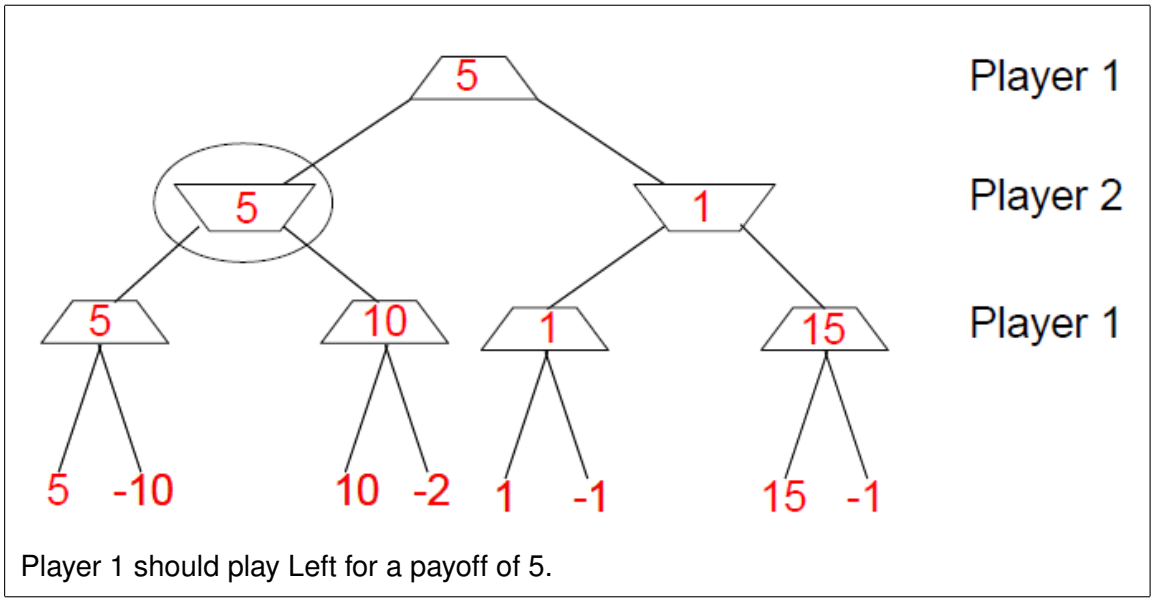
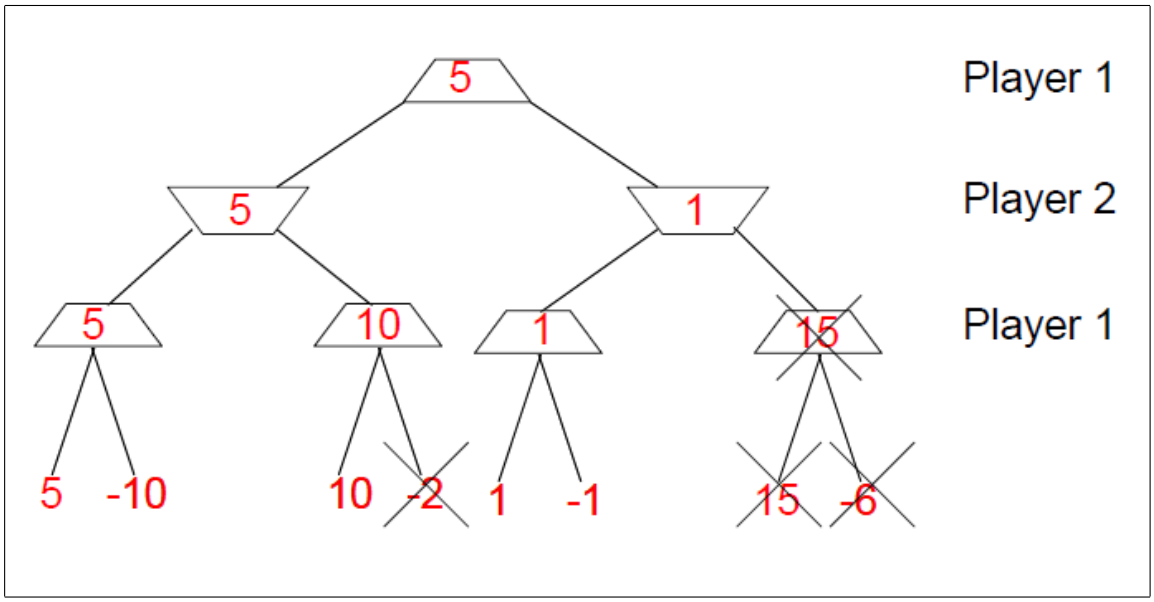
1. Consider the following zero-sum game with 2 players. At each leaf we have labeled the payos Player 1 receives. It is Player 1's turn to move. Assume both players play optimally at every time step (i.e., Player 1 seeks to maximize the payout, while Player 2 seeks to minimize the payout). Circle Player 1's optimal next move on the graph, and state the minimax value of the game. Show your work.

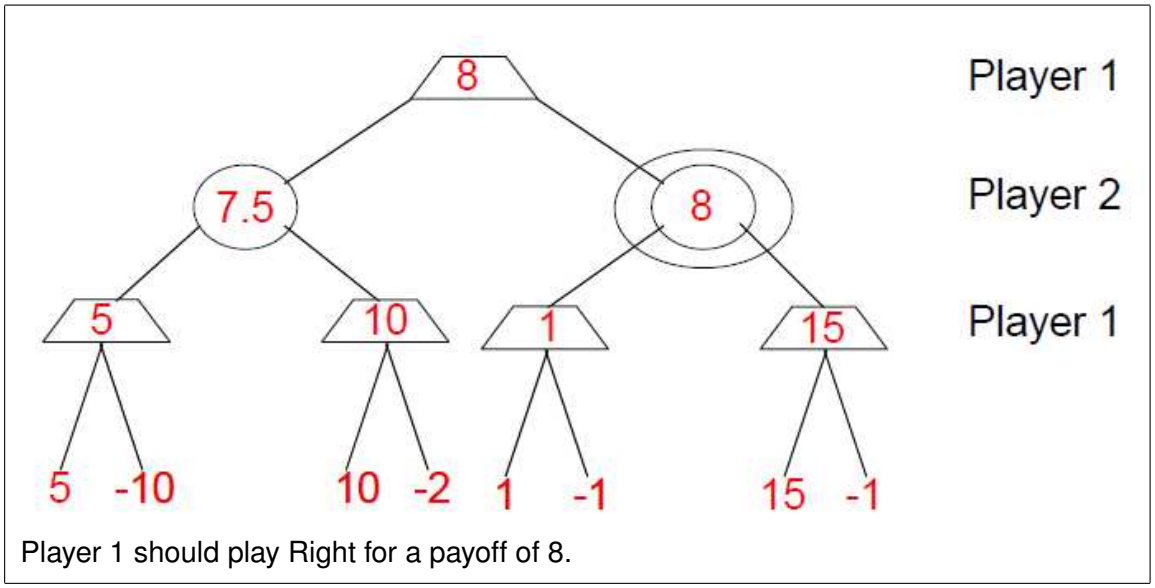


Player 1 should play Left for a payoff of 5.

2. For the same game tree, player 1 again moves first and attempts to maximize the expected payoff. Player 2 moves second, and attempts to minimize the expected payoff. Expanding nodes left to right, cross out nodes pruned by alpha-beta pruning.

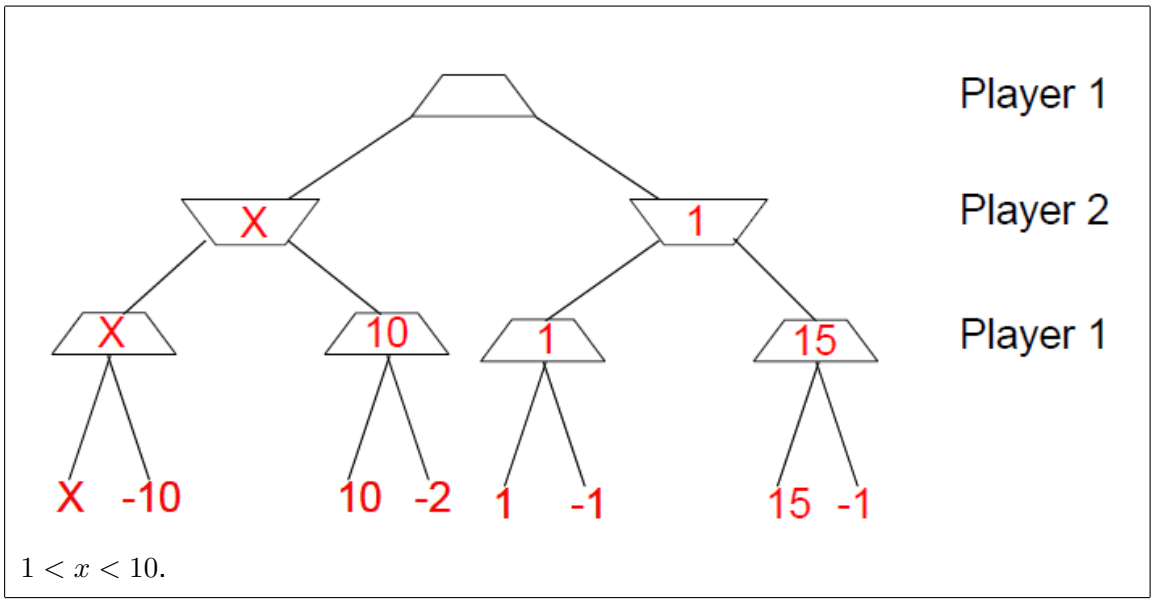


3. Now assume that Player 2 chooses an action uniformly at random every turn (and Player 1 knows this). Player 1 still seeks to maximize the payoff. Circle Player 1's optimal next move, and give the expected payoff. Show your work.



Player 1 should play Right for a payoff of 8.

4. Consider the following modified game tree, where one of the leaves has an unknown payoff x. Player 1 moves first, and attempts to maximize the value of the game. Assume player 2 is a minimizing agent (and Player 1 knows this). For what values of x does Player 1 choose the action to reach node with x?



$1 < x < 10$.

5. Assume Player 2 chooses actions at random (and Player 1 knows this). For what values of x does Player 1 choose the left action?

Running the expectimax calculation we find that the left branch is worth $(10+x)/2$ while the right branch is worth 8. Calculation shows that left branch has higher payoff $x > 6$.

6. Assume Player 2 chooses actions at random (and Player 1 knows this). For what values of x does Player 1 choose the action to reach node with x?

The minimax value of the tree can never exceed the expectimax value of the tree, because the only chance nodes are min nodes. The value of the min node is (weakly) less than the value of the corresponding chance node, so the value the max player receives at the root is (weakly) less under minimax than expectimax. Common mistakes:

- If you assume the minimax value of the tree is x for $x > 1$ and the expectimax value of the tree is $(x + 10)/2$ for $x > 6$ and solve the inequality, you get $x > 10$ as the critical value for x, corresponding to a minimax payoff of x. However, the minimax value can not exceed 10, or the min player will choose the branch with value 10, so you cannot get payoffs $x > 10$.

- If you calculate the value of the left branch under minimax rules and the value of the right branch under expectimax, you may get $x > 8$.