Classification: KNNs, Decision Trees, and Neural Nets ... oh my!
Classification

Input $(x, y) \subset \mathbb{R}^d \times \{-1, +1\}$

$(x_i, y_i) \subset \mathbb{R}^d \times \{-1, +1\}$

Goal Build function $f: \mathbb{R}^d \to \{-1, +1\}$

$f(x) = \text{sign}(g(x))$

$g(x): \mathbb{R}^d \to \mathbb{R}$

So on new data $(x, y') - \mathcal{U}$

that $f(x) = y'$ w/ high probability
K - Nearest Neighbor Classifier (KNN)

\[ d = 5 \]

Vote on signs in \[ \Sigma_f(x_i) \]

\[ f(x) = + \]

Choice of distance
Decision Trees

$X \subset \mathbb{R}^d$

$f(x) = f$

How to split?
- choose best

Gini Impurity Index

$$P_+ = \frac{\# + + \# + \#}{\text{total} \#} = \frac{2}{8}$$
$$P_- = \frac{\# - + \# - \#}{\text{total} \#} = \frac{3}{8}$$

$$G = P_+ (1 - P_+) + P_- (1 - P_-) = \frac{2}{16} + \frac{3}{16} = \frac{5}{16}$$
Not interpretable

Neural Nets

Polynomial Classifier

Linear Classifier

Decision Tree

Random Forest
Neural Nets

Neuron

\[ \phi(y) = \text{ReLU}(s) = \max \{ 0, y \} \]

\[ \phi(s) = \text{Sigmoid}(s) = \frac{1}{1 + \exp(-s)} \in (0, 1) \]

activation function

linear classifier

\[ y = \Phi_{w,b}(x) \]

\[ \langle w, x \rangle - b \geq 0 \]

\[ y = g_{w,b}(x) \]
Neural Network

\[ x \]
\[ h_1 = \phi(x) \]
\[ h_2 = \phi(h_1) \]
\[ h_3 = \phi(h_2) \]
\[ o = h_3 \]

\[ a_1 = w_1^T x \]
\[ a_2 = w_2^T a_1 \]
\[ a_3 = w_3^T a_2 \]

Activation function:
\[ \phi(x) = \begin{cases} 1 & \text{if } x > \theta \vspace{1em} \\ 0 & \text{otherwise} \end{cases} \]

Backpropagation:
\[ \delta_3 = (o - \hat{o}) \sigma'(a_3) \]
\[ \delta_2 = \delta_3 w_3^T \sigma'(a_2) \]
\[ \delta_1 = \delta_2 w_2^T \sigma'(a_1) \]

Weights:
\[ w_1 \in \mathbb{R}^d \]
\[ w_2 \in \mathbb{R}^m \]
\[ w_3 \in \mathbb{R}^p \]