

Bob has found an unfair 4-sided die. The die comes up even twice as often as it comes up odd; i.e.,  $P(x \text{ even}) = \frac{1}{3}$ , and  $P(x \text{ odd}) = \frac{1}{6}$ . After a little thought Bob decides that he can devise a game which he can never lose in hopes that he can trick his friend Tom out of soda.

Tom pays Bob the value of the dice he wants to bet on (\$1 for 1, \$2 for 2, \$3 for 3, \$4 for 4). If Tom guesses right, Bob will pay him double his bet.

1. What is Tom's expected reward for each bet?
2. Tom is a clever grad student and convinces Bob to alter the game so that instead of doubling the bet, Bob should square the bet for a winning throw. What are the new expected rewards?
3. Because Tom is clever he'll always choose bet 4, but suppose Bob plays this game with someone who is less clever and follows the policy: bet randomly (i.e., bet each number with equal probability). How much money does Bob expect to win playing with this agent?