CS 6300 | **Probability Practice** | **February 1, 2022**

1. Consider the joint distribution $P(X, Y)$ below.

X	Y	$P(X, Y)$
$+x$	$+y$	0.2
$+x$	$-y$	0.3
$-x$	$+y$	0.4
$-x$	$-y$	0.1

Events

(a) What is $P(+x, +y)$?

(b) What is $P(+x)$?

(c) What is $P(-y \vee +x)$?

Marginal Distributions Find $P(X)$ and $P(Y)$.

X	$P(X)$
$+x$	
$-x$	

Y	$P(Y)$
$+y$	
$-y$	

Conditional Probabilities

(a) What is $P(+x| + y)$?

(b) What is $P(-x| + y)$?

(c) What is $P(-y| + x)$?

Normalization Trick What is $P(X| - y)$?

2. **Bayes' Rule.** Consider the probability distributions below. What is $P(W|dry)$?

X	$P(W)$
sun	0.8
$rain$	0.2

D	W	$P(D|W)$
wet	sun	0.1
dry	sun	0.9
wet	$rain$	0.7
dry	$rain$	0.3

3. Marijuana legalization has been in the news, and one of the states is having a gubernatorial election. The Libertarian candidate (random variable L) is more likely to legalize marijuana (random variable M) than the other candidates, but legalization may happen if any candidate is elected. The probabilities are modeled below.

	$+l$	$-l$
$P(L)$	0.1	0.9

Libertarian governor elected

	$P(+m\|L)$	$P(-m\|L)$
$+l$	0.667	0.333
$-l$	0.25	0.75

Marijuana legalized

(a) What is $P(+m)$?

(b) What is $P(+l|+m)$?

(c) Fill in the joint distribution table below.

L	M	$P(L,M)$
$+l$	$+m$	
$+l$	$-m$	
$-l$	$+m$	
$-l$	$-m$	

(d) More information is provided with new random variables B (balanced budget) and A (workplace absenteeism).

	$P(+b\|M)$	$P(-b\|M)$
$+m$	0.4	0.6
$-m$	0.2	0.8

Balanced Budget

	$P(+a\|M)$	$P(-a\|M)$
$+m$	0.75	0.25
$-m$	0.5	0.5

Absenteeism

Fill in the joint distribution table below.

L	M	B	A	$P(L,M,B,A)$
$+l$	$+m$	$+b$	$+a$	
$+l$	$+m$	$+b$	$-a$	
$+l$	$+m$	$-b$	$+a$	
$+l$	$+m$	$-b$	$-a$	
$+l$	$-m$	$+b$	$+a$	
$+l$	$-m$	$+b$	$-a$	
$+l$	$-m$	$-b$	$+a$	
$+l$	$-m$	$-b$	$-a$	

L	M	B	A	$P(L,M,B,A)$
$-l$	$+m$	$+b$	$+a$	
$-l$	$+m$	$+b$	$-a$	
$-l$	$+m$	$-b$	$+a$	
$-l$	$+m$	$-b$	$-a$	
$-l$	$-m$	$+b$	$+a$	
$-l$	$-m$	$+b$	$-a$	
$-l$	$-m$	$-b$	$+a$	
$-l$	$-m$	$-b$	$-a$	

(e) Compute the following.

i. $P(+b|+m)$

ii. $P(+b|+m,+l)$

iii. $P(+b)$

iv. $P(+a|+b)$