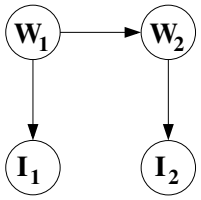
1 Sampling

The diagram below describes a person's ice cream eating habits based on the weather. The nodes W_i stand for the weather on a day i, which can either be rainy R or sunny S. The nodes I_i represent whether or not the person ate ice-cream on day i, and the node takes values T (for truly eating ice cream) or F. The conditional probability distributions relevant to the graphical model are also given to you.

W_1	$P(W_1)$
S	0.6
R	0.4

I_i	W_i	$P(I_i\|W_i)$
T	S	0.9
T	R	0.2

W_2	W_1	$P(W_2\|W_1)$
S	S	0.7
S	R	0.5



Suppose we want to answer the query $P(W_2|I_1 = T, I_2 = F)$ using likelihood weighting.

1. Generate 6 samples using the following random numbers left to right.

   0.41 0.85 0.93 0.67 0.13 0.81 0.05 0.33 0.58 0.49 0.61 0.49

Sample number	Sample
1	S,T,R,F
2	R,T,R,F
3	S,T,R,F
4	S,T,S,F
5	S,T,S,F
6	R,T,S,F

2. Derive the weights w for each sample.

Sample number	weight
1	0.9 * 0.8 = 0.72
2	0.2 * 0.8 = 0.16
3	0.72
4	0.9 * 0.1 = 0.09
5	0.09
6	0.2 * 0.1 = 0.02

3. Use likelihood weighting to estimate $P(W_2|I_1 = T, I_2 = F)$.

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\begin{aligned}
P(W_2 = R|I_1 = T, I_2 = F) &= \frac{0.72 + 0.16 + 0.72}{0.72 + 0.16 + 0.72 + 0.09 + 0.09 + 0.02} = 0.889 \\
P(W_2 = S|I_1 = T, I_2 = F) &= 1 - 0.889 = 0.111
\end{aligned}
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