CS3130/ECE3530: Probability and Statistics for Engineers **Due Tu 3/14**

Name:

uID:

Homework 4: Continuous Random Variables, Expectation, Variance

Instructions: Write your answers directly on this pdf (via an editor, iPad, or pen/pencil). The answers should be in the specified place. Students will be responsible for loading their assignments to GradeScope, and identifying what page contains each answer.

The assignment should be uploaded by 11:50pm on the date it is due. There is some slack built into this deadline on GradeScope. Assignments will be marked late if GradeScope marks them late.

If the answers are too hard to read you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

1. A lecture hall is having wiring issues, and the projector occasionally powers down randomly following an exponential distribution with rate λ (power downs per minute).

   (a) A professor plans to give an 80 minute lecture. What is the minimal power-down rate λ that the projector could have so that there is a probability of 0.05 that the lecture will finish before the projector shuts down?

   (b) If the lecture is only 50 minutes, how does this change the minimal rate needed to get a probability of 0.05 of finishing the lecture without an outage?

   (c) Given the rates λ that you solved for in parts (a) and (b), what is **expected value** of how much time occurs between power-down events under the exponential distribution mode?

2. Suppose X is a binomial random variable where $n = 10$ is the number of trails and $p = 0.5$ is the probability of success of any trial.

   (a) Use the `plot` function in R to plot a histogram/stick plot using type="h" representing the distribution of X in RED. **Provide R code and plot in your response.**

(b) Use the `lines` function in R to superimpose a normal curve in BLUE on the plot from part (a) using the mean and standard deviation of X, i.e. use $\mu = np$ and $\sigma = \sqrt{np(1-p)}$. **Provide R code and plot in your response.**

(c) Use `pbinom` to calculate $\Pr(X \leq 4)$ and use `pnorm` to calculate $\Pr(X \leq 4)$. In this case, is the normal distribution a good approximation of the binomial distribution? **Provide R code in your response.**

(d) Repeat parts (a), (b) with $n = 100$ trials. Also, use `pbinom` to calculate $\Pr(X \leq 60)$ and use `pnorm` to calculate $\Pr(X \leq 60)$. In this case, is the normal distribution a good approximation of the binomial distribution? **Provide R code and plots in your response. You only need one plot (with two distributions) in your response.**

3. You are playing a game where you roll a die and win \$0 for rolling a 1, 2, or 3; \$1 for rolling a 4 or 5; and \$3 for rolling a 6. Each time you play the game, you must pay \$1.

   (a) How much money are you expected to win each round (including the one you pay)?

   (b) What is the variance of the amount of money that you win?

   (c) Assuming you like to make money, is this a game you want to play?

4. Let $X \sim Unif(0, 3)$. What is $\mathbb{E}[e^{\frac{2X}{3}} - 3]$