# Supporting Human Interaction with Robust Robot Swarms


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***Abstract*—In this paper we propose a bio-inspired model for a decentralized swarm of robots, similar to the model proposed by Couzin [5], that allows for dynamic task assignment and is robust to limited communication from a human. We provide evidence that the model has two fundamental attractors: a torus attractor and a flock attractor. Through simulation and mathematical analysis we investigate the stability of these attractors and show that a control input can be used to force the system to change from one attractor to the other. Finally, we generalize another of Couzin's ideas [4] and present the idea of a *stakeholder* agent. We show how a human operator can use stakeholders to responsively influence group behavior while maintaining group structure.**


## I. Introduction

Recently there have been large efforts to create control systems suitable for human control of multi-agent system. Often times in such systems it is difficult or impossible for the human to communicate with all the agents in the system, particularly when communication links are unreliable, or when bandwidth and power constraints limit communication to only a small subset of agents. One way of approaching these difficulties that has become popular in the literature is to apply principles found in biological swarm systems to robot systems.

Despite the limited intelligence and abilities of each individual agent, swarms are able to achieve collective intelligence greater than the sum of their parts [12]. Some examples are flocks of birds [1] and schools of fish [5]. Swarm models have been explored by researchers from a wide variety of fields including computer science, engineering, physics, and biology. Many of these swarm models fall into one of two categories, flocking [11], [13], [6], [10] or cycles [8], [7].

Flocking is characterized by all agents moving cohesively in approximately the same direction. Reynolds' seminal work in [11] modeled flocks using three fundamental regions of interaction: repulsion, orientation, and attraction. Using these three simple rules he was able to simulate realistic-looking flocking behavior. In [13] a simple consensus model is presented, with an associated proof of convergence to a flock in [6]. In [10] the authors provide a model using Reynolds' three rules of interaction, and provide mathematical guarantees that the group will converge to a lattice structure with all agents moving in the same direction.

Cycles are characterized by all agents circling around a stationary point. A cycle is often called a torus. One prominent model is the cyclic pursuit model [8]. This model is characterized by agents in a ring topology pursuing one another to create a balanced cyclic group.

A few models of particular interest [5], [9] demonstrate both flocking and cyclic behaviors. The model proposed by Couzin *et al.* [5] uses three regions of interaction similar to those proposed by Reynolds. Couzin showed that by varying the sizes of these three regions different group structures emerge including a torus structure—a type of cyclic group—and a flock. Couzin In [4] he also explored leading a flock with a small number of informed agents. This paper extends Couzin's work by exploring dynamic switching between group types and exploring leading torus and flock group types with an eye toward human input. Other work on leading swarms can be found in [10], [3].

In this paper we describe a decentralized model which is similar to Couzin's model [5] except we have smoothed out some of the switching nonlinearities and changed the dynamics to be more suitable for robot systems. We assume that a human can only influence a subset of the agents in the swarm. In Section II we present our model. In Section III we discuss the torus and flock attractors and through simulation demonstrate when the system converges to these attractors. In Section IV we consider a simplified model that uses only attraction and provide mathematical guarantees for convergence to a cyclic group. Finally, in Section V we present the notion of a *stakeholder* and show how a human operator can use stakeholders to cause the swarm to switch from one group type to another. We also demonstrate how stakeholders can be used to cause the swarm to track a time varying reference.

## II. Model

Let $i = 1, 2, ..., N$ be a set of homogeneous agents with nonholonomic dynamics given by

$$\begin{aligned} \dot{x_i} &= s \cdot \cos(\theta_i) \\ \dot{y_i} &= s \cdot \sin(\theta_i) \\ \dot{\theta}_i &= w_i \end{aligned} \tag{1}$$

where $[x_i, y_i]^T \in \mathbb{R}^2$ is the $i$th agent's position, $\theta_i \in [-\pi, \pi]$ is the angular heading of the agent, $s$ is the constant agent speed, and $w_i$ is the angular velocity control input.

For simplicity we define:

$$\begin{aligned} v_i = & \quad [\cos(\theta_i), \sin(\theta_i)]^T \\ c_i = & \quad [x, y]^T. \end{aligned} \tag{2}$$

Let $A(t) = a_{ij}(t)$ denote the sensory adjacency matrix where $a_{ij}(t) = 1$ means that agent $j$ is visible to agent $i$ at time $t$. Each $a_{ij}(t)$ is determined at time $t$ according to a Bernoulli random variable with parameter

$$p_{ij}(t) = \begin{cases} 1/d_{ij}(t) & \text{if } d_{ij}(t) \geq 1 \\ 1 & \text{otherwise} \end{cases}$$

where $d_{ij}(t)$ is the Euclidean distance between agents $i$ and $j$ at time $t$. This method of choosing neighbors is similar to the random neighbor model used in [2] which replicated field observations of starlings [1]. This is relevant for robot systems because occlusions make visibility less likely with growing distance for visual sensors and interference makes sensing less likely with growing distance for radio or wifi-based sensors.

Agents react to neighbors within three different zones: repulsion, orientation, and attraction. The neighbors in these zones are determined by

$$\begin{aligned} n_i^r &= \{j : \|c_i - c_j\| \leq R_r, a_{ij} = 1\} \\ n_i^o &= \{j : \|c_i - c_j\| \leq R_o, a_{ij} = 1\} \\ n_i^a &= \{j : a_{ij} = 1\} \end{aligned} \tag{3}$$

where $n_i^r$, $n_i^o$, and $n_i^a$ are the sets of agent $i$'s neighbors in the regions of repulsion, orientation, and attraction, respectively. The parameters $R_r$ and $R_o$ are the associated radii of repulsion and orientation. Note that this model allows for overlapping regions of repulsion, orientation, and attraction. This eliminates the hard switch between the repulsion, orientation, and attraction forces seen in [5] that may be sensitive to sensor transients in real robots.

The control input $w_i$ is determined by first computing the repulsion, orientation, and attraction vectors

$$u_i^r = -\sum_{n_i^r} \frac{c_j - c_i}{\|c_j - c_i\|^2} \tag{4}$$

$$u_i^o = \frac{v_i + \sum_{n_i^o} v_j}{\|v_i + \sum_{n_i^o} v_j\|} \tag{5}$$

$$u_i^a = \frac{\sum_{n_i^a}(c_j - c_i)}{\|\sum_{n_i^a}(c_j - c_i)\|}. \tag{6}$$

Next, the desired heading vector $u_i$ is computed as

$$u_i = u_i^r + u_i^o + u_i^a. \tag{7}$$

Note that, because of the normalization in (5) and (6), orientation and attraction forces are always equally weighted in the model. This keeps one of the two fundamental forces from overpowering the other. It also allows the exponentially growing repulsion vector to overpower the orientation and attraction forces as agents move closer together, which aids in collision avoidance.

Finally, angular velocity, $w_i$, is computed as

$$w_i = k\alpha \tag{8}$$

$$\alpha = \text{atan2}(u_i) - \theta_i \tag{9}$$

where $k$ is a scalar gain and $\alpha$ is the heading error. Using modulo $2\pi$ arithmetic we limit $\alpha \in [-\pi, \pi]$. Since $\max|\text{atan2}(u_i)| = \pi$, $w_i$ is bounded by $k\pi$.

## III. ATTRACTORS

In order to define the two different attractors of the model we use two metrics of group behavior described in [5], namely group angular momentum, $m_{group}$, and group polarization, $p_{group}$. Group angular momentum is a measure of the degree of rotation of the group about the group centroid and is calculated as

$$m_{group}(t) = \frac{1}{N}\left|\sum_{i=1}^{N} \det[r_{ic}(t)|v_i(t)]\right|. \tag{10}$$

The vector $r_{ic}(t)$ is a unit vector pointing from the group centroid to the position of agent $i$ and is given by

$$r_{ic}(t) = \frac{c_i(t) - c_g(t)}{\|c_i(t) - c_g(t)\|} \tag{11}$$

$$c_g(t) = \frac{1}{N}\sum_{i=1}^{N} c_i(t) \tag{12}$$

where $c_g(t)$ is the group centroid. The term $\det[r_{ic}(t)|v_i(t)]$ is the determinant of the $2 \times 2$ matrix with columns $r_{ic}(t)$ and $v_i(t)$ and is a two-dimensional analogue of the cross product. The $m_{group}$ of a swarm reaches a maximum value of 1 if all the agents are rotating around the group centroid in the same direction.

Group polarization measures the degree of alignment among individuals within the group and is calculated as

$$p_{group}(t) = \frac{1}{N}\left|\sum_{i=1}^{N} v_i(t)\right|. \tag{13}$$

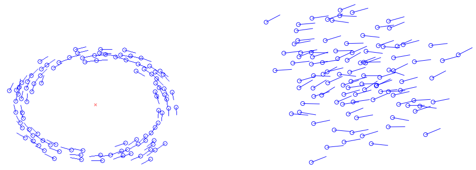
The $p_{group}$ of a swarm reaches a maximum value of 1 when all the agents have the same heading.

### A. Torus Attractor

This model can produce a torus formation as shown in Figure 1(a). A torus is characterized by $p_{group}$ close to zero, $m_{group}$ close to one, and a relatively stationary group centroid. One potential application of a torus is perimeter monitoring.

### B. Flock Attractor

The other attractor that the model exhibits is a flock; see Figure 1(b). A flock is characterized by $p_{group}$ close to one and $m_{group}$ close to zero. Flock groups are useful for moving the swarm quickly from one location to another.



(a) Torus (b) Flock

Fig. 1. 100 agents in (a) torus formation; (b) flock formation. The agents' directions of travel are represented by straight lines emanating from the center of each agent.

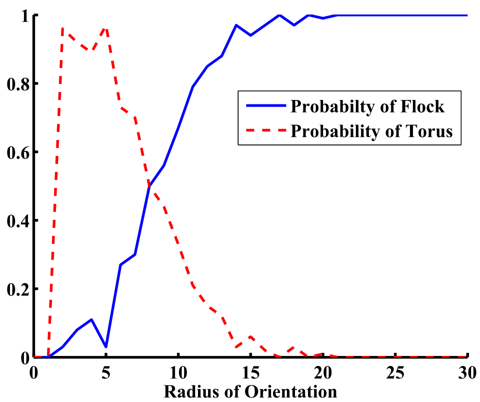



Fig. 2. Average group moment and polarization as a function of radius of orientation.

### C. Group Expressiveness and Parameter Selection

We desire to be able to switch between the torus and flock attractors without globally broadcasting parameters to the agents. To determine parameter values that allow both group types to emerge we ran a series of simulations using $N = 100$, $k = .5$, $R_r = 1$ and varied the the radius of orientation. Simulations were run for 200 seconds with a step size of $\Delta t = 0.1$. The radius of orientation was varied from 0 to 30 in 1 unit increments. One hundred simulations were performed for each value of $R_o$. For each iteration agents were given random initial positions uniformly distributed over a 10x10 square centered at the origin. Agents were also given random initial headings. The percentage of trials that converged to the torus attractor and the flock attractor were calculated for each value of $R_o$ and are shown in Figure 2. As can be seen in the figure, the value $R_0 = 8$ had an equal probability of converging to either attractor. Figures 1(a) and 1(b) show a torus and a flock formed with the parameter values listed above and $R_0 = 8$.

## IV. Analysis of Attraction Dynamics

The previous section provided empirical evidence that both torus and flock behaviors can emerge using the system. In this section, we provide an example argument that these behaviors are formal attractors of the dynamic system. These attractors have associated regions of stability and are the foundation for claims of robustness of swarm behavior. To do this we consider a system based only on attraction ($u_i = u_i^a$) and prove convergence to a stable cycle. We assume a complete agent topology ($a_{ij} = 1 \ \forall \ i \neq j$). Using this assumption the desired heading of agent $i$ is

$$u_i = \frac{1}{N} \sum_{j \neq i}^{N} (c_j - c_i) \tag{14}$$

which reduces to $c_g - c_i$, where $c_g$ is the group centroid. Therefore, $u_i$ points toward the centroid.

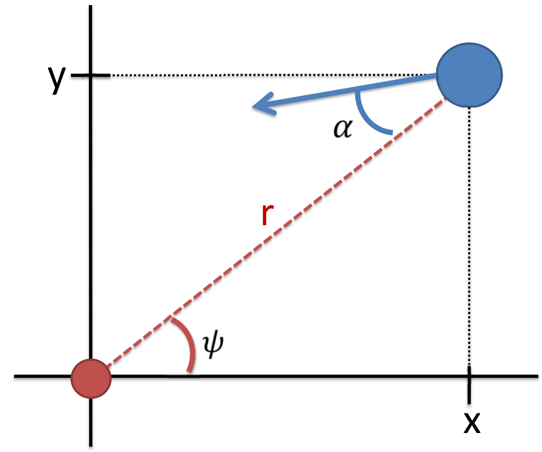



Fig. 3. Coordinates for an agent (blue) attracted toward the centroid (red).

### A. Change of Variables

We assume a stationary group centroid at the origin and, using a method similar to [8], we perform a change of variables

$$r = \sqrt{x^2 + y^2} \tag{15}$$
$$\alpha = \psi_i + \pi - \theta \tag{16}$$

where $r$ is the distance from the group centroid and $\psi = \arctan\left(\frac{y}{x}\right)$. Figure 3 shows how these variables relate to each agent.

Computing equations for $\dot{r}$ and $\dot{\alpha}$ we find that

$$\dot{r} = -s \cos \alpha \tag{17}$$
$$\dot{\alpha} = \frac{1}{r} s \sin \alpha - \omega \tag{18}$$

which describe an agent's dynamics in terms of its distance from the centroid $r$ and the desired angle of orientation $\alpha$.

### B. Stability of Equilibrium Points

Solving for the equilibrium points of (17) and (18) we get

$$r = \frac{2s}{k\pi}, \qquad \alpha = \pm\frac{\pi}{2} \tag{19}$$

where we have restricted $\alpha$ to be in the interval $[-\pi, \pi]$. These two equilibria define a clockwise and counterclockwise orbit about the fixed centroid with radius $r = 2s/k\pi$.

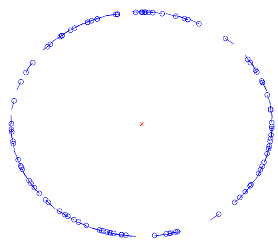


Fig. 4. A cyclic group formed by an attraction only swarm. The group centroid is marked by an 'x'.

We now investigate the stability of the equilibria by linearizing equations (17) and (18) about the equilibrium points. Evaluating the Jacobian at $r = 2s/k\pi$ and $\alpha = \pi/2$ and letting $\omega = k\alpha$ we have

$$\begin{bmatrix} 0 & s\sin\alpha \\ -\frac{s}{r^2}\sin\alpha & \frac{s}{r}\cos\alpha - k \end{bmatrix}\Bigg|_{(2s/k\pi,\pi/2)} = \begin{bmatrix} 0 & v \\ -\frac{k^2\pi^2}{4s} & -k \end{bmatrix} \quad (20)$$

which has eigenvalues with negative real parts. Therefore the equilibrium point is locally asymptotically stable. Linearizing about the other equilibrium point

$$r = \frac{2s}{k\pi}, \qquad \alpha = -\frac{\pi}{2} \quad (21)$$

gives a similar result.

This indicates that for a stationary group centroid all agents converge to either a clockwise or counterclockwise orbit about the group centroid with a fixed radius $r = 2s/k\pi$.

To verify these results we simulated the simplified dynamics with $k = 0.5$, $s = 5$. The resulting formation is shown in Figure 4 and had a radius of approximately $20/\pi$ as expected. This simulation suggests that the assumption of a stationary group centroid was well founded.

## V. Leadership

### A. Stakeholders

To allow a human to influence a swarm's behavior we introduce a type of agent called a stakeholder. In [4], Couzin showed how a limited number of stakeholders can be used to influence the behavior of a flock. In this section we show that not only can stakeholders allow a human to control the direction of a flock, but that stakeholders can also be used to control a torus and dynamically switch between attractors. A stakeholder is an agent that is influenced by the group and also influenced by its interaction with a human operator. We assume that the human operator can broadcast a desired location to $M \leq N$ stakeholder agents. By broadcasting a reference input to a limited number of agents a human may influence the movement of the swarm and even cause the group to switch attractors.

A stakeholder has dynamics given by (1) and control input (8). The difference is that $u_i$ is modified to allow a human operator to influence the stakeholder's behavior. This influence is added into the stakeholder attraction dynamics through a weighted sum as follows:

$$u_i^{sa} = \frac{\rho\hat{q}_i + (1-\rho)u_i^a}{\|\rho\hat{q}_i + (1-\rho)u_i^a\|}, \quad (22)$$

$$\hat{q}_i = \frac{q - c_i}{\|q - c_i\|} \quad (23)$$

where $q \in \mathbb{R}^2$ is the reference, $\rho \in [0,1]$ is the priority of the reference input, and $u_i^a$ is given by Equation (6). Orientation and repulsion are given by equations (5) and (4) with

$$u_i = u_i^{sa} + u_i^o + u_i^r. \quad (24)$$

This allows human input to the system without eliminating inter-agent dynamics.

### B. Controlled Switch

As mentioned above, one potential application of the torus group is perimeter monitoring and one potential application of the flock is to move the swarm quickly from one location to another. It is therefore desirable to be able to dynamically switch between attractors so that the human may have access to both capabilities. We investigated how $M$ and $\rho$ affect the potential for a human to cause the swarm to switch from one attractor to another. Simulations were run varying $M$ over the interval 10 to 100 in 10 unit increments and $\rho$ over the interval 0.1 to 1 in 0.1 unit increments. Ten 200 second trials were performed for each parameter pair. Other parameters were $N = 100$, $k = .5$, $R_r = 1$, $R_o = 8$.

To investigate switching from a torus to a flock we first initialized the group into a torus formation and then gave $M$ stakeholders a distant fixed reference input to move toward. The final group polarization for each trial was recorded to determine if the group had successfully switched from a torus to a flock. The results of these simulations are shown in Figure 5 where the probability of the group successfully switching from a torus to a flock by the end of any given 200 second trial is plotted as a function of $M$ and $\rho$. When the torus did not switch to a flock, it either moved in formation in the general direction of the reference input or the group fragmented.

To investigate switching from a flock to a torus we first initialized the group into a flock formation and then gave $M$ stakeholders a fixed reference input at the origin. The final group moment for each trial was recorded to determine if the swarm had successfully switched from a flock to a torus. The results of these simulations are shown in Figure 6 where the probability of the group successfully switching from a flock to a torus is plotted as a function of $M$ and $\rho$. When the flock did not switch to a torus, it either flew in a large clover like pattern centered at the reference point or the group fragmented.

Examining Figures 5 and 6 we see that it is easier to switch from a torus to flock than vice versa. We also see from Figure 6 that it is not necessarily desirable to set $\rho = 1$ because the lack of agent interaction induced by the coercive leadership strategy made it difficult for the agents to form a torus. However, if $\rho < 0.4$ then there was almost zero probability of transitioning

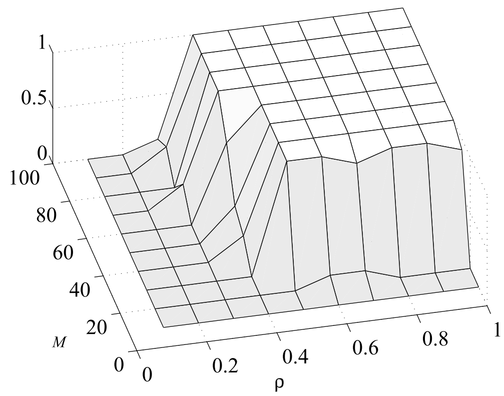



Fig. 5. Probability of swarm switching from torus to flock when under human control for various values of $M$ and $\rho$.

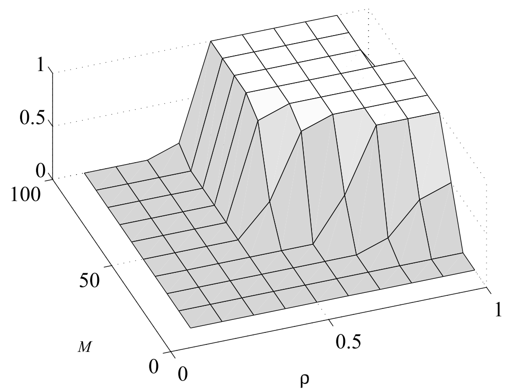



Fig. 6. Probability of swarm switching from flock to torus when under human control for various values of $M$ and $\rho$.

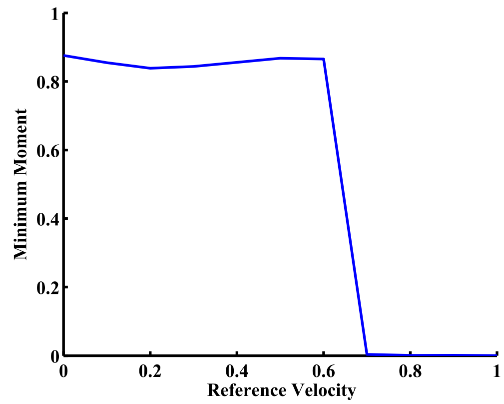



Fig. 7. Average minimum group moment as a function of reference velocity when leading a torus.

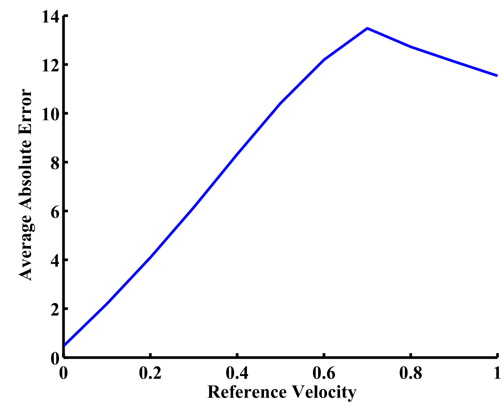



Fig. 8. Average distance from group centroid to reference input as a function of reference velocity when leading a torus.

from one attractor to the other. This provides evidence that the attractors are resilient in the presence of perturbations. We also notice that a human only needs to control 30 percent of the agents in the swarm to transition from a torus to a flock and 40 percent to transition from a flock to a torus. This indicates that the control strategy is robust in the presence of communication dropouts.

### *C. Path Following*

In the previous section we discussed switching from one attractor to the other. However, for many applications it is desirable to lead the swarm while keeping the group structure intact. We ran a series of simulations to determine whether or not these group types could be effectively moved without breaking formation. To do so we choose $M = 50$ and $\rho = 0.8$ to ensure sufficient influence on the group without communicating with all the agents. We again use agent parameters $N = 100$, $k = .5$, $R_r = 1$, $R_o = 8$, but this time we introduce a time varying reference input with velocity

$$\dot{q} = s_q \cdot [1, 0]^T \tag{25}$$

where $s_q$ is the speed of the reference input.

*1) Torus Path Following :* For torus formations $s_q$ was varied from 0 to 1 in 0.1 unit intervals. Ten trials were performed for each parameter set. The lowest group moment achieved during each trial was recorded. If the minimum group moment was low it indicates that the torus did not successfully maintain formation during the trial. The average absolute distance from the group centroid to the reference input was also recorded. The average minimum group moment over the ten trials is plotted in Figure 7. From the figure we see that the torus formation was consistently maintained for $s_q \leq 0.6$. The average absolute distance from the group centroid over the 10 trials is plotted in Figure 8. We see that error increases linearly with the speed. The decrease in error for a reference velocity greater than 0.7 is due to the torus briefly switching to a flock to catch up with the reference. Further investigation revealed that most of the error was due to a constant steady state error where the torus lagged consistently behind the reference input. These results indicate that torus groups are difficult to lead but are still responsive to human input for very low reference speeds.

*2) Flock Path Following :* For flock formations $s_q$ was varied from 4 to 5 in 0.1 unit intervals. Ten trials were

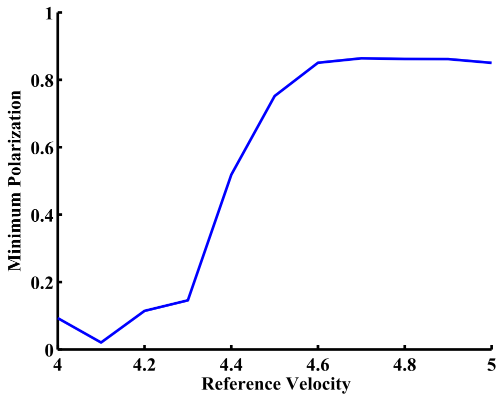



Fig. 9. Average minimum group moment as a function of reference velocity when leading a flock.

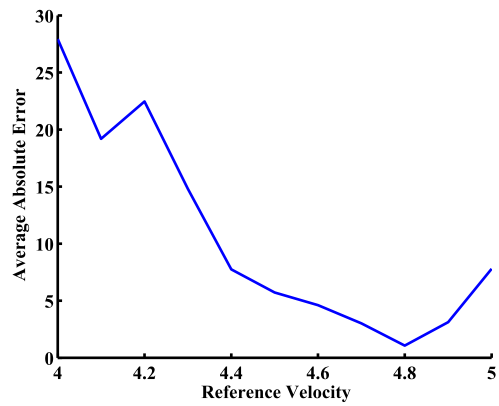



Fig. 10. Average distance from group centroid to reference input as a function of reference velocity when leading a flock.

performed for each parameter set. The lowest group polarization achieved during each trial was recorded. If the minimum group polarization was low it indicates that the flock did not successfully maintain formation during the trial. The average minimum group polarization is plotted in Figure 9. From the figure we see that the flock formation was consistently maintained for $s_q \geq 4.6$. We investigated the lower values of $s_q$ and found that the group briefly circled back to allow the reference input to catch up to the swarm. The average absolute distance from the group centroid over the 10 trials is plotted in Figure 10. We see that error is minimized when $s_q = 4.8$. These results suggest that flocks are responsive to human input and maintain their group structure for reference speeds near the agent speed.

## VI. Summary

In this paper we demonstrated a model with single integrator rotational dynamics. We demonstrated that this model has both a flock and torus attractor. We identified parameter values that allow the swarm to converge to either attractor with equal probability when starting from random initial conditions. Through a linear stability analysis we showed that for an attraction-only model the swarm converges to a cycle around a stationary centroid. We further showed that by using a limited number of stakeholders the swarm could be dynamically switched from one attractor to the other. We also identified control parameters that allowed stakeholders to lead both flock and torus group types while maintaining group structure. This satisfies the requirement of robust HSI systems, namely that a human can lead and switch between attractors of the system while only communicating with a subset of the agents. Future areas of research include more thorough mathematical guarantees on convergence to attractors, stability criteria, more advanced control strategies, exploration of model behavior for a wider range of parameter values, including the lowest number of agents for which the results in this paper hold, and implementation with real robot platforms.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