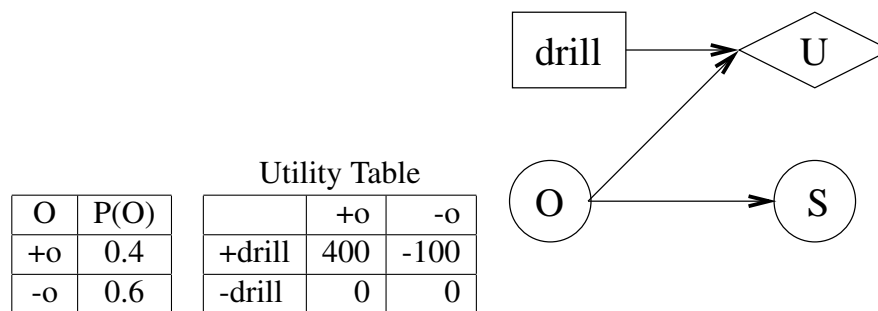


An oil prospector can decide to carry out a seismic test at a given site, and the result of the test will help him decide whether to drill or not. The test results can be “closed pattern” (more likely when the site holds the oil) or “diffuse pattern” (more likely when empty). The seismic test costs \$20K. The site might contain an oil deposit ($O = +o$) or might not ($O = -o$). The prospector estimates that the chance of hitting an oil deposit is 40%. The cost of drilling is \$100K and the payoff for hitting an oil deposit is \$400K (including the cost of drilling).

1. Draw the decision network that represents this problem (the only action is to drill or not drill on the site). Specify the $P(O)$ and utility U tables.



2. Calculate the expected net gain from drilling on the site, given no test.

$$\begin{aligned} EU(+drill|\emptyset) &= P(+o)U(+o, drill) + P(-o)U(-o, drill) \\ &= 0.4 * 400K + 0.6 * (-100K) = 100K \end{aligned}$$

This is also the $MEU(\emptyset)$, since $EU(-drill|\emptyset) = 0$.

3. Suppose that the seismic test S tells you that $P(S = closed|O = +o) = 0.8$ and $P(S = closed|O = -o) = 0.1$. Calculate (1) the probability that the test reveals a closed pattern and (2) the probability that the site contains an oil deposit, given each possible test outcome.

$$\begin{aligned} P(S = closed) &= P(S = closed|+o)P(+o) + P(S = closed|-o)P(-o) \\ &= 0.8 * 0.4 + 0.1 * 0.6 = 0.38 \end{aligned}$$

$$P(S = diffuse) = 1 - 0.38 = 0.62$$

$$P(+o|S = closed) = \frac{P(S = closed|+o)P(+o)}{P(S = closed)}$$

$$\begin{aligned}
&= \frac{0.8 * 0.4}{0.38} = 0.842 \\
P(+o|S = diffuse) &= \frac{P(S = diffuse|+o)P(+o)}{P(S = diffuse)} \\
&= \frac{(1 - 0.8) * 0.4}{0.62} = 0.129
\end{aligned}$$

4. Calculate the maximum expected utility given a test of closed.

$$\begin{aligned}
EU(+drill|S = closed) &= P(+o|S = closed)U(+drill, +o) + \\
&\quad P(-o|S = closed)U(+drill, -o) \\
&= 0.842 * 400 + (1 - 0.842) * (-100) = 321
\end{aligned}$$

$$\begin{aligned}
EU(-drill|S = closed) &= P(+o|S = closed)U(-drill, +o) + \\
&\quad P(-o|S = closed)U(-drill, -o) \\
&= 0.842 * 0 + (1 - 0.842) * 0 = 0
\end{aligned}$$

$$MEU(S = closed) = 321$$

5. Calculate the maximum expected utility given a test of diffuse.

$$\begin{aligned}
EU(+drill|S = diffuse) &= P(+o|S = diffuse)U(+drill, +o) + \\
&\quad P(-o|S = diffuse)U(+drill, -o) \\
&= 0.129 * 400 + (1 - 0.129) * (-100) = -35.5
\end{aligned}$$

$$\begin{aligned}
EU(-drill|S = diffuse) &= P(+o|S = diffuse)U(-drill, +o) + \\
&\quad P(-o|S = diffuse)U(-drill, -o) \\
&= 0.129 * 0 + (1 - 0.129) * 0 = 0
\end{aligned}$$

$$MEU(S = diffuse) = 0$$

6. Calculate the VPI.

$$\begin{aligned} VPI(S) &= P(S = closed)MEU(S = closed) + P(S = diffuse)MEU(S = diffuse) - \\ &\quad MEU(\emptyset) \\ &= 0.38 * 321 + 0.62 * 0 - 100 = 21.98 \end{aligned}$$