FoDA L5

Bayesian Inference

Sep 6, 2022

Diversity and Inclusivity

It is my intent that students from all diverse backgrounds and perspectives be well served by this course, that students learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength and benefit. It is my intent to present materials and activities that are respectful of diversity: gender, sexuality, disability, age, socioeconomic status, ethnicity, race, and culture. Your suggestions are encouraged and appreciated. Please let me know ways to improve the effectiveness of the course for you personally or for other students or student groups. In addition, if any of our class meetings conflict with your religious events, please let me know so that we can make arrangements for you.

Immigration is a complex phenomenon with broad impactthose who are directly affected by it, as well as those who are indirectly affected by their relationships with family members, friends, and loved ones. If your immigration status presents obstacles to engaging in specific activities or fulfilling specific course criteria, confidential arrangements may be requested from the Dream Center. Arrangements with the Dream Center will not jeopardize your student status, your financial aid, or any other part of your residence. The Dream Center offers a wide range of resources to support undocumented students (with and without DACA) as well as students from mixed-status families. To learn more, please contact the Dream Center at 801.213.3697 or visitdream.utah.edu.

Simplified Bayes' Rule

$$\Pr(M \mid D) \propto \Pr(D \mid M) \cdot P(M)$$

\propto "proportional to"

$$f(m) \propto g(m) \Rightarrow f(m) = c \cdot g(m)$$

possibly unknown constant
positive $c > 0$

$$\underbrace{p(M \mid D)}_{\text{posterior}} \propto \underbrace{f(D \mid M)}_{\text{likelihood}} \cdot \underbrace{r(M)}_{\text{prior}}$$

R.V. H estimate height of average student at the U.

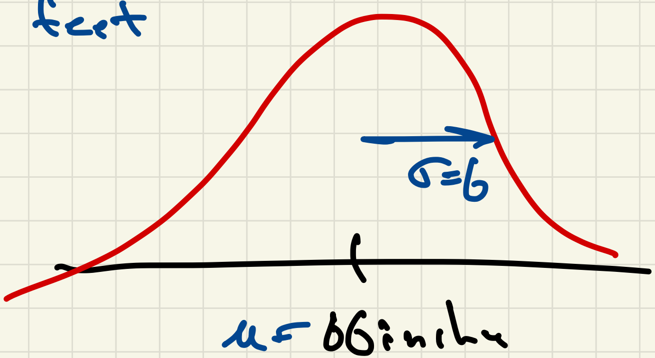
$$D = \{x_1, x_2, \ldots x_n\} \sim \text{active } o/D$$

prior $p(m) = N_{\mu=66,\,\sigma=6}(m)$

5.5 = average (D)
feet

$$= \frac{1}{\sqrt{\pi 2\sigma^2}} \cdot \exp\left(-\frac{(m-\mu)^2}{2(\sigma)^2}\right)$$

$$= \frac{1}{\sqrt{\pi\ 72}} \cdot \exp\left(\frac{-(m-66)^2}{72}\right)$$



Data $D = \{x_1, x_2, \ldots x_n\}$ inches

$$g(x_i) = N_{\mu=m, \sigma=2}(x_i)$$

likelihood $f(D \mid M) = \prod_{x_i \in D} g(x_i)$ $= g(x_i \mid M = m)$

$$= \prod_{x_i \in D} \left(\frac{1}{\sqrt{8\pi}} \cdot \exp\left(-\frac{1}{8}(m - x_i)^2\right)\right)$$

posterior

$$p(M \mid D) \propto f(D \mid M) \cdot r(M)$$

some constant

$$\ln(p(M \mid D)) \propto \ln(f(D \mid M)) + \ln(r(M)) + C$$

$$\propto \sum_{x_i \in D} \left(-\frac{1}{8}(m - x_i)^2\right) + -\frac{1}{72}(m - 66)^2 + C$$

$$\propto 9 \underbrace{\sum_{x_i \in D} (m - x_i)^2} - (m - 66)^2 + C$$

$\varpi = |D| \cdot 9 + 1$

$$\text{wave}(D, r) = \frac{\left(\sum_{x_i} 9x_i\right) + 66}{\varpi}$$

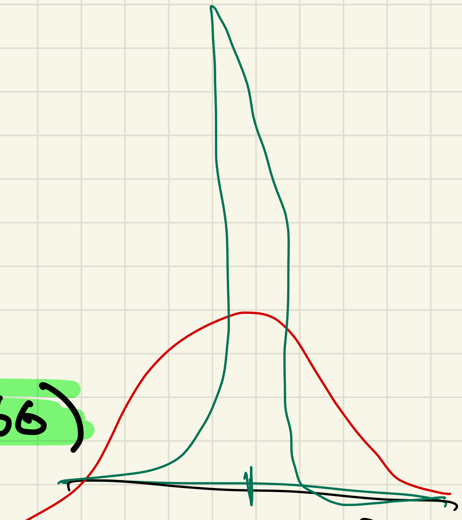
What if prior $r = \mathcal{N}_{66,\,0.1}$

$\sigma = 2 \qquad Var(D) = 4$

$Var(prior) = \overline{0.01}$

2100

$$\underset{m}{\operatorname{argmin}} = \left(\frac{1}{400} \sum_{x_i \in D} (x_i - m)\right)^2 + (m - 66)$$



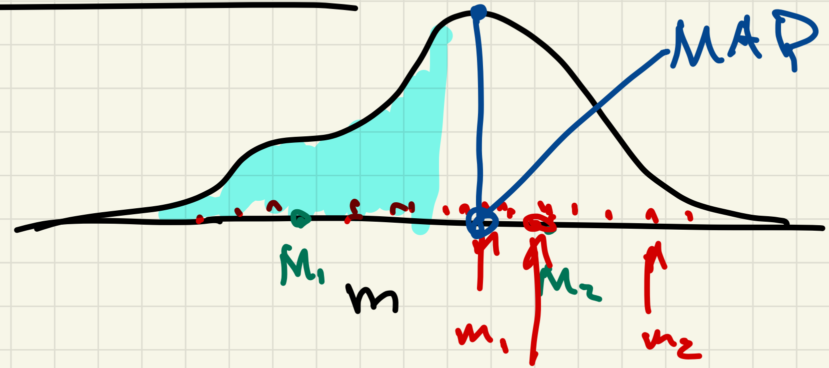
What happens with more data?

$|D| = n \rightarrow \infty$

$$\frac{100{,}000}{400} = 250$$

Power of Posterior

$p(M \mid D) \longrightarrow$



$$\frac{p(M_1 \mid D)}{p(M_2 \mid D)}$$

prediction $f(M)$ $\quad \int_{M \in \Omega} f(M)\, p(M \mid D)\, dm$

set $D = \{x_1 \ldots x_n\}$

$f : D \to \mathbb{R}$

$$\underset{x_i \in D}{\operatorname{argmax}} \, f(x_i)$$

\prod Π