You are trying to locate the ghost in the maze below, where walls are in bold. The ghost makes a clanging noise, and your acoustic sensor $S$ can tell you the number of walls near the ghost ( $W \in 0,1,2,3$ ), with some error.


- For squares with 1 or 2 walls, the sensor $S$ is correct $2 / 3$ rds of the time, but $1 / 6$ th of the time it overestimates by 1 or underestimates by 1 .
- For squares with 0 or 3 walls, the sensor is correct $5 / 6$ ths of the time, but gets the estimate wrong by 1 in $1 / 6$ th of the time.

With equal probability, the ghost moves up, right, down or left (URDL). If there is a wall in the way, the ghost stays put. To simulate this movement, a 4 -sided die maps 1-U, 2-R, 3-D and 4-L. There are 16 states ( $1 \mathrm{~A}, 1 \mathrm{~B}$, etc.).

1. For convenience, states are grouped by the number of walls $W$ surrounding each square in column 1. Fill out the CPT for $P(S \mid W)$. Row 1 contains the number of sensed walls. The interior of the table is to be filled with the $P(S \mid W)$ values.

| $W \backslash S$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |

2. Assume you do not know where the ghost is intially. Initialize the map by placing one particle in each of the 16 squares; show this on the leftmost map. Starting with the top left, moving left to right, then top to bottom, update the location of the particles for 1 time step. Assume that the numbers rolled randomly by the die are given for each of the 16 particles as:

$$
2,3,3,1,2,4,4,3,1,4,1,2,3,1,4,2
$$

Fill in the "New Location" row of the table below, where the particle number is 1-16 in the $1-4$, A-D ordering as above. Also, show the particles in the 2nd map above.

| Particle | 1 | 2 | 3 | 4 |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |  | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| New Location |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Re-weight |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Normalize |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

3. After this time step, your sensor reads $S=3$. Complete the sensor step of the filtering. First re-weight your particles from the previous question, then normalize them based on the sum of all the weights * 16. (I.e, if the sum of all the weights becomes 4.5, divide each particle weight by 4.5 and multiply by 16). Fill in the "Re-weight" and "Normalize" rows of the table above.
4. Now re-sample the particles with 16 new, equally weighted (1.0) particles. To do the resampling, use the following pseudo-code:
```
for \(i=1 \rightarrow 16\) do
    rand \(\leftarrow \operatorname{rand}[0,16]\)
    sum \(\leftarrow 0\)
    for all weighted (normalized) particles \(j\) do
            sum \(\leftarrow\) sum + weight of particle \(j\)
            if sum \(>\) rand then
                place a new particle onto new map at this particle's location
                break
            end if
        end for
end for
```

Use the following randomly generated numbers for the outer for loop. Show the results in the 3rd map above and the 2nd line of the table below.
$\{6.1049,12.2483,12.7232,2.9900,7.8362,7.1294,10.3410,11.3498$, $12.0750,4.4164,10.8752,10.4816,2.6018,1.9040,7.9738,15.3559\}$

| Particle | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Location | 1 A | 1 B | 1 C | 1 D | 2 A | 2 B | 2 C | 2 D | 3 A | 3 B | 3 C | 3 D | 4 A | 4 B | 4 C | 4 D |
| New Location |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| New Location |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

5. Complete another time step for the distribution completed in the last step. Use these following random 'dice rolls'. Show the results on the last map and the 3rd line of the table above.

$$
3,1,3,4,3,2,1,2,4,1,4,2,4,1,2,1
$$

