \{1, 2, 3, 4, 2\}$

multiset

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$(1,2,3) \neq (3,1,2)$

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- When x is an element of A, we denote this by:
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- The “empty” or “null” set has no elements:
$$\emptyset = \{\ \}$$

Some Important Sets

- Integers:

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

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$$5 \in \mathbb{R}, \quad 17.42 \in \mathbb{R}, \quad \pi = 3.14159\ldots \in \mathbb{R}$$

Building Sets Using Conditionals

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- Alternate way to define natural numbers:

such that

$$\mathbb{N} = \{x \in \mathbb{Z} : x \geq 0\}$$

$\{0, 1, 2, \ldots\}$

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- Set of even integers:

$$\{x \in \mathbb{Z} : x \text{ is divisible by } 2\}$$

$\{\ldots, -4, -2, 0, 2, 4, 6, \ldots\}$

Building Sets Using Conditionals

$e = 2.71\ldots$

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- Rationals:

$$\mathbb{Q} = \{\, p/q : p, q \in \mathbb{Z}, q \neq 0\}$$

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- $\emptyset \subseteq A$ for any set A
- $A \subseteq A$ for any set A (but $A \not\subset A$)

no bar

$A \subset B$

$\subset$ strict subset

some $x \in B$ s.t. $x \notin A$

$A \subseteq B$

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heads

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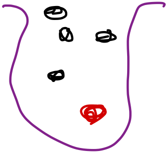

Examples:

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- Roll a 6-sided die: $\Omega = \{1, 2, 3, 4, 5, 6\}$
- Pick a ball from a ~~bucket~~ urn of red/black balls: $\Omega = \{R, B\}$

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4 x 6-sided die

6^4 outcomes

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- Tossing 2 coins? $\{HH, TT, HT, TH\}$

10 coins size$(\Omega) = 2^{10}$

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- Roll a 6-sided die: $\Omega = \{1, 2, 3, 4, 5, 6\}$
- Pick a ball from a bucket of red/black balls: $\Omega = \{R, B\}$
- Tossing 2 coins?
- Shuffling deck of 52 cards? $= \prod_{i=1}^{52} i$

$52! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 51 \cdot 52$

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- You flip a coin and it comes up "heads":
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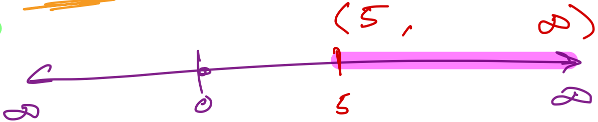
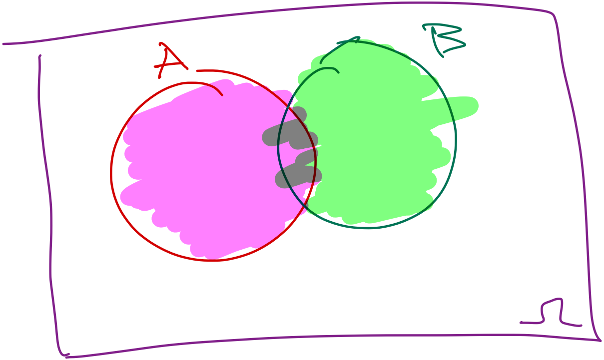
- Your code takes longer than 5 seconds to run:
  $(5, \infty) \subseteq \mathbb{R}$



Set Operations: Union

Definition

The **union** of two sets A and B, denoted $A \cup B$ is the set of all elements in either A or B (or both).



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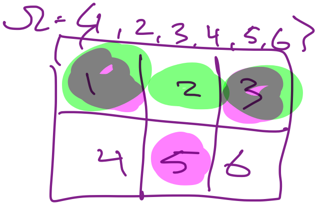

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$A = \{1, 3, 5\}$ “an odd roll”

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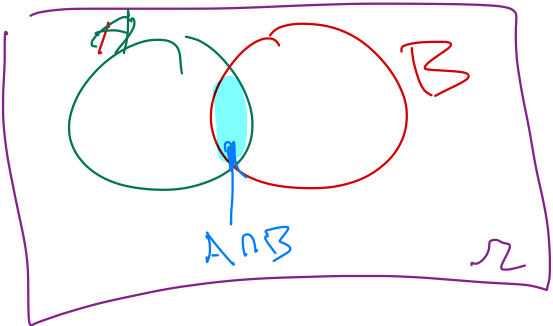

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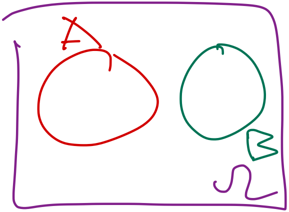

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Note: If $A \cap B = \emptyset$, we say A and B are **disjoint**.

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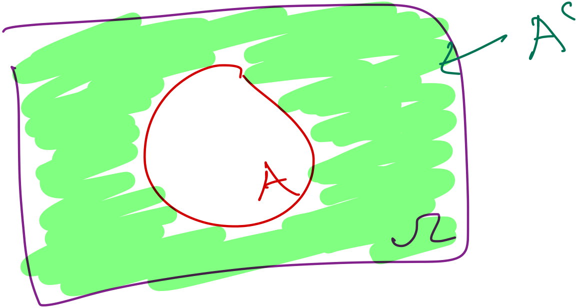

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$A^c = \{2, 4, 6\}$ "an even roll"

Set Operations: Difference

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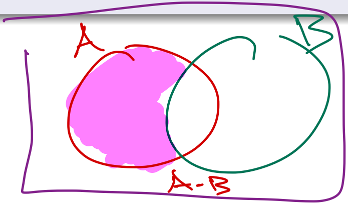
The **difference** of a set $A \subseteq \Omega$ and a set $B \subseteq \Omega$, denoted $A - B$, is the set of all elements in Ω that are in A and are not in B.



Example:

$A = \{3, 4, 5, 6\}$
$B = \{3, 5\}$
$A - B = \{4, 6\}$

Note: $A - B = A \cap B^c$