CS3130/ECE3530: Probability and Statistics for Engineers **Due Tu 3/28**

Name:

uID:

Homework 5: Joint Probability, Independence, Covariance, and Correlation

Instructions: Write your answers directly on this pdf (via an editor, iPad, or pen/pencil). The answers should be in the specified place. Students will be responsible for loading their assignments to GradeScope, and **ensuring the answers align with the GradeScope template**.

The assignment should be uploaded by 11:50pm on the date it is due. There is some slack built into this deadline on GradeScope. Assignments will be marked late if GradeScope marks them late.

If the answers are too hard to read you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

1. The US Women's National Soccer team schedules a home-and-home series with the German team, where each team plays one game in their home country. A win is worth 3 points, a loss worth 0 points, and a tie 1 point. The expected outcome for how many points team USA will earn in each game (home or away) is given in the following probability table.

		Home		
		0 (= loss)	1 (= tie)	3 (= win)
Away	0 (= loss)	0.05	0.10	0.25
	1 (= tie)	0.05	0.05	0.05
	3 (= win)	0.05	0.10	0.30

Answer the following questions, showing and explaining your work:

(a) What is the probability that the USA does not win at least one game?

(b) What is the marginal distribution for the USA's points at Home?

(c) Is the event that USA loses at home independent of the event that they tie away?

(d) Is the result from Home independent from the result from Away?

(e) What is the expectation of the total points that the USA earns (the sum of Home and Away)?

(f) If we know there are no ties, what is the conditional probability? (express as a probability table)

(g) Does the USA expect to earn more points if we know there will be no ties, (compared to if we do not have the knowledge)?

2. In January, let X represent the fraction of time it is snowing at Snowbird, and Y represent the fraction of time it is snowing at the SLC airport. Say they have the following joint density of X and Y

$$f(x,y) = \begin{cases} \frac{2}{11}(10x+y) & \text{for } x,y \in [0,1]\times[0,1] \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the marginal probability it is snowing at Snowbird $f_X(x)$?

(b) What is the probability that it is snowing at Snowbird less than half of the time (i.e., $\Pr(\{X \leq \frac{1}{2}\})$)?

(c) What is the conditional probability it is snowing at Snowbird less than half the time, if it is snowing at SLC exactly half the time?

(d) What is the covariance $\mathrm{Cov}(X, Y)$?

(e) What is the correlation $\rho(X, Y)$?