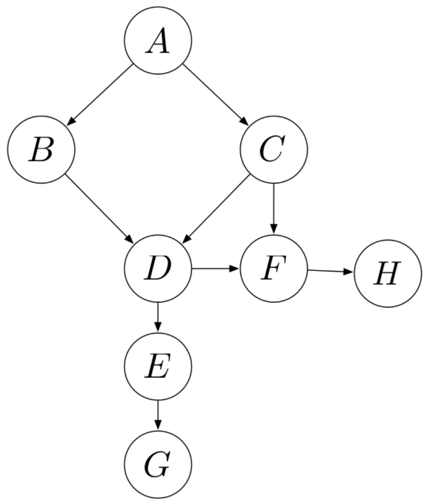
# 1 D-Separation

For the Bayes net below, determine if each independence assertion is guaranteed to be true.



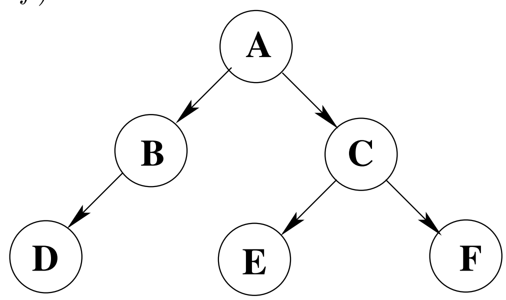

1. $B \perp\!\!\!\perp C$
2. $B \perp\!\!\!\perp C \mid G$
3. $B \perp\!\!\!\perp C \mid H$
4. $A \perp\!\!\!\perp D \mid G$
5. $A \perp\!\!\!\perp D \mid H$
6. $B \perp\!\!\!\perp C \mid A, F$
7. $F \perp\!\!\!\perp B \mid D, A$
8. $F \perp\!\!\!\perp B \mid D, C$

## 2 Inference by Enumeration

Consider the following Bayes' net. Derive $P(A|+e, -f)$.

| A | $P(A)$ |
|---|---|
| +a | 0.3 |
| -a | 0.7 |

| B | A | $P(B\|A)$ |
|---|---|---|
| +b | +a | 0.7 |
| +b | -a | 0.6 |




| C | A | $P(C\|A)$ |
|---|---|---|
| +c | +a | 0.2 |
| +c | -a | 0.9 |

| D | B | $P(D\|B)$ |
|---|---|---|
| +d | +b | 0.3 |
| +d | -b | 0.4 |

| E | C | $P(E\|C)$ |
|---|---|---|
| +e | +c | 0.4 |
| +e | -c | 0.5 |

| F | C | $P(F\|C)$ |
|---|---|---|
| -f | +c | 0.9 |
| -f | -c | 0.2 |

We have

$$P(A|+e,-f) \propto P(A,+e,-f) = \sum_{B,C,D} P(A,B,C,D,+e,-f)$$

From the Bayes' net,

$$P(A,B,C,D,+e,-f) = P(A)P(B|A)P(C|A)P(D|B)P(+e|C)P(-f|C)$$

The order in which to joint factors is arbitrary, and B, C, A is used below.

$$\begin{aligned} f_1(A,B,D) &= P(B|A)P(D|B) \\ f_2(A,C,+e,-f) &= P(C|A)P(+e|C)P(-f|C) \\ f_3(A,B,C,D,+e,-f) &= P(A)f_1(A,B,D)f_2(A,C,+e,-f) \end{aligned}$$

where the factors are:

| A | B | D | $f_1(A,B,D)$ |
|---|---|---|---|
| +a | +b | +d | |
| +a | +b | -d | |
| +a | -b | +d | |
| +a | -b | -d | |
| -a | +b | +d | |
| -a | +b | -d | |
| -a | -b | +d | |
| -a | -b | -d | |

| A | C | $f_2(A,C,+e,-f)$ |
|---|---|---|
| +a | +c | |
| +a | -c | |
| -a | +c | |
| -a | -c | |

Using $f_1$ and $f_2$, compute $f_3$:

| A | B | C | D | $f_3(A, B, C, D, +e, -f)$ |
|---|---|---|---|---|
| +a | +b | +c | +d | |
| +a | +b | +c | -d | |
| +a | +b | -c | +d | |
| +a | +b | -c | -d | |
| +a | -b | +c | +d | |
| +a | -b | +c | -d | |
| +a | -b | -c | +d | |
| +a | -b | -c | -d | |
| -a | +b | +c | +d | |
| -a | +b | +c | -d | |
| -a | +b | -c | +d | |
| -a | +b | -c | -d | |
| -a | -b | +c | +d | |
| -a | -b | +c | -d | |
| -a | -b | -c | +d | |
| -a | -b | -c | -d | |

Then we marginalize over B,C, and D

| A | $\sum_{B,C,D} f_3(A, B, C, D, +e, -f)$ |
|---|---|
| +a | |
| -a | |

Finally, we normalize to get our answer

| A | $P(A\| + e, -f)$ |
|---|---|
| +a | |
| -a | |