L11: Linear Regression

Basics in 1 dimension

Feb 11, 2026
FoDA
Jeff M. Phillips

{UCBPC}

Galentine's

INDUSTRY PANEL

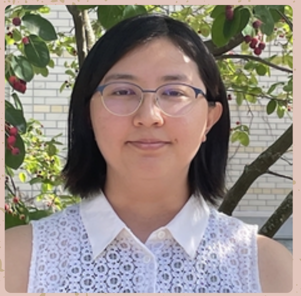
dinner | networking | panel

Join UCBPC for a Galentine's Panel featuring women in the computing industry! Grab a slice of pizza & network with panelists!

Gardner Commons | 2900
February 11 | 5 - 6 PM

Tasnim Rahman
PhD Student

Mou Nandi
CEO of Monere



Jenny Lin
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Questions? Contact @ucbpc_soc on Instagram

Input Data $(X, y) = \{(x_1, y_1), (x_2, y_2), \dots (x_n, y_n)\}$

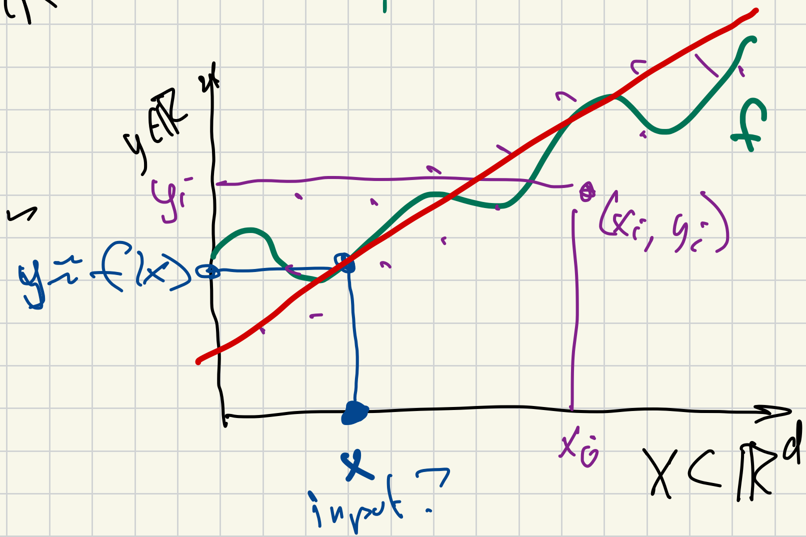
$X = \{x_1, x_2 \dots x_n\} \subset \mathbb{R}^d$ $d = 1$ explanatory variables

$y = \{y_1, y_2 \dots y_n\} \subset \mathbb{R}$ dependent variables

Goal learn a function

$$f(x) \to y$$

$$f : \mathbb{R}^d \to \mathbb{R}$$

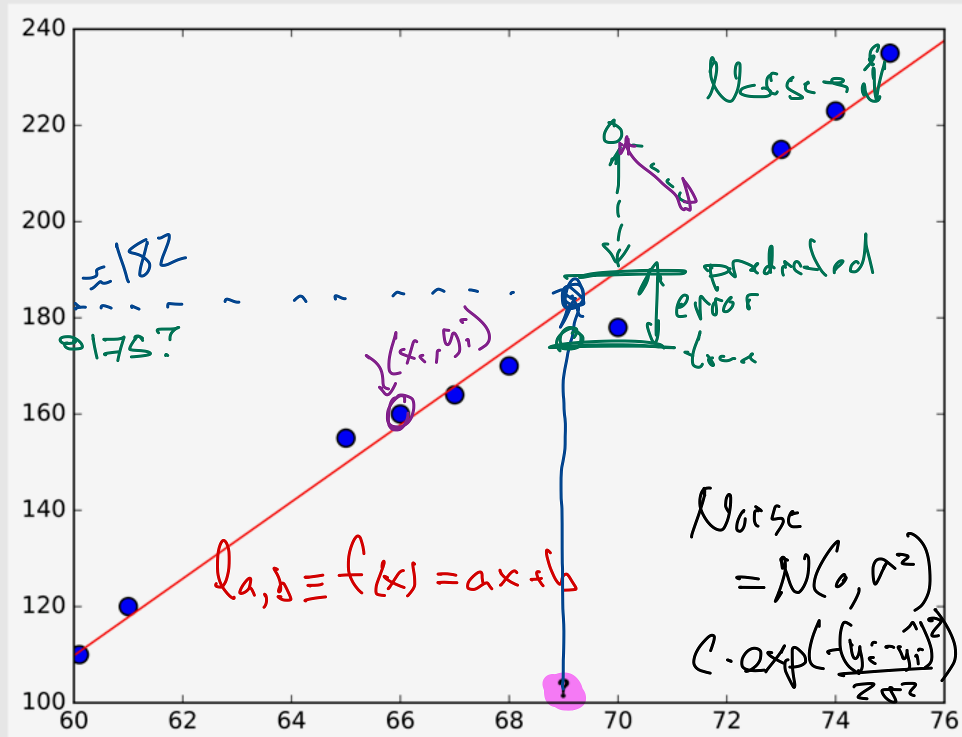


linear function

$$f(x) = ax + b \to y$$

X height (in)	y weight (lbs)
$x_i =$ 66	$y_i =$ 160
68	170
60	110
70	178
65	155
61	120
74	223
73	215
75	235
67	164
69	?

$\Rightarrow l_{a,b}$



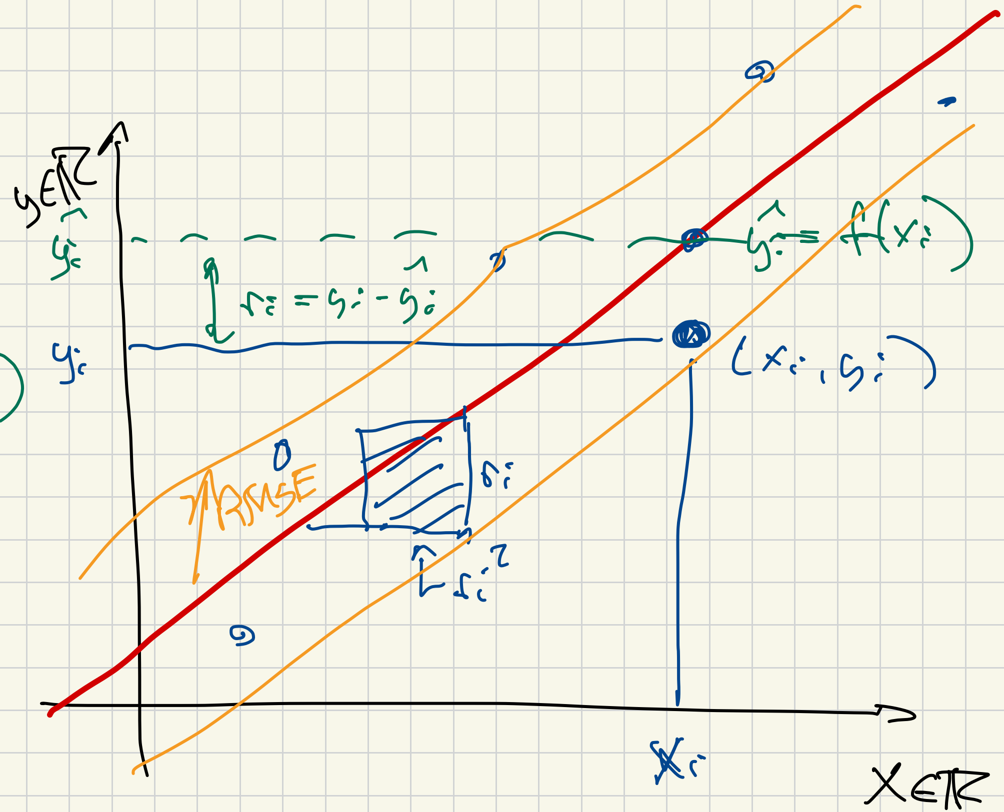
$f(69) = 182$

$y_i = f(x_i) + \varepsilon_i \qquad \varepsilon_i \sim \text{Noise}$

residual

$r_i = y_i - \hat{y}_i$

$= y_i - \ell(x_i) = y_i - f(x_i)$



Cost function g for choice ℓ, input (X, y)

Sum of Squared Errors

$$SSE((X,y), f) = \sum_{i=1}^{n} (y_i - f(x_i))^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} r_i^2$$

residual

$$f(x) = ax + b = \ell_{a,b}$$

- MLE of all lines, Normal noise on $\hat{y}_i + \varepsilon_i = y_i$
  $\varepsilon_i \sim N(0, \sigma^2)$
- looks like variance.
- $SSE((X,y), f) = \sum_{i=1}^{n} (r_i)^2 = \|r\|^2$
- simple "closed form" solution ← AlgG

Root Mean Squared Error

RMSE

root

$$\mathrm{RMSE}\left((x_i, y_i), f\right) = \sqrt{\underbrace{\frac{1}{n} \underbrace{\sum_{i=1}^{n} \boxed{\left(y_i - f(x_i)\right)^2}^{\text{sq. residual}}}_{SSE}}_{MSE}}$$

mean

MSE = average squared errors

in proportion to actual data

↳ same units as y_i

Input $(x, y) \subset \mathbb{R} \times \mathbb{R}$ $\quad (x, y)$ fixed

Goal $f: l: ax + b \qquad l_{a,b}(x_i) = \hat{y}_i$

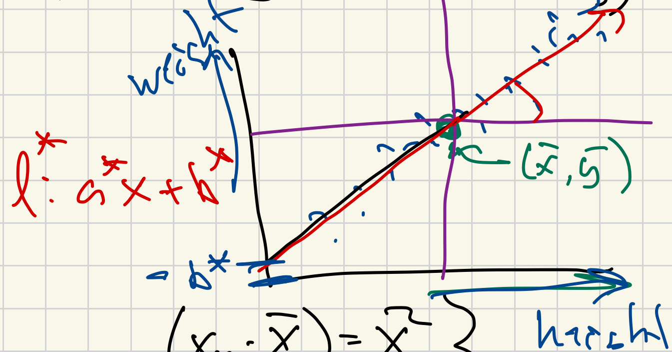
$$\underset{a,b}{\text{minimize}} \quad SSE\left((x,y), l_{a,b}\right) = \underbrace{\sum_{i=1}^{n} \left(y_i - (a x_i + b)\right)^2}_{\text{cost function } g}$$

parameters

$$(a^*, b^*) = \underset{(a,b) \in \mathbb{R}^2}{\operatorname{argmin}} \; SSE\left((x,y), l_{a,b}\right)$$

Algorithm

Solve for a^*, b^* given (x, y)



1. $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

2. centering

$$\tilde{X} = \{(x_1-\bar{x}) = \tilde{x}_1,\ (x_2-\bar{x}) = \tilde{x}_2,\ \dots\ (x_n-\bar{x}) = \tilde{x}_n\}$$

$$\tilde{y} = \{(y_1-\bar{y}),\ (y_2-\bar{y})\ \dots\}$$

3. $$a^* = \frac{\langle \tilde{Y}, \tilde{X}\rangle}{\|\tilde{X}\|^2} = \frac{\|\tilde{y}\| \cdot \cancel{\|\tilde{X}\|} \cdot \cos(\theta_{\tilde{x},\tilde{y}})}{\|\tilde{X}\|^{\cancel{2}}}$$

4. $$b^* = \bar{y} - a^* \bar{x}$$