1 Variable Elimination

A mouse is reasoning about the behavior of a cat. Whether the cat will attack (A) depends on whether the cat is hungry (H) and whether the cat is sleepy (S). The mouse can observe two things, whether the cat is sleepy (S) and whether the cat has a collar (C). The cat is more often sleepy (S) when it's either full (f) or starved (v) than when it is peckish (p), and the collar (C) tends to indicate that the cat is not starved. Derive $P(A)$ by variable elimination by executing the following steps.

$P(C)$	
C	$P(C)$
c	0.4

$P(H\|C)$		
H	C	P
f	c	0.70
v	c	0.10
p	c	0.20
f	$\neg c$	0.20
v	$\neg c$	0.50
p	$\neg c$	0.30

$P(S\|H)$		
S	H	P
s	f	0.90
s	v	0.60
s	p	0.30

$P(A\|H,S)$			
A	H	S	P
a	f	s	0.01
a	f	$\neg s$	0.10
a	v	s	0.40
a	v	$\neg s$	0.90
a	p	s	0.20
a	p	$\neg s$	0.70

1. Write the equation for $P(A, C, H, S)$ using the Bayes net structure.

$$P(A, C, H, S) = P(C)P(H|C)P(S|H)P(A|H, S)$$

2. Write the formula to compute $P(A)$ using marginalization of hidden variables S, H, C in that order (left to right). Pull factors out of summations when there is no dependence.

$$P(A) = \sum_S \sum_H P(S|H)P(A|H, S) \sum_C P(C)P(H|C)$$

3. Join on C to yield $P(C, H)$.

H	C	$P(C, H)$
f	c	$0.7 * 0.4 = 0.28$
v	c	$0.1 * 0.4 = 0.04$
p	c	$0.2 * 0.4 = 0.08$
f	$\neg c$	$0.2 * 0.6 = 0.12$
v	$\neg c$	$0.5 * 0.6 = 0.30$
p	$\neg c$	$0.3 * 0.6 = 0.18$

4. Sum out C to yield $P(H)$.

H	$P(H)$
f	$0.28 + 0.12 = 0.40$
v	$0.04 + 0.30 = 0.34$
p	$0.08 + 0.18 = 0.26$

5. Join on H to yield $f_1(A, H, S)$.

H	S	A	$f_1(A, H, S)$
f	s	a	$0.9 * 0.01 * 0.40 = 0.0036$
v	s	a	$0.6 * 0.40 * 0.34 = 0.0816$
p	s	a	$0.3 * 0.20 * 0.26 = 0.0156$
f	s	$\neg a$	$0.9 * 0.99 * 0.40 = 0.3564$
v	s	$\neg a$	$0.6 * 0.60 * 0.34 = 0.1224$
p	s	$\neg a$	$0.3 * 0.80 * 0.26 = 0.0624$
f	$\neg s$	a	$0.1 * 0.10 * 0.40 = 0.0040$
v	$\neg s$	a	$0.4 * 0.90 * 0.34 = 0.1224$
p	$\neg s$	a	$0.7 * 0.70 * 0.26 = 0.1274$
f	$\neg s$	$\neg a$	$0.1 * 0.90 * 0.40 = 0.0360$
v	$\neg s$	$\neg a$	$0.4 * 0.10 * 0.34 = 0.0136$
p	$\neg s$	$\neg a$	$0.7 * 0.30 * 0.26 = 0.0546$

6. Then sum out H to yield $f_2(A, S)$, and finally sum out S.

S	A	$f_2(A, S)$
s	a	$0.0036 + 0.0816 + 0.0156 = 0.1008$
s	$\neg a$	$0.3564 + 0.1224 + 0.0624 = 0.5412$
$\neg s$	a	$0.0040 + 0.1224 + 0.1274 = 0.2538$
$\neg s$	$\neg a$	$0.0360 + 0.0136 + 0.0546 = 0.1042$

$\rightarrow$

A	$P(A)$
a	$0.1008 + 0.2538 = 0.3546$
$\neg a$	$0.5412 + 0.1042 = 0.6454$

It doesn't seem like the cat is likely to attack.