FoDA : L2

Probability Review

#1

Sample Space Ω \Omega

possible outcome $\omega \in \Omega$

event $A \subseteq \Omega$

roll die $\Omega = \{1, 2, 3, 4, 5, 6\}$

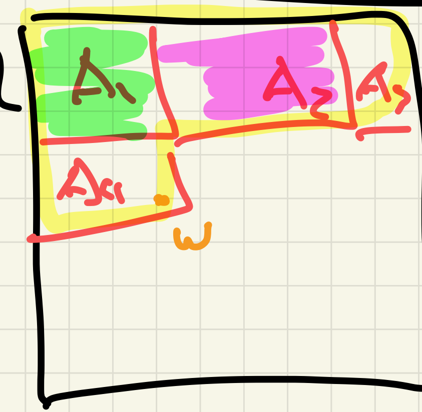
$\omega = 3$

event odd $A = \{1, 3, 5\}$

$$Pr(A) = \frac{|A|}{|\Omega|} = \frac{|\{1,3,5\}|}{|\{1,2,3,4,5,6\}|} = \frac{3}{6} = \frac{1}{2}$$

probability of A

Probability $Pr(A)$ Ω



- $0 \le Pr(A) \le 1$
- $Pr(\Omega) = 1$
- Sequence events $A_1, A_2, \dots$

  $i \ne j$ $\quad A_i \cap A_j = \emptyset$

$$Pr\left(\bigcup_{i=1} A_i\right) = \sum_{i=1} Pr(A_i)$$

bias coin $Pr(T) = 0.4$

$Pr(H) = 0.6$

$\Omega = \{H, T\}$

$Pr(1)$

Continuous Sample Spaces

time, water, land

start class at 10:45:00

10:45:59

<u>event</u>: A : 10:45:20 - 10:45:59.$\overline{99}$

$Pr(A) = 0.75$

Random Variable

X : function between to sample spaces

$X : \Lambda \to \Omega$

a variable that has not yet been defined or not given value

$\omega = X(\lambda)$

↑ do not know λ

Coin w/ points

$\Lambda = \{H, T\}$

points $\Omega = \mathbb{R}$ real numbers

random variable

$X : \Lambda \to \Omega$

$X(H) = 1$

$X(T) = 5$

Conditional (and Joint) Probabilities

events A, B

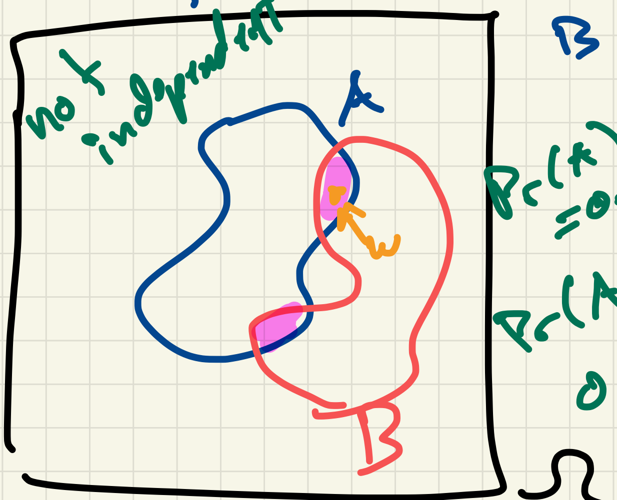
$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$\Pr(A \cap B) = 0.1$ $\quad A = \{1, 2, 3\}$

$B = \{1, 3, 5\}$

$\Pr(B) = 0.4$ $\quad A \cap B = \{1, 3\}$

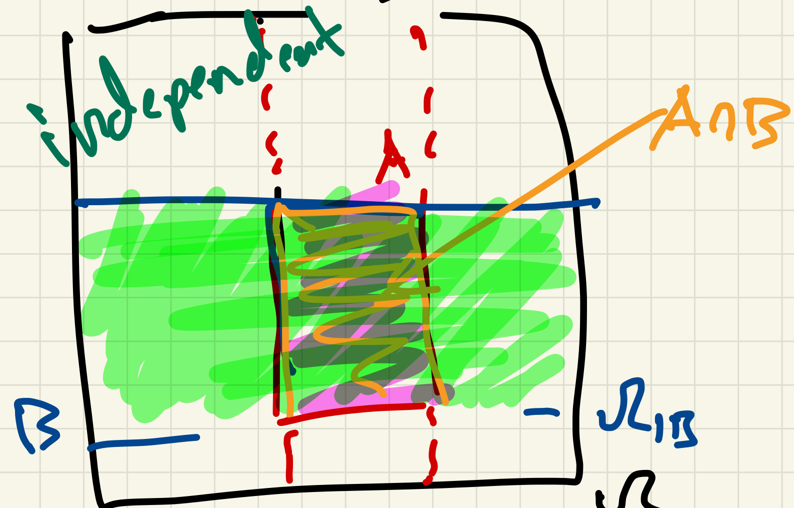
prob A, given know B true



$\Pr(A) = 0.2$

$\Pr(A \mid B)$

0.1



independence

two events A, B **independent** iff

$$\Pr(A \mid B) = \Pr(A)$$

$\Pr(B \mid A) = P(B)$ and $\Pr(A) \cdot \Pr(B) = \Pr(A \cap B)$

two Random Variables X, Y are **independent**

$X : \Delta_x \to \Omega_x$ $\qquad$ $Y : \Delta_Y \to \Omega_Y$

iff all events $A \subset \Omega_x$ $B \subset \Omega_Y$

A, B independent events

$$\Omega = \{(c=1, v=1), (c=1, v=0), (c=0, v=1), (c=0, v=0)\}$$

vaccine

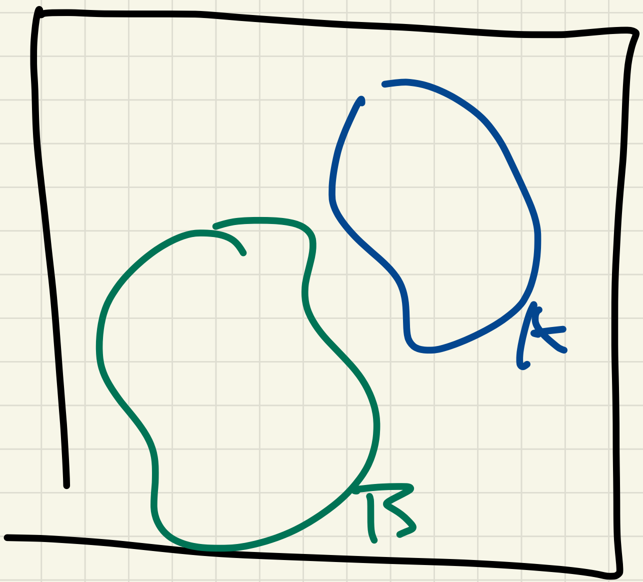
		$v=1$	$v=0$
$c=1$	Severe COVID	0.01	0.2
$c=0$	no/mild COVID	0.54	0.25

$$\Pr(v=1) = 0.55$$

$$\Pr(c=1 \mid v=1) = \frac{\Pr(c=1 \wedge v=1)}{\Pr(v=1)} = \frac{0.01}{0.55} \approx 0.019$$

$$\Pr(c=1) = 0.01 + 0.2 = 0.21$$

$$\Pr(c=1) \stackrel{?}{=} \Pr(c=1 \mid v=1)$$



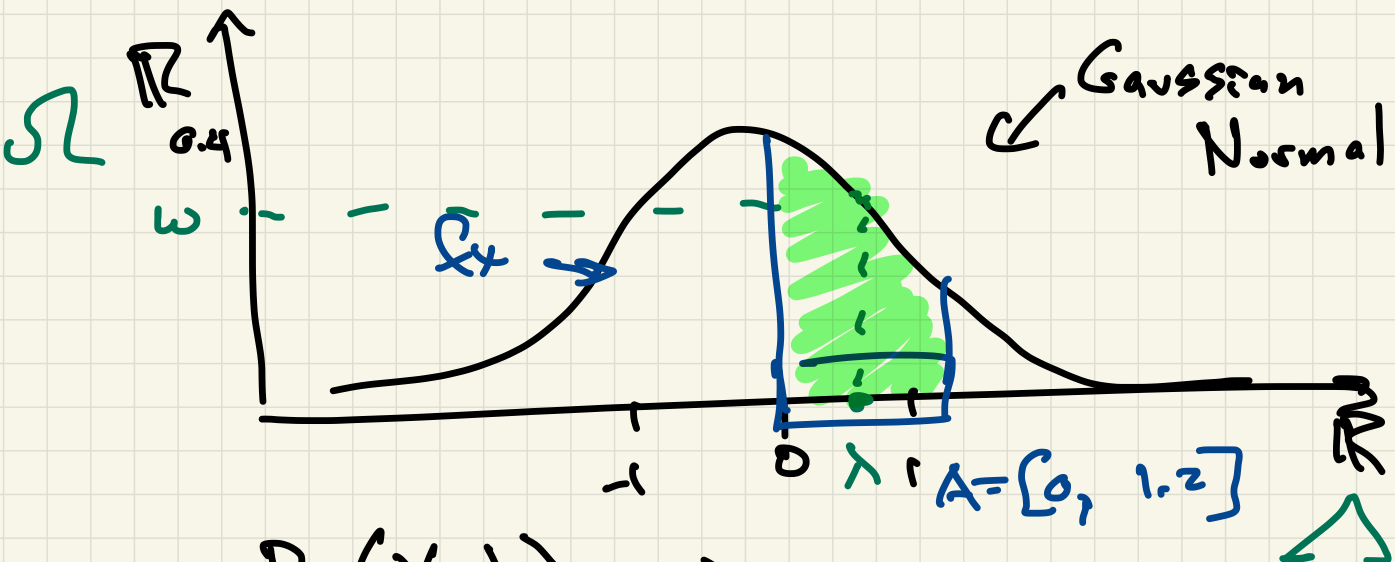
$Pr(A) = .2$

$Pr(A|B)$

$= \dfrac{Pr(A \cap B)}{Pr(B)}$ $= 0$

Density function

R.V. X → function $f_X : \mathbb{R} \to \mathbb{R}$



$Pr(X=\lambda) = 0$

$Pr(A) = P(X \in A) = 0.35$