CS 6300 **Utility Practice** **February 11, 2021**

Bob has found an unfair 4-sided die. The die comes up even twice as often as it comes up odd; i.e., P(x even) $= \frac{1}{3}$, and P(x odd) $= \frac{1}{6}$. After a little thought Bob decides that he can devise a game which he can never lose in hopes that he can trick his friend Tom out of soda.

> Tom pays Bob the value of the dice he wants to bet on ($1 for 1, $2 for 2, $3 for 3, $4 for 4). If Tom guesses right, Bob will pay him double his bet.

1. What is Tom's expected reward for each bet?

   (a) -1 + 1/6 * 2 + 5/6 * 0 = -4/6 = -2/3

   (b) -2 + 1/3 * 4 + 2/3 * 0 = -2/3

   (c) -3 + 1/6 * 6 + 5/6 * 0 = -2

   (d) -4 + 1/3 * 8 + 2/3 * 0 = -4/3

2. Tom is a clever grad student and convinces Bob to alter the game so that instead of doubling the bet, Bob should square the bet for a winning throw. What are the new expected rewards?

   (a) -1 + 1/6 * 1 + 5/6 * 0 = -4/6 = -5/6

   (b) -2 + 1/3 * 4 + 2/3 * 0 = -2/3

   (c) -3 + 1/6 * 9 + 5/6 * 0 = -1.5

   (d) -4 + 1/3 * 16 + 2/3 * 0 = +4/3

3. Because Tom is clever he'll always choose bet 4, but suppose Bob plays this game with someone who is less clever and follows the policy: bet randomly (i.e., bet each number with equal probability). How much money does Bob expect to win playing with this agent?

$$\begin{aligned}(1-1/6*1+5/6*0)/4 &+ (2-1/3*4+2/3*0)/4+(3-1/6*9+5/6*0)/4\\ &+ (4-1/3*16+2/3*0)/4\\ &= (5/6+2/3+3/2-4/3)/4\\ &= 5/12\end{aligned}$$