

Notes: Bayes' Rule

CS 3130/ECE 3530: Probability and Statistics for Engineers

January 31, 2023

Bayes' Rule: So far we've talked about conditional probabilities $P(A|B) = P(A \cap B)/P(B)$. Bayes' Rule allows us to switch the order of A and B , that is, to compute $P(B|A)$. Here's how it works. First, use the multiplication rule two different times:

$$P(A \cap B) = P(A|B)P(B)$$

and

$$P(A \cap B) = P(B \cap A) = P(B|A)P(A)$$

This tells us that the two right hand sides above are equal:

$$P(A|B)P(B) = P(B|A)P(A)$$

Solving for $P(B|A)$, we get **Bayes' Rule:**

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

In-Class Problem: You have two urns, one with 4 black balls and 3 white balls, the other with 2 black balls and 2 white balls. You pick one urn at random and then select a ball from the urn. What is the probability the ball is white?

If you picked a black ball, what is the probability that you had picked the first urn (the 4 black, 3 white urn)?

In-Class Problem: You have a system with a main power supply and auxiliary power supply. The main power supply has a 10% chance of failure. If the main power supply is running, the auxiliary power supply also has a 10% chance of failure. But if the main supply fails, the auxiliary supply is more likely to be overloaded and has a 15% chance to fail. What is the probability that the auxiliary power will fail?

If the auxiliary power fails, what is the probability that the main power also failed?

Bayes' in Machine Learning: Bayes' Rule is crucial in Machine Learning, where a main problem is to learn a model that describes something in the real world. There are typically many possible models, and the question becomes: What is the best model to describe the data that I have?

We can write this using Bayes' Rule as:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

Here M is the event "my model describes the real world", and D is the event "this is the data I've collected." So, in English the left-hand side, $P(M|D)$, is "the probability that my model describes the real world given that this is the data I've collected."