Models of Computation for Massive Data

Jeff M. Phillips

August 28, 2013

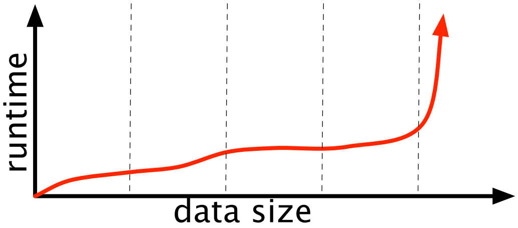
Outline

Sequential:

- External Memory / (I/O)-Efficient
- Streaming

Parallel:

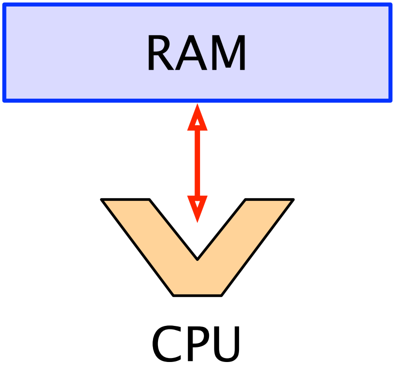
- PRAM and BSP
- MapReduce
- GP-GPU
- Distributed Computing



RAM Model

RAM model (Von Neumann Architecture):

- CPU and Memory
- CPU Operations ($+$, $-$, $*$, $\ldots$) constant time
- Data stored as *words*, not *bits*.
- READ, WRITE take constant time.

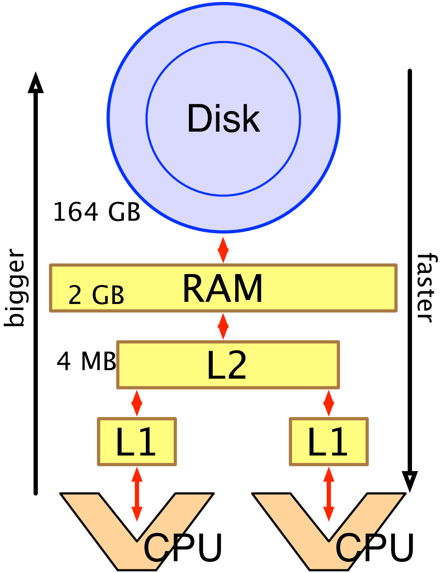


Today's Reality

What your computer actually looks like:

- 3+ layers of memory hierarchy.
- Small number of CPUs.

Many variations!



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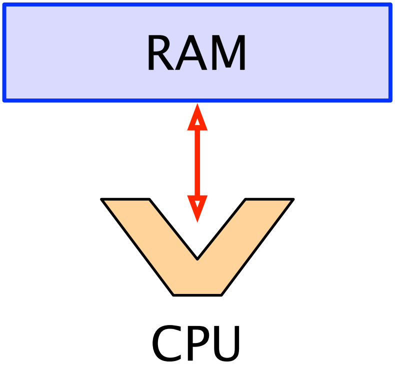
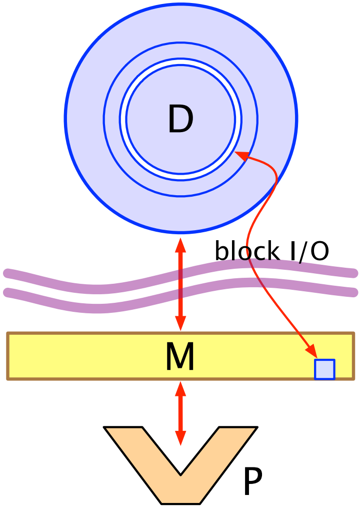


External Memory Model



- N = size of problem instance
- B = size of disk block
- M = number of items that fits in Memory
- T = number of items in output
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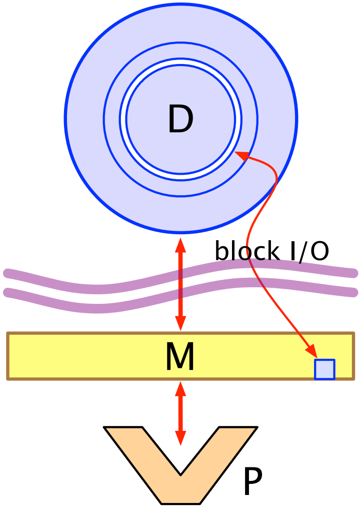

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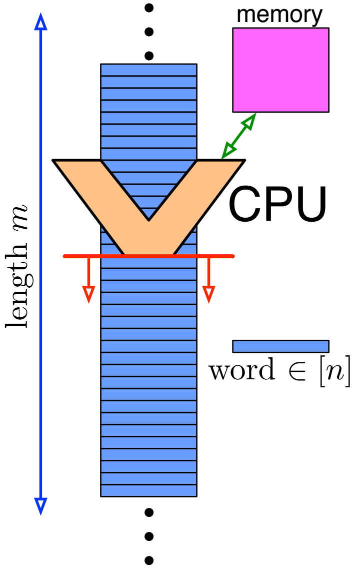
Advanced Data Structures: Sorting, Searching

Streaming Model



CPU makes "one pass" on data

- Ordered set $A = \langle a_1, a_2, \ldots, a_m \rangle$
- Each $a_i \in [n]$, size $\log n$
- Compute $f(A)$ or maintain $f(A_i)$ for $A_i = \langle a_1, a_2, \ldots, a_i \rangle$.
- Space restricted to $S = O(\text{poly}(\log m, \log n))$.
- Updates $O(\text{poly}(S))$ for each a_i.

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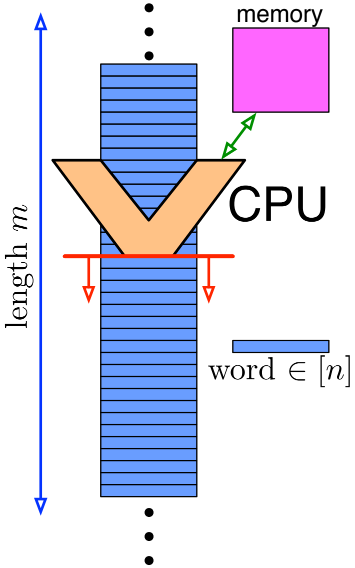

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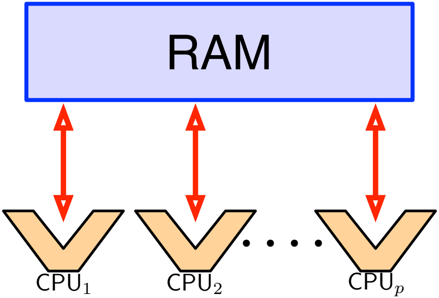
PRAM

Many (p) processors. Access shared memory:

- EREW : Exclusive Read Exclusive Write
- CREW : Concurrent Read Exclusive Write
- CRCW : Concurrent Read Concurrent Write

Simple model, but has shortcomings...
...such as Synchronization.



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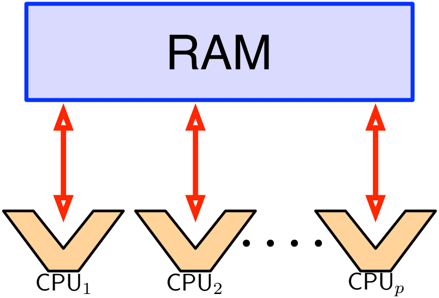
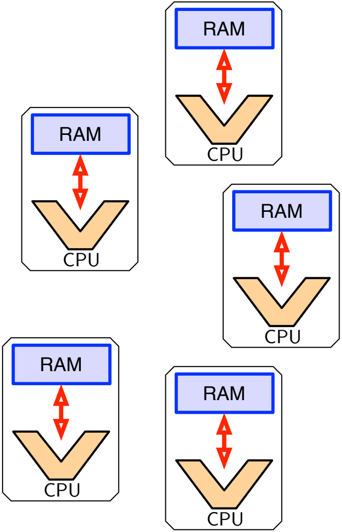


Bulk Synchronous Parallel

Each Processor has its own Memory
Parallelism Procedes in Rounds:

1. Compute: Each processor computes on its own Data: w_i.
2. Synchronize: Each processor sends messages to others: $s_i = \textsc{MessSize} \times \textsc{CommCost}$.
3. Barrier: All processors wait until others done.

Runtime: $\max w_i + \max s_i$



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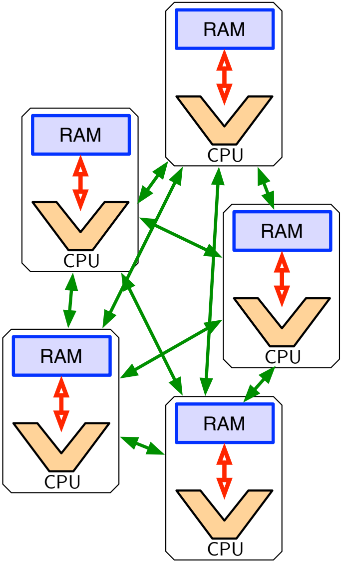

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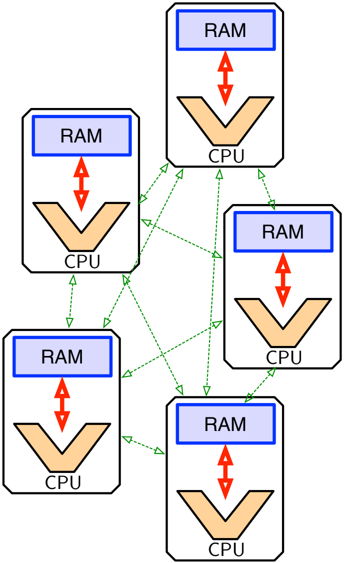

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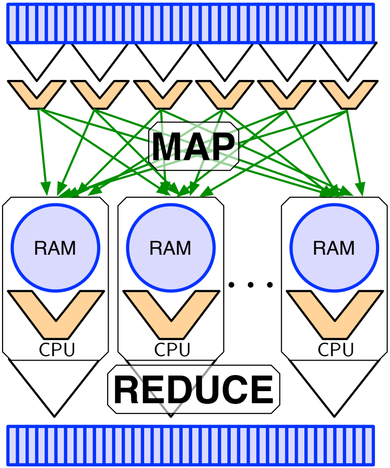
Repeat M+R a constant number of times, often only one round.

- Optional post-processing step.



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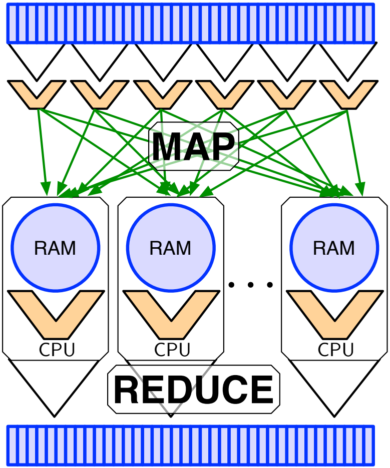

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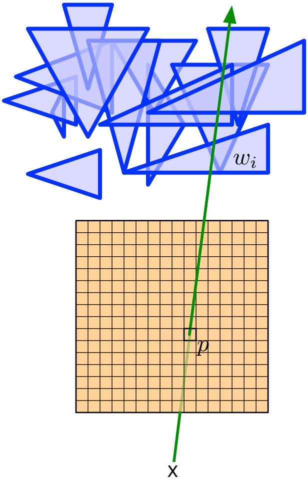
General Purpose GPU

Massive parallelism on your desktop.
Uses **G**raphics **P**rocessing **U**nit.
Designed for efficient video rasterizing.
Each *processor* corresponds to pixel p

- depth buffer:
  $D(p) = \min_i ||x - w_i||$
- color buffer: $C(p) = \sum_i \alpha_i \chi_i$
- ...

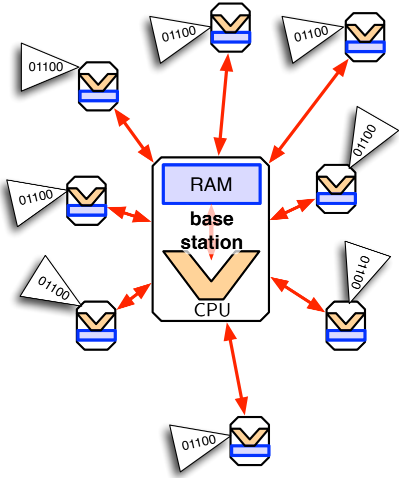


Pro: Fine grain, massive parallelism. Cheap. Harnesses Locality.
Con: Somewhat restrictive model, hierarchy. Small memory.

Distributed Computing

Many small slow processors with data.
Communication very expensive.

- Report to *base station*
- Merge tree
- Unorganized (peer-to-peer)



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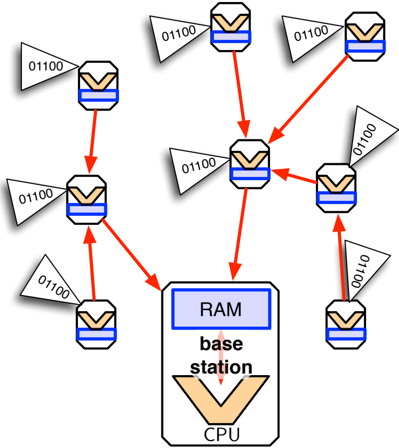

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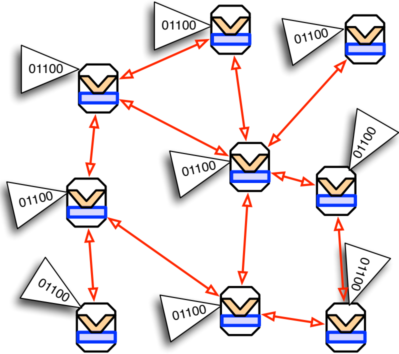

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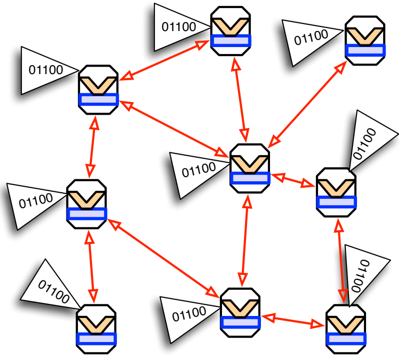

Data collection or Distribution

Advanced Algorithms: Approximate, Randomized

Themes

What are course goals?

- How to analyze algorithms in each model
- Taste of how to use each model
- When to use each model

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Work Plan:

- 1-3 weeks each model.
  - Background and Model.
  - Example algorithms analysis in each model.

I/O	Stream	Parallel	MapReduce	GPU	Distributed
4	5	4	4	3	3

Class Work

1 Credit Students:

- Attend Class. (some Fridays less important)
- Ask Questions.
- If above lacking, may have quizzes.
- Scribing Notes, Video-taping Lectures, or Giving Lectures.

3 Credit Students:
Must also do a project!

- Project Proposal (Aug 30).
  Approved or Rejected by Sept 4.
- Intermediate Report (Oct 23).
- Presentations (Dec 11 or 13).

Sequential Review

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- Single Tape: MoveL, MoveR, read, write
- each constant time
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How large (in seconds) is:

- Searching ($\log n$)
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- EXP: $\exp(n) = c^n$ (usually hopeless ... but 0.00001^n not bad)
- NP: verify solution in P, find solution conjectured EXP
  (If EXP number parallel machines, then in P time)

Data Group

Data Group Meeting

Thursdays @ 12:15-1:30pm in LCR

(to be confirmed)

http://datagroup.cs.utah.edu/dbgroup.php