Date: March 5, 1998
Due Date: March 19, 1998

CS 522: Advanced Methods in Scientific Computing

Assignment 6

Define the error of the gradient of a finite element solution to be:

$$e_g = \nabla u - \nabla u_h \tag{1}$$

Then the corresponding L_2 norm can be defined as:

$$\|e_g\| = (\int_\Omega (\nabla u - \nabla u_h)^T (\nabla u - \nabla u_h)\, d\Omega)^{\frac{1}{2}} \tag{2}$$

This is equivalent to,

$$\|e_g\|^2 = \|\nabla u\|^2 - \|\nabla u_h\|^2. \tag{3}$$

This quantity can also be expressed as a percentage error,

$$\eta = \frac{\|e_g\|}{\|\nabla u\|} \times 100\%. \tag{4}$$

1. Write a program that estimates the error and refines elements that fall outside a predefined error tolerance.

2. For the L-problem as described in Chapter 5 of the CS 522 notes with exact solution given by $u(r,\theta) = r^{2/3} sin(2\theta/3)$ in polar coordinates, solve this equation using the finite difference (FD) method and the finite element (FE) method. Compare your results with the analytic solution for different regular grids. Then refine the FD grid using regular subdivisions and refine the FE grid using the adaptive program in part 1. Continue to compare each solution with the analytic solution.

3. Using the series of adaptive FE grids and solutions, compute the values of h versus r for each grid/solution. Compare this to the a priori error estimate $h \approx r^{1/3}\delta$.

4. (Optional) Use the error estimate described in section 5.5 of the CS 522 notes and compare your results with the error estimate from part 1.