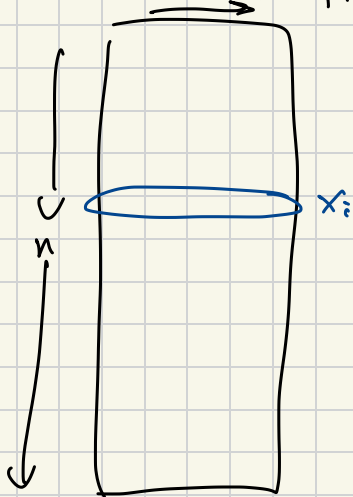


# L21: Privacy

Jeff M. Phillips

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Data  $\times$   $d$ -attributes



What if this  
was gov.?

Ethics  $\approx$  Empathy

Early 2000s

big companies

collected lots of data  
usually on customers

make data public (sometimes)

Late 2000s, this stopped.

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- ▶ **Dr.** Sweeney now teaches at Harvard.

How can we release data anonymously  
while preserving information?

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Goal: Represent info so we can  
do a generalization.

k-anonymity: Remove attributes (from subset)  
until each combination of attributes  
available, maps to at least  
 $k$  different records.

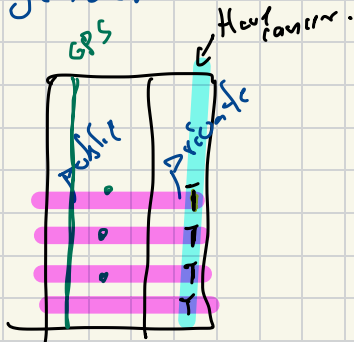
# Divide data

public attributes

- zip code
- age
- gender

private attributes

- has cancer
- has diabetes
- search for Ben Hur Spass



l-diversity :  $k$ -anonymity (+)

each group, had private attributes diverse values.

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What if all in group had  
either cancer or diabetes

---

$t$ -closeness :  $l$ -diversity (+)

the private keys were close  
(in distribution) to all data.

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1, 2, 3, 4, 5

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And another set  $D_2 = \{\langle \text{user-id}, \text{movie}, \text{date of grade} \rangle\}$ .  
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- ▶ Netflix Prize had proposed sequel, dropped in 2010 for more privacy concerns.

# Differential Privacy

• Guarantee on released  $D$ ,  $\epsilon$ -DP

so if  $D_1 \neq D_2$  so  $\text{diff}(D_1, D_2) = 1$

$$\frac{\Pr[g(D_1)]}{\Pr[g(D_2)]} \leq e^{\pm \epsilon}$$

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Goal release  $D$  w/ guess w/ noise

so all guess error (w/ noise)  $\leq f(\epsilon)$

$$\frac{Pr[S=s]}{Pr[S=s']} = 2 \Rightarrow \text{twice as likely to predict } S \text{ not } S'$$

$x_i \neq \Rightarrow$  correct

$$\frac{Pr(S)}{Pr(S')} = 99 \Rightarrow 99\% \text{ was } X \text{ not } X'$$

$\epsilon = 0.01$

$$\frac{Pr(S)}{Pr(S')} \leq 1.01 \Rightarrow \text{not sure whether } X \text{ or } X'$$

if  $\epsilon$  small  $\Rightarrow$  more private

# Laplacian Mechanism

adds Laplace noise  $\text{Lap}(x; w)$

$$\text{Lap}(x; w) = \frac{1}{2w} \exp\left(-\frac{|x|}{w}\right)$$



$S = \{s_i\}$  each

$$s_i = x_i + \text{Lap}(1/k = w)$$

$$X = \{x_1, x_2, \dots, x_m\}$$

$$S = \{s_1, s_2, \dots, s_m\}$$

$\downarrow \text{Lap}(1/k)$



Height

$x = 66$  inches

$X = \{x\}$

$x' = 67$  inches

$X' = \{x'\}$

$$S \sim N(x) = x + \text{Lap}(1/\omega) \quad \omega = \frac{1}{\epsilon}$$

$$\frac{\Pr[S = 70]}{\Pr[S' = 70]} = \frac{\frac{1}{\omega} \exp(-|66-70|/\omega)}{\frac{1}{\omega} \exp(-|67-70|/\omega)}$$
$$= \exp\left(\frac{1}{\omega} (|66-70| - |67-70|)\right)$$
$$= \exp\left(\frac{1}{\omega} (1)\right) = \exp(\epsilon)$$

## Binary Example

$$X = \{x\} \quad x \in \{0, 1\}$$

$$X = \{x=1\} \quad X' = \{x'=0\}$$

$$S = \{s\} \quad s = x + \text{Lap}(1/\epsilon)$$

$$\frac{P_c [g(s) = 1]}{P_c [g(s) = 1]} = \frac{\exp((1-1)/(1/\epsilon))}{\exp((0-1)/(1/\epsilon))}$$

$$= \exp\left(\frac{1}{1/\epsilon}\right) \left( (1-1) - (0-1) \right)$$

$$= \exp(\epsilon \cdot (1)) = \exp(\epsilon)$$

Binary database

$$X = \{x_1, x_2, \dots, x_n\} \quad x_i \in \{0,1\}$$

$$X' = \{x'_1, x'_2, \dots, x'_n\}$$

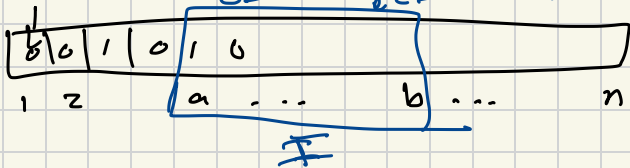
some  $x_i \neq x'_i$

$R = \text{Interval}$

$$I = [a, b]$$

$$I \cap X = \{x_{a_1}, x_{a_2}, x_{a_3}, \dots, x_b\}$$

$$g_I(x) = \sum_{i \in I} x_i = |X \cap I|$$



Laplacian Mechanism

$$S \leftarrow \mathcal{L}(X)$$

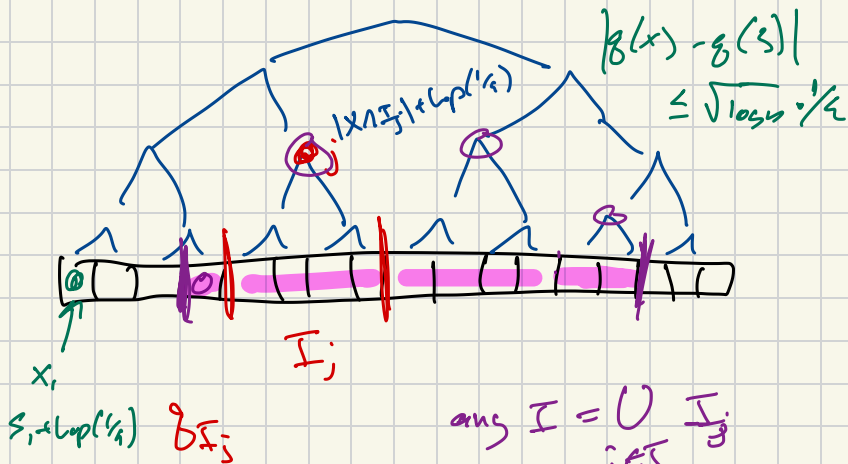
$$\{s_1, \dots, s_n\} \quad s_j = x_j + \text{Lap}(1/n)$$

How about  $g_{\mathbb{I}}(s)$

option 1  $g_{\mathbb{I}}(s) = |X \cap \mathbb{I}| + \text{Lap}(1/4)$

option 2  $g_{\mathbb{I}}(s) = \sum_{i \in \mathbb{I}} s_i = \sum_{i \in \mathbb{I}} (x_i + \text{Lap}(1/4))$   
 $= |X \cap \mathbb{I}| + |\mathbb{I}| \cdot \text{Lap}(1/4)$

$$\left| g_{\mathbb{I}}(s) - g_{\mathbb{I}}(x) \right| \leq \sqrt{|\mathbb{I}|} \cdot \text{Lap}(1/4) \\ \leq \sqrt{n} \cdot \left( \frac{1}{4} \right)$$



$$\begin{aligned}
 g(s) &= \sum_{j \in J} g_{I_j}(s) = \sum_{j \in J} (|X \cap I_j| + \text{Lop}(1/4)) \\
 &= |X \cap I| + \sqrt{|J|} \text{Lop}(1/4)
 \end{aligned}$$