

# A Misconception-Driven Adaptive Tutor for Linear Temporal Logic

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**Abstract.** Linear Temporal Logic (LTL) is used widely in verification, planning, and more. Unfortunately, users often struggle to learn it. To improve their learning, they need drill, instruction, and adaptation to their strengths and weaknesses. Furthermore, this should fit into whatever learning process they are already part of (such as a course).

In response, we have built a *misconception-based* automated tutoring system. It assumes learners have a basic understanding of logic, and focuses on their understanding of LTL operators. Crucially, it takes advantage of multiple years of research (by our team, with collaborators) into misconceptions about LTL amongst both novices and experts.

The tutor generates questions using these known learner misconceptions; this enables the tutor to determine which concepts learners are strong and weak on. When learners get a question wrong, they are offered immediate feedback in terms of the concrete error they made. If they consistently demonstrate similar errors, the tool offers them feedback in terms of more general misconceptions, and tailors subsequent question sets to exercise those misconceptions.

The tool is hosted for free on-line, is available open source for self-hosting, and offers instructor-friendly features.

**Keywords:** LTL, misconceptions, adaptive tutor

## 1 Introduction

Linear Temporal Logic is a cornerstone of verification [60], and is also used for synthesis [3,4,9,12,43,47,56,64], robotics [6,7,11,23,31,33,34,39,54,61,63], embedded systems [10,57], business processes [1,14,16,17,42], and more [45,58].

Since an incorrect specification can cause bugs to go undetected or derail a system’s functionality, it is critical that users (from students to professionals) can write and understand LTL specifications correctly. This has spurred a growing body of research focused on improving the process of authoring and interpreting LTL specifications. These efforts include tools designed to represent specifications in terms of alternate formalisms (e.g., Büchi automata [8,19]), explain formulae via visualization [35,62], and generate specifications from natural language [15,24].

Another way to tackle the problem is to better understand *what* aspects of LTL are difficult. For several years [27,30], with collaborators, we have focused on this question. Our findings have resulted in a catalog of common misconceptions, and these have been distilled into multiple-choice question/answer sets.

However, simply deploying a question/answer sheet as, say, an electronic survey is not enough. To learn well, learners need lots of examples and periodic drill (even refreshers). When they make mistakes, they need feedback to understand what they got wrong. This feedback should occur at two levels. On individual questions, learners may have made a *mistake*. Though they need concrete feedback (e.g., showing traces), the error could also be due to a lack of attention or even a slip of the finger. If, however, they make the same kind of error consistently, they may have a *misconception*; addressing that requires correcting conceptual knowledge. In turn, they need additional questions that target their errors to confirm that they have internalized the feedback.

We have operationalized all of the above into an on-line, adaptive *tutor*, the LTL Tutor. It is hosted for free online:

<https://ltl-tutor.xyz>

To be privacy-protecting, the tutor does not gather any identifying information. Educators can create a “course” and get aggregated data of how students in it are performing. For those who have additional data privacy concerns, the software is also available open source and designed for self-hosting. The tutor also supports both the classical LTL syntax (used in this paper) as well as variants used by recent tools [13,44] that use keywords (e.g., **always**, **after**).

Some tutoring systems are designed to teach a topic from scratch. As educators, we recognize this can be very disruptive because it assumes a specific context, preparation, amount of available time, and so on. Instead, the LTL Tutor is designed to be a *companion* that complements whatever pedagogy is already in use, rather than a substitute. We assume students have a basic grounding in formal logic, and may have heard a lecture or two about LTL in the instructor’s preferred style. What the LTL Tutor does is save the instructor from having to provide *drill*, feedback, and corrections; and it leverages our extensive catalog of LTL difficulty without the instructor needing to learn it in depth themselves. Effectively, the LTL Tutor tries to learn, and then correct, the *latent* conceptual model of LTL that the student has in their mind—however it is obtained.

Finally, we note the increased interest in LTL in industrial settings. It can be especially difficult for industrial practitioners to get assistance the way a student in a course can from instructors and teaching assistants. Thus, the LTL Tutor should be of particular value to practitioners.

This paper describes the design and implementation of the LTL Tutor. After providing a high-level overview (section 3), it especially focuses on two aspects:

- A misconception driven process for generating novel question sets tailored to individual learners (section 4).
- Mechanisms designed to provide learners with insight both into the questions they get wrong and the underlying misconceptions that may be driving these errors (section 5).

## 2 Related Work

Our work is inspired by the seminal work on *concept inventories* [32] from physics education. A concept inventory is a collection of multiple-choice questions where each wrong answer (often called a *distractor*) is not merely wrong, but corresponds to a *specific misconception*. Thus, if students choose a certain distractor, the instructor can be confident about what the student’s confusion is. Our prior work [27,28,29,30] effectively created such an instrument, alongside a catalog of misconceptions, for LTL, which this work leverages to make generative.

To make it generative, we need a way to not only create new problems but also create misconception-based distractors. While mutation testing [2] is appealing here, the mutants created may be trivial, redundant [52], or even functionally identical to the original [41]. We thus draw inspiration from Prasad et al. [49]’s work on “conceptual” mutation to address these problems. Their work (not for LTL!), however, is only partially automated and requires significant expert intervention. A key technical contribution of this work (section 4) is to perform conceptual mutation in a completely automated way.

When learners make conceptual errors, we have to provide high-level feedback (section 5.2). We draw on the literature of conceptual change [48], specifically using *refutation texts*, which has been found effective in many settings [53]. (The SMoL Tutor for programming language semantics [40] also uses these, and is also driven by misconceptions.)

Our tutor is inspired by vanLehn’s two-loop model for tutoring systems [59]. In both, the inner loop provides immediate feedback during a task. There, the outer loop chooses the next task; ours additionally targets misconceptions (fig. 5).

We are also inspired by the work on cognitive tutoring [5,46,55], which capture how an expert would solve a problem and try to get learners to mimic that approach. This approach is too resource-intensive for our lightweight setting, as it requires extensive effort to model and encode expert problem-solving methods.

Finally, we describe existing tutors for formal logic. These systems rely on hand-crafted questions or generate questions without a guiding principle (akin to conventional mutation testing). In contrast, the LTL Tutor stands out by generating novel questions based on an inventory of conceptual errors.

Itlis is a web-based tutor designed to teach learners about the logical foundations of computer science [25,26]. The system has two primary focuses: allowing instructors to easily construct, compose, and pipeline questions and question sets, and the ability to provide students with instant meaningful feedback and explanations for errors. Unlike the LTL Tutor, this means that Itlis modules are designed to be closely tied to specific courses of study (e.g., a modal logic module [18]).

Lodder et al. have developed logic tutors [36,37,38]. Rather than requiring experts to specify the steps of a solution, the tutors automatically generate authoritative step-by-step proofs for instructor-specified problems. Students are given feedback when their proof steps diverge from the generated authoritative proof. However, this work is (a) proof-, not model-theoretic and (b) not for LTL.



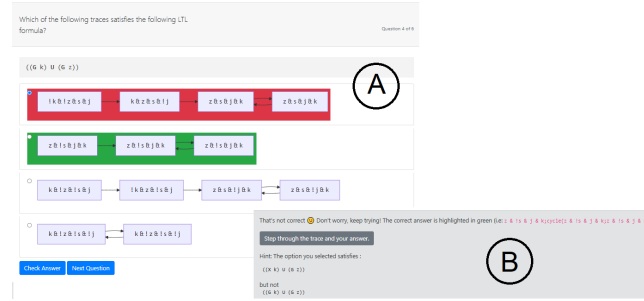


Fig. 2: A Trace Satisfaction (Multiple Choice) (A) question, with (B) feedback about a learner's mistake.

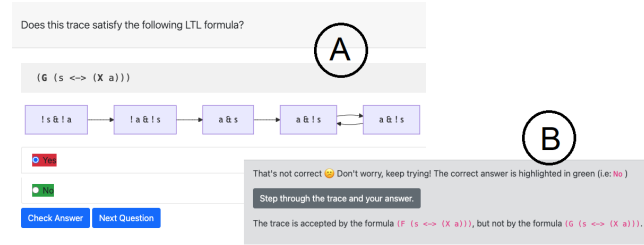


Fig. 3: A Trace Satisfaction (Y/N) (A) question, with (B) feedback about a learner's mistake. Figure 4 shows how the stepper can help shed further light on the mistake.

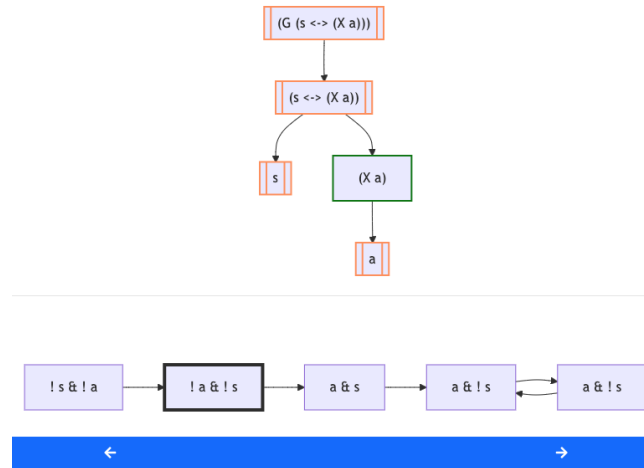


Fig. 4: LTL Stepper: The syntax tree (top) shows how each sub-formula of  $G(s \leftrightarrow (Xa))$  is satisfied (green border) or not satisfied (orange double border) at a given trace step. The trace (bottom) highlights the trace step under study and shows the assignment of truth values to literals at each step.

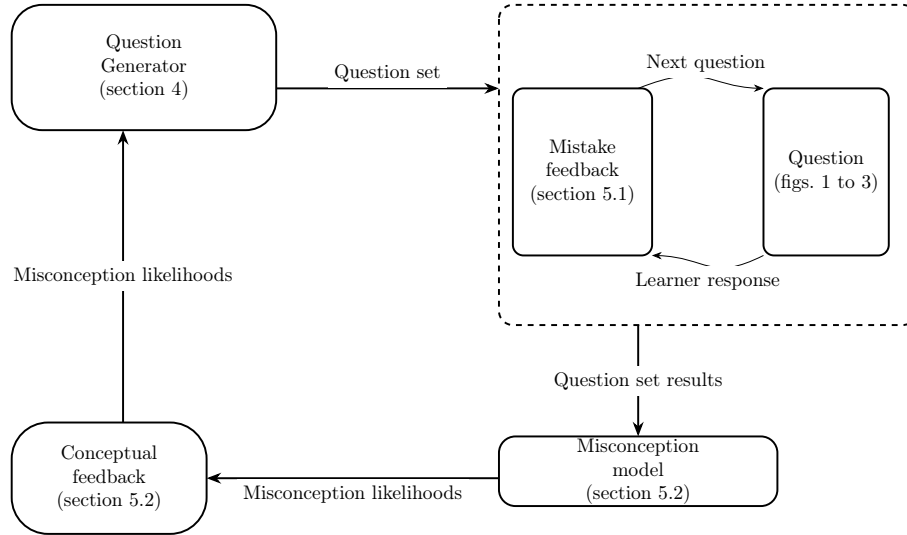


Fig. 5: The overall flow of the LTL Tutor.

## 4 Generating Problems

From the above description, we can see that the heart of the tutor lies in generating good sets of related formulas. Thus, both kinds of questions are generated from a *seed formula*. This formula is randomly generated by the **SPOT randltl** tool [19], with the likelihood of each operator’s occurrence determined by the learner’s predicted likelihood of having a related misconception. For example, if the learner has a high likelihood of the Implicit **G** misconception, the formula generation process will bias the seed formula towards the **G** operator.

For English-to-LTL, the LTL Tutor generates a simple English description of the seed formula, which is used as the question prompt. We describe this translation process in section 7. The seed formula represents the correct answer to the question. It is mutated to create distractors, as we describe below.

For trace satisfaction, the seed formula serves as the question prompt. Traces accepted by the seed formula and its mutants are then used as candidate answers and distractors, respectively. These traces are generated by the **SPOT** tool for  $\omega$ -automata manipulation [19,20].

*Generating Good Mutants* The key to generating useful problems comes down to creating good mutants. Doing so well is a central contribution of the LTL Tutor.

An English-to-LTL question, for example, might be founded in the following seed formula and English sentence pair:

$$\mathbf{G}(e \implies (\mathbf{F}h))$$

*Whenever the engine is on, the headlight will be on then or at some point in the future.*

Table 1: Conceptual mutation rules for the Implicit  $\mathbf{G}$ , Implicit  $\mathbf{F}$ , and Bad State Quantification misconceptions. Arbitrary LTL formulae are represented by  $\alpha$  and  $\beta$ . Conceptual mutation rules for all misconceptions are available in appendix A.

Misconception	Mutation Rules
Implicit $\mathbf{G}$	$\mathbf{G}\alpha \xrightarrow{\text{mutate}} \alpha$
Implicit $\mathbf{F}$	$\mathbf{F}\alpha \xrightarrow{\text{mutate}} \alpha$
Bad State Quantification	$\mathbf{G}\alpha \xrightarrow{\text{mutate}} \mathbf{F}\alpha$
	$\mathbf{F}\alpha \xrightarrow{\text{mutate}} \mathbf{G}\alpha$
	$\alpha\mathbf{U}\beta \xrightarrow{\text{mutate}} (\mathbf{F}\alpha)\mathbf{U}\beta$
	$\alpha\mathbf{U}\beta \xrightarrow{\text{mutate}} \alpha\mathbf{U}(\mathbf{F}\beta)$
	$\alpha\mathbf{U}\beta \xrightarrow{\text{mutate}} (\mathbf{G}\alpha)\mathbf{U}\beta$
	$\alpha\mathbf{U}\beta \xrightarrow{\text{mutate}} \alpha\mathbf{U}(\mathbf{G}\beta)$
	$\alpha\mathbf{U}\beta \xrightarrow{\text{mutate}} (\beta\mathbf{U}\alpha)$

As we have noted, it is natural to mutate this seed formula to create mutants. Because LTL equality is decidable [60], we can easily rule out syntactic variants that are not semantically different. Thus, we could adapt typical syntactic mutation techniques from programming to LTL formulae: for instance, we might randomly change logical operators (eq. (1)), change operands (eq. (2)), or swap operand order (eq. (3)):

$$\mathbf{G}(e \implies (\mathbf{F}h)) \xrightarrow{\text{mutate}} \mathbf{G}(e \wedge (\mathbf{F}h)) \quad (1)$$

$$\mathbf{G}(e \implies (\mathbf{F}h)) \xrightarrow{\text{mutate}} ((\mathbf{G}e) \implies (\mathbf{F}h)) \quad (2)$$

$$\mathbf{G}(e \implies (\mathbf{F}h)) \xrightarrow{\text{mutate}} \mathbf{G}((\mathbf{F}h) \implies e) \quad (3)$$

While these syntactic mutants of the original formula could be used as distractors in a multiple choice question, they are very unlikely to capture the actual difficulties that learners have. For example, since the *and* operator is nowhere in the English sentence to be translated, it is highly unlikely that a learner would pick  $\mathbf{G}(e \wedge (\mathbf{F}h))$  (the mutant in eq. (1)) as an answer. Furthermore, if a learner were to select this option, it is unclear *why* they did so. This limits the kinds of feedback that can be provided to the learner.

Instead, the LTL Tutor uses our well-substantiated catalogs of LTL misconceptions [27,30] to guide the mutation process. This process of *conceptual* mutation is achieved by associating each misconception with mutation rules (a sample of these is given in table 1, while all are available in appendix A). Applying any of these mutation rules to a given LTL formula (or sub-formula) generates a mutant that embodies the corresponding misconception. The seed formula above, for example, could be mutated to explicitly embody multiple misconceptions, with the misconception given as a label:

$$\begin{array}{lll}
G(e \implies (Fh)) & \xrightarrow{\text{mutate}} & e \implies (Fh) & (\text{Implicit } G) \\
G(e \implies (Fh)) & \xrightarrow{\text{mutate}} & G(e \implies h) & (\text{Implicit } F) \\
G(e \implies (Fh)) & \xrightarrow{\text{mutate}} & F(e \implies h) & (\text{Bad State Quantification})
\end{array}$$

Crucially, each distractor is now associated with a known misconception.<sup>3</sup> Not only do these distractors reflect the actual difficulties that learners are known to have, when chosen, they also provide insight into the underlying misconceptions. Thus, for instance, if a learner selects the first conceptual mutant above (as shown in fig. 1), it is likely that they have the Implicit  $G$  misconception. We next discuss how we operationalize this insight.

## 5 Helping Learners Learn

As mentioned in section 1, we draw a meaningful distinction between learner *mistakes* and *misconceptions*. While mistakes can be addressed via feedback in terms of the problem at hand (section 5.1), misconception feedback must be provided in terms of the misunderstood concept (section 5.2).

### 5.1 Addressing Learner Mistakes

The first thing the LTL tutor does is give feedback at the question level. When a learner selects a distractor to an English-to-LTL question (e.g., fig. 1), they are shown a concrete example of why their answer is incorrect and the relationship between their answer and the correct solution. Feedback for trace satisfaction questions involves the formula used to generate the (incorrect) trace alongside the correct formula (fig. 2, fig. 3). If the learner wants further insight, they can walk through the evaluation of the trace they selected over the correct formula trace using an interactive trace stepper. Figure 4 shows how the stepper can help shed light on the learner’s mistake in fig. 3.

### 5.2 Addressing Learner Misconceptions

A single mistake, however, is not enough to identify a pervasive misconception. A learner could have misread the question, mis-clicked, or just had a minor misunderstanding. However, if a learner consistently makes the same mistake, it is likely that they have a misconception.

As described in section 3, the LTL Tutor models the likelihood of a learner having a misconception based on their previous mistakes. This model informs seed formula generation, and thus the likelihood of a learner encountering a question that exercises a particular misconception.

<sup>3</sup> When multiple mutants are *syntactically* equal, we present only one to the learner, but associate all relevant misconceptions with that distractor.



**Globally****Review**

You might believe that the  $G$  (Globally) operator implicitly applies at some point in time, meaning it automatically holds at a specific state. However, the  $G$  operator has a stronger implication: it asserts that the proposition  $G \ x$  must hold true at every future state, starting from the current one and onward. In contrast, simply stating  $x$  only requires that  $x$  be true in the current state, with no guarantee or constraint on future states.

For example, if  $x$  represents 'the system is secure', the statement  $x$  means that the system is secure in the current state, but it doesn't ensure that the system will remain secure in the future. The statement  $G \ x$ , however, asserts that the system is secure in the current state and will remain secure in every future state.

Fig. 6: Feedback for the Implicit  $G$  misconception.

Once the learner has got at least 5 questions incorrect, the tutor also provides them textual feedback about their most highly likely misconceptions. Crucially, this feedback does not refer to a specific question encountered by the learner, but rather the general misconception itself. Using the refutation text format (section 2), this feedback confronts the learner with their misconception and provides a rebuttal to it. For example, the feedback for the Implicit  $G$  misconception (fig. 6) presents a hypothesis of the learner's idea of how the  $G$  operator behaves, explains the correct semantics of the operator, and provides an example to illustrate the difference.

## 6 Instructor Support

Many of our existing users are instructors who employ the LTL Tutor in the context of a course. They would benefit from having feedback on how their students are doing, not only to track progress but also to detect class-wide persistent misconceptions (which may suggest weaknesses in their materials).

Therefore, the LTL Tutor provides instructors the option of creating a notion of a "course". This generates a code that students use when submitting work. Instructors can then use the course instance to track student progress by identifier or class progress via aggregate statistics.

Because we host the LTL Tutor, this can create discomfort or problems for some instructors regarding student privacy. For that reason, the tutor is available as an open-source system [51] with instructions for running local copies [50]. While we appreciate instructors providing us with summary statistics (which help us keep track of both global student understanding of LTL<sup>4</sup> and the tutor's performance), we do not require this.

<sup>4</sup> Because LTL operators are tied to natural language, it is conceivable that different linguistic backgrounds would have different performance and misconceptions.

Table 2: Example patterns used to translate LTL to English.

LTL Pattern	English Translation
$\mathbf{G}(\alpha \implies (\mathbf{F}\beta))$	Whenever $\alpha$ (holds), eventually $\beta$ will (hold)
$\mathbf{G}(\mathbf{F}\alpha)$	There will always be a point in the future where $\alpha$ (holds)
$\mathbf{F}(\mathbf{G}\neg\alpha)$	Eventually, it will never be the case that $\alpha$ (holds)
$\mathbf{X}\mathbf{X}\dots\mathbf{X}\alpha$	In $n$ states, $\alpha$ (will hold)
$\mathbf{G}(\alpha \implies (\mathbf{X}(\beta\mathbf{U}\delta)))$	Whenever $\alpha$ (happens), $\beta$ (will hold) until $\delta$ (holds)

## 7 LTL to English

The creation of English-to-LTL questions requires the translation of a seed formula into English. To do this, the LTL Tutor first attempts to match the LTL formula to a set of common patterns (inspired by Dwyer et al.’s work on patterns in property specifications [21,22]), some of which are described in table 2.

All remaining untranslated sub-formulae are then recursively translated via a description of their logical operators. For instance, a formula like  $\mathbf{G}(p \wedge \mathbf{X}q)$  might be broken into “Globally,  $p$  and  $\mathbf{X}q$ ,” where  $\mathbf{X}q$  is further translated as “in the next state,  $q$ .” This ensures even complex or unconventional formulae receive a systematic, if literal, English description. The demands of the educational context require that English translations never be incorrect. Thus, while stilted or non-idiomatic English could be improved with language models, we are wary of potential “hallucinations” that could lead to incorrect translations.

## 8 Limitations

Given the tutor’s support for English-to-LTL, it is natural to wonder why it does not also support LTL-to-English. This is particularly relevant since the work we build upon [28,29] identified misconceptions in both directions. However, LTL-to-English requires the ability to check English output. Naturally, it may be possible to employ language models for this. However, we have not done this because of our desire for reliability in evaluating output, which seems hard to achieve. Furthermore, in prior work, people demonstrated strong performance in this direction [30], reducing its priority. In addition, language models can significantly drive up computational costs (complicating our hosting) or require use of external paid services (which is difficult at scale).

A natural weakness of the current tutor is that it centers around our existing catalog of misconceptions. Though this has been built up over many years, there may yet be other misconceptions in the wild. One of our goals is to adapt the tutor to be more open to these: e.g., using some of the purely syntactic mutants that we rejected earlier (section 4) to see whether they yield unexpected answers. The reason we have not done this already is that turning these mistakes into misconceptions ideally requires learners to provide textual explanations of their choices, and making the interface for this useful to us while not irritating to them is a challenge.

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## A Conceptual Mutation Rules

This section describes all conceptual mutation rules used by the LTL Tutor, alongside their associated misconceptions. Arbitrary LTL formulae are represented by  $\alpha$ ,  $\beta$ , and  $\delta$ . Arbitrary binary operators are represented by  $\bowtie$ , and arbitrary unary operators are represented by  $\bowtie$ .

Misconception	Mutation Rules	
Implicit $G$	$G\alpha$	$\xrightarrow{\text{mutate}} \alpha$
Implicit $F$	$F\alpha$	$\xrightarrow{\text{mutate}} \alpha$
Bad State	$G\alpha$	$\xrightarrow{\text{mutate}} F\alpha$
Quantification	$F\alpha$	$\xrightarrow{\text{mutate}} G\alpha$
	$\alpha U \beta$	$\xrightarrow{\text{mutate}} (F\alpha)U\beta$
	$\alpha U \beta$	$\xrightarrow{\text{mutate}} \alpha U (F\beta)$
	$\alpha U \beta$	$\xrightarrow{\text{mutate}} (G\alpha)U\beta$
	$\alpha U \beta$	$\xrightarrow{\text{mutate}} \alpha U (G\beta)$
	$\alpha U \beta$	$\xrightarrow{\text{mutate}} (\beta U \alpha)$
Precedence	$\alpha \bowtie (\beta \bowtie \delta)$	$\xrightarrow{\text{mutate}} (\alpha \bowtie \beta) \bowtie \delta$
Exclusive $U$	$\alpha U (\neg \alpha \wedge \beta)$	$\xrightarrow{\text{mutate}} \alpha U \beta$
Weak $U$	$\alpha U \beta$	$\xrightarrow{\text{mutate}} (\alpha U \beta) \wedge F\beta$
Bad State	$\delta U (\alpha \wedge (F\beta))$	$\xrightarrow{\text{mutate}} (\delta U \alpha) \wedge (F\beta)$
Index	$\delta U (\alpha \wedge (G\beta))$	$\xrightarrow{\text{mutate}} (\delta U \alpha) \wedge (G\beta)$
	$\delta U (\alpha \wedge (F\beta))$	$\xrightarrow{\text{mutate}} (\delta U \alpha) \wedge (F\beta)$
	$\delta U (\alpha \vee (G\beta))$	$\xrightarrow{\text{mutate}} (\delta U \alpha) \vee (G\beta)$
	$\delta U (\alpha \implies (F\beta))$	$\xrightarrow{\text{mutate}} (\delta U \alpha) \implies (F\beta)$
	$\delta U (\alpha \implies (G\beta))$	$\xrightarrow{\text{mutate}} (\delta U \alpha) \implies (G\beta)$
	$X(\alpha \wedge \beta)$	$\xrightarrow{\text{mutate}} (X\alpha) \wedge \beta$
	$X(X(X \dots X\alpha))$	$\xrightarrow{\text{mutate}} X\alpha$
Other Implicit	$(\neg \alpha U \alpha)$	$\xrightarrow{\text{mutate}} F\alpha$
	$(\alpha U (G\alpha))$	$\xrightarrow{\text{mutate}} \alpha \wedge (F(G\alpha))$
	$(F\alpha) \wedge (G(\beta \implies (X(G\beta))))$	$\xrightarrow{\text{mutate}} \alpha \wedge (X(G\beta))$
	$X\alpha$	$\xrightarrow{\text{mutate}} F\alpha$
	$\alpha \bowtie \beta$	$\xrightarrow{\text{mutate}} \alpha$
	$\alpha \bowtie \beta$	$\xrightarrow{\text{mutate}} \beta$
	$\neg \alpha$	$\xrightarrow{\text{mutate}} \alpha$