FoDA : L3

Probability Review:

pdf, cdf, Expectation, Variance

Joint & Marginal Distr.

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Density Functions

RV X

$f_X : \Omega \to \mathbb{R}$

sample space

$[0, \infty)$

"likelihood"

probability density function

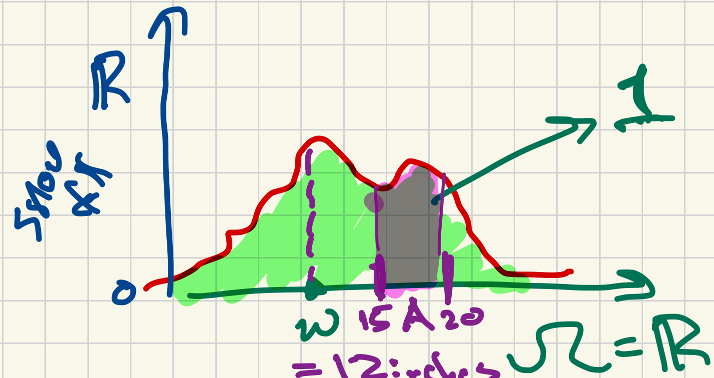
pdf

$$\int_{w \in \Omega} f_X(w)\, dw = 1$$

$$\Pr[X \in [15,20]] = 0.25$$

$$= \int_{w \in [15,20]} f_X(w)\, dw$$

$$\Pr[X = w] = 0$$



Cummulative Density Function cdf

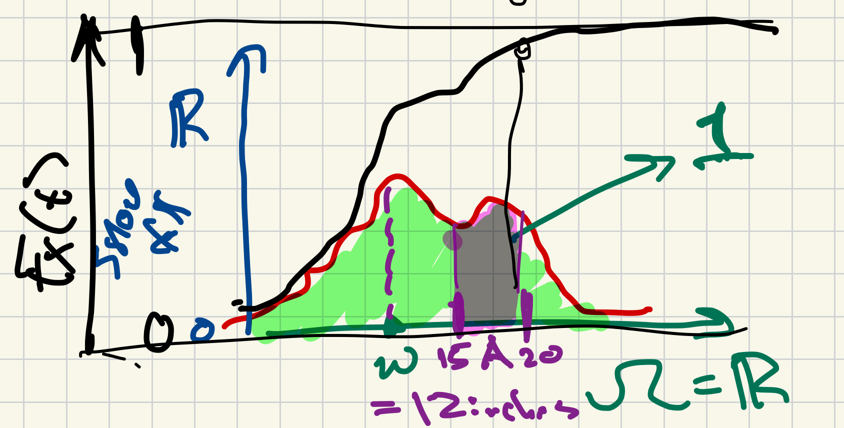
$$F_X : \Omega \to [0,1]$$

$\mathbb{R}$

$$F_X(t) = \int_{w=-\infty}^{t} f_X(w)\, dw$$

$$Pr\left[X \in [15, 20]\right] = F_X(20) - F_X(15)$$

t = Probability of 20 inches or less.



Expected Value

R.V. X

$$E[X] = \sum_{\omega \in \Omega} \left(\omega \cdot \Pr[X = \omega] \right)$$

die

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$\Pr[\omega = X] = \frac{1}{6}$

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$$

$$= 3.5$$

$\Pr[X = 3] = \frac{1}{6}$

Continuous X

$$E[X] = \int_{\omega \in \Omega} \omega \cdot f_X(\omega) \, d\omega$$

$\exp(z) = e^z$

Normal $\quad f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(- \frac{(x - \mu)^2}{\sigma^2} \right)$

$X \sim N(\mu, \sigma)$

Linearity of Expectation

2 R.V.s X, Y

$$E[X+Y] = E[X] + E[Y]$$

fixed scalar value $\alpha \in \mathbb{R}$

$$E[\alpha X] = \alpha \cdot E[X]$$

but

multiply

$$E[X \cdot Y] \neq E[X] \cdot E[Y]$$

does not work.

height : true height basketball player H

: height shoes S

Shoes S cm	$S=1$	$S=2$	$S=3$	$S=4$
	0.1	0.1	0.5	0.3

$E[S] = 3\,cm$

$$H = \mathcal{N}(\mu = 1.755\,m, \sigma = 0.1\,m)$$

$$E[H] = 1.755\,m$$

$$E[H + S/100] = E[H] + \frac{1}{100}E[S] = 1.755\,m + \frac{1}{100}(3\,cm) = \boxed{1.785\,m}$$

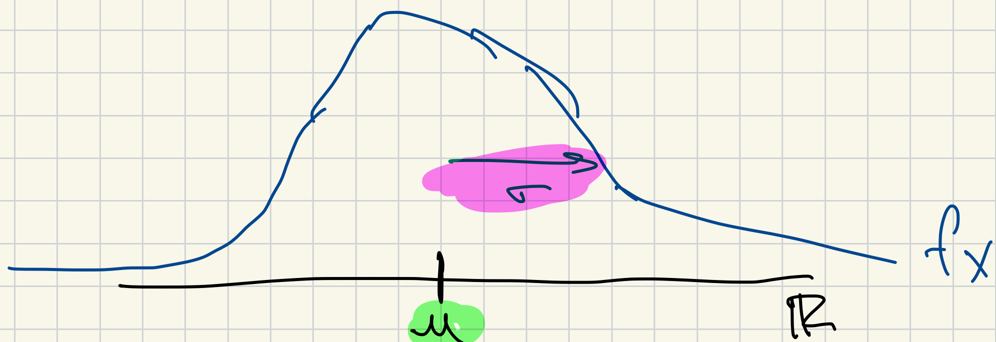
Variance

R.V. X

$$Var[X] = E[(X - \underbrace{E[X]}_{\text{constant}})^2]$$

$$= E[X^2] - (E[X])^2 \neq 0$$

$\sigma = \sqrt{Var[X]}$ = standard deviation.



$X \sim N(\mu, \sigma)$

Joint Random Variables

R.V. X, Y

Covariance $\quad Cov[X,Y] = E\left[(X - E[X])(Y - E[Y])\right]$

pdf

not exactly

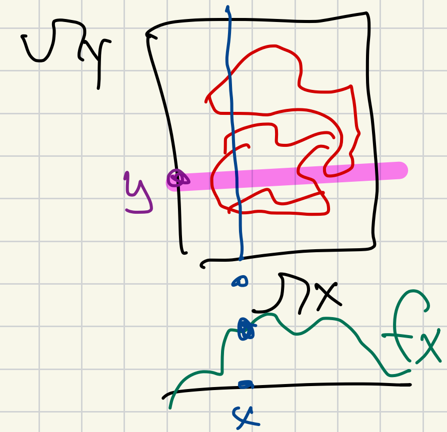
$$f_{X,Y}(x,y) \simeq Pr[X = x, Y = y] = 0$$

if continuous

$$\int_{x \in \Omega_X} \int_{y \in \Omega_Y} f_{X,Y}(x,y)\, dx\, dy = 1$$

Marginal Distribution

$$f_X(x) = \int_{y=-\infty}^{\infty} f_{X,Y}(x,y)\, dy$$



Ω_X

Conditional Distribution

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$