CS 6300 **HMM Practice** **April 7, 2022**

You are trying to diagnose whether your computer is broken or not. On a given day, your computer's hidden state is either *broken* or *working*. Each day you make one of the following observations: *blue-screen*, *slow*, or *snappy*, depending on the state of your computer. You decide to use the following HMM to model your daily observations:

Initial Distribution

State	$P(X_\bullet)$
working	0.9
broken	0.1

Transition Distribution

State	Next State	$P(X_{t+1}\|X_t)$
working	*working*	0.9
working	*broken*	0.1
broken	*broken*	1.0
broken	*working*	0.0

Emission Distribution

State	Observation	$P(O_t\|X_t)$
working	*snappy*	0.7
working	*slow*	0.2
working	*blue-screen*	0.1
broken	*snappy*	0.1
broken	*slow*	0.4
broken	*blue-screen*	0.5

1. What is the posterior distribution of X_1, your computer's state on day one, given the observation (*slow*) on day 1?

   The equation for starting out is:

$$P(X_1|e_1) = \alpha P(e_1|X_1) \sum_{x_0} P(X_1|x_0)P(x_0)$$

   Particularized to this problem, and using the relevant CPTs:

$$\begin{aligned}
P(X_1|slow) &= \alpha P(slow|X_1) \sum_{x_0} P(X_1|x_0)P(x_0) \\
&= \alpha \begin{bmatrix} P(slow|working) \\ P(slow|broken) \end{bmatrix} \left\{ P(working) \begin{bmatrix} P(working|working) \\ P(broken|working) \end{bmatrix} + \right. \\
&\qquad \left. P(broken) \begin{bmatrix} P(working|broken) \\ P(broken|broken) \end{bmatrix} \right\} \\
&= \alpha \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \left\{ 0.9 \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} + 0.1 \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} \right\} \\
&= \alpha \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \begin{bmatrix} 0.81 \\ 0.19 \end{bmatrix} \\
&= \alpha \begin{bmatrix} 0.162 \\ 0.076 \end{bmatrix}
\end{aligned}$$

Normalizing,

$$P(X_1|slow) = \frac{1}{119}\left[\begin{array}{c} 81 \\ 38 \end{array}\right]$$

2. What is the posterior distribution of X_2, your computer's state on day two, given the observation sequence (*slow, slow*)?

   For day 2, the general expression is:

$$P(X_2|e_1, e_2) = \alpha P(e_2|X_2) \sum_{x_1} P(X_2|x_1)P(x_1|e_1)$$

   For this problem,

$$\begin{aligned}
P(X_2|slow, slow) &= \alpha P(slow|X_2) \sum_{x_1} P(X_2|x_1)P(x_1|slow) \\
&= \alpha \left[\begin{array}{c} P(slow|working) \\ P(slow|broken) \end{array}\right] \Bigg\{ P(working|slow) \left[\begin{array}{c} P(working|working) \\ P(broken|working) \end{array}\right] + \\
& \quad P(broken|slow) \left[\begin{array}{c} P(working|broken) \\ P(broken|broken) \end{array}\right] \Bigg\} \\
&= \alpha \left[\begin{array}{c} 0.2 \\ 0.4 \end{array}\right] \left\{ \frac{81}{119} \left[\begin{array}{c} 0.9 \\ 0.1 \end{array}\right] + \frac{38}{119} \left[\begin{array}{c} 0.0 \\ 1.0 \end{array}\right] \right\} \\
&= \alpha \left[\begin{array}{c} 0.2 \\ 0.4 \end{array}\right] \left[\begin{array}{c} 72.9 \\ 46.1 \end{array}\right] /119 \\
&= \alpha \left[\begin{array}{c} 0.1225 \\ 0.1550 \end{array}\right]
\end{aligned}$$

   Normalizing,

$$P(X_2|slow, slow) = \left[\begin{array}{c} 0.4414 \\ 0.5586 \end{array}\right]$$

3. If you observe that your computer is *slow* every day, what is the first day for which the computer is most likely *broken*?

   Day 2 is the first time when *broken* is more likely (denominator greater than the numerator).