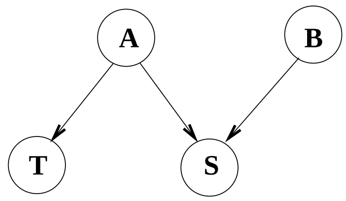
**CS 6300** **Bayes Nets Practice** **March 15, 2022**

Suppose that a patient can have a symptom $S$ that can be caused by two different diseases $A$ and $B$. Disease $A$ is much rarer, but there is a test $T$ that tests for the presence of $A$. The Bayes' Net and corresponding conditional probability tables are shown below.




| $A$ | $P(A)$ |
|---|---|
| $+a$ | 0.1 |
| $-a$ | 0.9 |

| $B$ | $P(B)$ |
|---|---|
| $+b$ | 0.5 |
| $-b$ | 0.5 |

| $A$ | $T$ | $P(T\|A)$ |
|---|---|---|
| $+a$ | $+t$ | 1 |
| $+a$ | $-t$ | 0 |
| $-a$ | $+t$ | 0.2 |
| $-a$ | $-t$ | 0.8 |

| $A$ | $B$ | $S$ | $P(S\|A,B)$ |
|---|---|---|---|
| $+a$ | $+b$ | $+s$ | 1 |
| $+a$ | $+b$ | $-s$ | 0 |
| $+a$ | $-b$ | $+s$ | 0.8 |
| $+a$ | $-b$ | $-s$ | 0.2 |
| $-a$ | $+b$ | $+s$ | 1 |
| $-a$ | $+b$ | $-s$ | 0 |
| $-a$ | $-b$ | $+s$ | 0 |
| $-a$ | $-b$ | $-s$ | 1 |

1. From the Baye's Net structure, what is $P(A, T, B, S)$?

$$P(A, T, B, S) = P(A)P(B)P(T|A)P(S|A, B)$$

2. What is $P(-a, -t, +b, +s)$?

$$P(-a, -t, +b, +s) = P(-a)P(+b)P(-t| - a)P(+s| - a, +b) = 0.9 \cdot 0.5 \cdot 0.8 \cdot 1 = 0.36$$

3. What is the probability that a patient has disease $+a$ given that they have disease $+b$?

$$P(+a| + b) = P(+a) = 0.1$$

4. What is the probability that a patient has disease $+a$ given that they have symptoms $+s$, disease $+b$, and test $+t$ returns positive?

$$P(+a| + t, +b, +s) \quad = \quad \frac{P(+a, +t, +b, +s)}{P(+t, +b, +s)} = \frac{P(+a, +t, +b, +s)}{P(+a, +t, +b, +s) + P(-a, +t, +b, +s)}$$

where

$$
\begin{aligned}
P(+a,+t,+b,+s) &= P(+a)P(+b)P(+t|+a)P(+s|+a,+b) \\
&= 0.1 \cdot 0.5 \cdot 1 \cdot 1 = 0.05 \\
P(-a,+t,+b,+s) &= P(-a)P(+b)P(+t|-a)P(+s|-a,+b) \\
&= 0.9 \cdot 0.5 \cdot 0.2 \cdot 1 = 0.09
\end{aligned}
$$

Hence

$$
P(+a|+t,+b,+s) = \frac{0.05}{0.05+0.09} = \frac{5}{14}
$$

5. What is the probability that a patient has disease $+a$ given that they have symptom $+s$ and test $+t$ returns positive?

$$
\begin{aligned}
P(+a|+t,+s) &= \frac{P(+a,+t,+s)}{P(+t,+s)} \\
&= \frac{P(+a,+t,+b,+s)+P(+a,+t,-b,+s)}{P(+a,+t,+s)+P(-a,+t,+s)} \\
&= \frac{P(+a,+t,+b,+s)+P(+a,+t,-b,+s)}{P(+a,+t,+b,+s)+P(+a,+t,-b,+s)+P(-a,+t,+b,+s)+P(-a,+t,-b,+s}
\end{aligned}
$$

where

$$
\begin{aligned}
P(+a,+t,-b,+s) &= P(+a)P(-b)P(+t|+a)P(+s|+a,-b) \\
&= 0.1 \cdot 0.5 \cdot 1 \cdot 0.8 = 0.04 \\
P(-a,+t,-b,+s) &= P(-a)P(-b)P(+t|-a)P(+s|-a,-b) \\
&= 0.9 \cdot 0.5 \cdot 0.2 \cdot 0 = 0
\end{aligned}
$$

Substituting,

$$
P(+a|+t,+s) = \frac{0.05+0.04}{0.05+0.04+0.09+0} = 0.5
$$