Little Tricky Logic:
Misconceptions in the Understanding of LTL

Ben Greenman
Sam Saarinen
Tim Nelson
Shriram Krishnamurthi

BROWN

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LTL = Linear Temporal Logic
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For systems that change over time
**LTL** = **Linear Temporal Logic**

For systems that change over time

- Expressive
- Supports good decision procedures
- Small

... and easy to learn?
RQ. In what ways is LTL tricky, and what can we do about it?
RQ. In *what ways* is LTL tricky, and *what can we do* about it?

+2 years of studies with researchers and students
4 survey rounds
Quiz Time!
Part 1:
Formulas vs. Traces
Q. Do the traces below satisfy this formula?
\{\text{eventually Red}\} \text{ and } \{\text{eventually Green}\}
Q. Do the traces below satisfy this formula?
{eventually Red} and {eventually Green}
Q. Do the traces below satisfy this formula?

\{\text{eventually Red}\} \text{ and } \{\text{eventually Green}\}
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\{\text{eventually Red}\} \text{ and } \{\text{eventually Green}\}
Q. Do the traces below satisfy this formula?

\{\text{eventually Red}\} \text{ and } \{\text{eventually Green}\}

![Incorrect Trace](Diagram of incorrect trace)

![Correct Trace](Diagram of correct trace)
Q. Do the traces below satisfy this formula?

\{\text{eventually Red}\} \text{ and } \{\text{eventually Green}\}

\[\begin{array}{c}
\text{X} \\
\begin{array}{cccc}
\text{R} & \text{G} & \text{B} \\
1 & 2 & 3 & 4 & 5+
\end{array}
\end{array}\]

\[\begin{array}{c}
\text{X} \\
\begin{array}{cccc}
\text{R} & \text{G} & \text{B} \\
1 & 2 & 3 & 4 & 5+
\end{array}
\end{array}\]

\[\begin{array}{c}
\checkmark \\
\begin{array}{cccc}
\text{R} & \text{G} & \text{B} \\
1 & 2 & 3 & 4 & 5+
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Q. Do the traces below satisfy this formula?

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\{\text{eventually Red} \} \text{ and } \{\text{eventually Green} \}
Q. Do the traces below satisfy this formula?
\{\text{eventually Red}\} \text{ and } \{\text{eventually Green}\}

\begin{align*}
\text{Not satisfied, because Green comes before Red} \\
\text{Bad Prop misconception}
\end{align*}
Q. Do the traces below satisfy this formula?

Red
Q. Do the traces below satisfy this formula?

Red

[Diagram showing the sequence of R, G, B colors with numbers 1 to 5+]

Red
Q. Do the traces below satisfy this formula?

Red

[Diagram showing a sequence of states with arrows between them, indicating the flow of the traces. The final state is marked with a red cross to indicate it does not satisfy the formula.]
Q. Do the traces below satisfy this formula?

Red

[Diagram showing different colored states connected by arrows for Red, with a cross indicating the traces do not satisfy the formula.]
Q. Do the traces below satisfy this formula?

Red

[X] Incorrect:

[X] Correct:
Q. Do the traces below satisfy this formula?

Red

[Diagram showing three sets of traces, with one marked with an 'X' and the other with a checkmark. Each set consists of a sequence of colors (R, G, B) and numbers (1, 2, 3, 4, 5+) indicating the progression of the traces.]
Q. Do the traces below satisfy this formula?

Red

[Diagrams with red circles labeled R, G, B and arrows indicating sequence 1, 2, 3, 4, 5+ with two sequences marked with a checkmark and three marked with an xmark]
Q. Do the traces below satisfy this formula?

Red
Q. Do the traces below satisfy this formula?

Red

Satisfied because Red is on at some point

Implicit F misconception
Q. Do the traces below satisfy this formula?

**Red until Blue**
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]

×

1 2 3 4 5+
Q. Do the traces below satisfy this formula?

**Red until Blue**

[ strong until ]

![Trace Diagram]

1. Red
2. Green
3. Blue
4. Red
5. Green
6. Blue

The traces do not satisfy the formula.
Q. Do the traces below satisfy this formula?

**Red until Blue**

[ strong until ]

![Diagram showing two sets of traces, one marked with an 'X' for incorrect and one with a checkmark for correct.](image-url)
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]

\[ \text{Red until Blue} \]
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]

1. Red
2. Red
3. Red
4. Red
5+. Red

1. Red
2. Red
3. Red
4. Blue
5+. Blue

1. Red
2. Red
3. Red
4. Red
5+. Red
Q. Do the traces below satisfy this formula?

Red until Blue [ strong until ]
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]

Satisfied because Blue may stay off
Even among researchers!

Weak U misconception
Part 2:
LTL to English
Q. Translate to English

{Red until Blue} and {always Red}
Q. Translate to English

{Red until Blue} and {always Red}

"Red is always on"
Q. Translate to English

{Red until Blue} and {always Red}

❌ "Red is always on"
Q. Translate to English

{Red until Blue} and {always Red}

X  "Red is always on"

"Red is always on and Blue is eventually on"
Q. Translate to English

{Red until Blue} and {always Red}

❌ "Red is always on"

✔ "Red is always on and Blue is eventually on"
Q. Translate to English

{Red until Blue} and {always Red}

X  "Red is always on"

✓  "Red is always on and Blue is eventually on"

"This statement can never be satisfied"
Q. Translate to English

\{\text{Red until Blue}\} \text{ and } \{\text{always Red}\}

\times \ "\text{Red is always on}"

\checkmark \ "\text{Red is always on and Blue is eventually on}"

\times \ "\text{This statement can never be satisfied}\"
Q. Translate to English

{Red until Blue} and {always Red}

X "This statement can never be satisfied"
Q. Translate to English
{Red until Blue} and {always Red}

When Blue turns on, Red **must** be off

Exclusive U  misconception

✗  "This statement can never be satisfied"
Q. Translate to English

\{eventually Red\} implies \{always Blue\}
Q. Translate to English

{eventually Red} implies {always Blue}

"if Red is ever on, then Blue is always on"
Q. Translate to English

{eventually Red} implies {always Blue}

✓ "if Red is ever on, then Blue is always on"
Q. Translate to English

{eventually Red} implies {always Blue}

✓ "if Red is ever on, then Blue is always on"

"Red is on at some point, after which Blue is on"
Q. Translate to English

{eventually Red} implies {always Blue}

✓ "if Red is ever on, then Blue is always on"

✗ "Red is on at some point, after which Blue is on"
Q. Translate to English

\{eventually Red\} implies \{always Blue\}

× "Red is on at some point, after which Blue is on"
Q. Translate to English
{eventually Red} implies {always Blue}

Red will turn on
Bad Prop misconception

"Red is on at some point, after which Blue is on"
Part 3:
English to LTL
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Impossible!
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Impossible!
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Impossible!

\{\text{eventually Red}\} \text{ and } \{\text{always } \{\text{Red} \Rightarrow \text{always } !\text{Red}\}\}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

✗ Impossible!

✗ \{\text{eventually Red}\} \text{ and } \{\text{always } \{\text{Red }\Rightarrow \text{ always } !\text{Red}\}\}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

- Impossible!

- \{ eventually Red \} and \{ always \{ Red => always !Red \} \}

- \{ eventually Red \} and \{ always \{ Red => after \{ always !Red \} \} \}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Red

Impossible!

{eventually Red} and {always {Red => always !Red}}

{eventually Red} and {always {Red => after {always !Red}}}


Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

✗ {eventually Red} and {always {Red ⇒ always !Red}}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

An implication constrains the **next state**

Bad State Index  misconception

✗ {eventually Red} and {always {Red => always !Red}}
All Done!
Tricky?

Obvious?!

Expert Blind Spot?
Quiz Q's Based on 3 Instruments

- Trace Satisfaction
- LTL to English
- English to LTL

---

Example Question: Is the formula
always (Engine or Light)
satisfied by this trace?

Example Answer: Yes, because either the engine (smoke) or the headlight is on in each state.

Does the example make sense to you?
- Yes
- No (please explain)

---

Q: Is the formula
(Red) until (Blue)
satisfied by this trace?

- Yes
- No
## Code Book for Analysis

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bad Prop</td>
<td>Implicit F</td>
</tr>
<tr>
<td>Bad State Index</td>
<td>Implicit G</td>
</tr>
<tr>
<td>Bad State Quantification</td>
<td>Other Implicit</td>
</tr>
<tr>
<td>Exclusive U</td>
<td>Weak U</td>
</tr>
</tbody>
</table>

**Coding Rubric** in paper, past versions in artifact
Software: Quizius

Class-sourcing to discover misconceptions

1. Answer Top Q's

Question
The above sentence should describe a set of traces over the variables x1, x2, etc. Encode it formally in LTL. Please ignore superficial mistakes like typos, and do not use external tools like Spin to help you.

2. Submit New Q's

Write a Question
We are asking you to describe, in English, a set of traces that are interesting, tricky or surprising to encode in LTL. Please try to give an English description that is no bigger than it needs to be in
What Next?
What Next?

1. Teach Better

our instruments can help!
What Next?

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our instruments can help!

... but learners are everywhere
not just in classrooms

amazon  Meta  Netflix
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NETFLIX
What Next?

1. Teach Better
   our instruments can help!

   ... but learners are everywhere
   not just in classrooms

2. Build Tools
   guard against misconceptions

...
What Next?

1. Teach Better
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   ... but learners are everywhere
   not just in classrooms

2. Build Tools
   guard against misconceptions

3. Design Logics
   our findings have inspired changes

   Alloy 6     Electrum
            Amazon   Meta
What's New?
logic for systems
eventually \{ Red and Green \}^*
Q. Translate to English

```plaintext
next_state { eventually { Red }}
```
Q. Translate to English

`next_state { eventually { Red }}`

"... doesn't make sense to call `next_state` on `eventually`. Eventually should already evaluate to true or false"
Q. Translate to English

```
next_state { eventually { Red }}
```

"... doesn't make sense to call next_state on eventually
eventually should already evaluate to true or false"

**Programmer intuitions!**
Two equations, for all formulas $a$:

$$!X(a) \equiv X(!a) \quad G(F(a)) \neq F(G(a))$$

Valid in LTL, invalid in LTLf
Thank You!

benjamin.l.greenman@gmail.com
What Next?

1. Teach Better
   our instruments can help!

2. Build Tools
   guard against misconceptions

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   our findings have inspired changes

... but learners are everywhere
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