Little Tricky Logic:
Misconceptions in the Understanding of LTL

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LTL = Linear Temporal Logic
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For systems that change over time
**LTL** = Linear **T**emporal **L**ogic

For systems that change over time

- Expressive
- Supports good decision procedures
- Small
- .... and easy to learn?
RQ. In what ways is LTL tricky, and what can we do about it?
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2 years of studies with researchers and students
4 survey rounds
Quiz Time!
Quiz Format:

one question, possible answers, **you decide** yes/no

- **✓** Possible Answer 1
- **✗** Possible Answer 2
- **✓** Possible Answer 3

LTL Operators:

- always (G)
- eventually (F)
- after (X)
- until (U)
Part 1:
Formulas vs. Traces
Q. Do the traces below satisfy this formula?
{eventually Red} and {eventually Green}
Q. Do the traces below satisfy this formula?

\{\text{eventually Red} \} \text{ and } \{\text{eventually Green} \}
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Q. Do the traces below satisfy this formula?
\{\text{eventually Red}\} \text{ and } \{\text{eventually Green}\}

\text{Not satisfied, because Green comes before Red}

Bad Prop misconception
Q. Do the traces below satisfy this formula?

Red
Q. Do the traces below satisfy this formula?

Red

\[ \text{Red} \]

\[ \begin{array}{c}
\text{R} \\
\text{G} \\
\text{B}
\end{array} \quad \begin{array}{c}
\text{R} \\
\text{G} \\
\text{B}
\end{array} \quad \begin{array}{c}
\text{R} \\
\text{G} \\
\text{B}
\end{array} \quad \begin{array}{c}
\text{R} \\
\text{G} \\
\text{B}
\end{array} \quad \begin{array}{c}
\text{R} \\
\text{G} \\
\text{B}
\end{array} \]

1 2 3 4 5+
Q. Do the traces below satisfy this formula?

Red

![Diagram with arrows indicating sequence of RGB values from 1 to 5+]

×
Q. Do the traces below satisfy this formula?

Red

X

1 2 3 4 5+
Q. Do the traces below satisfy this formula?

Red

X

Г
Г
Г
Г
Г
1 2 3 4 5+

Г
Г
Г
Г
Г
1 2 3 4 5+

✓
Q. Do the traces below satisfy this formula?

**Red**

![Diagram showing traces with Red, Green, and Blue data points]

The traces on the left do not satisfy the formula, while the traces on the right do.
Q. Do the traces below satisfy this formula?

**Red**

![Diagram showing traces](Image)

1. **X**
   - Red, Green, Blue
   - 1 → 2 → 3 → 4 → 5+

2. **✓**
   - Red, Green, Blue
   - 1 → 2 → 3 → 4 → 5+

3. **X**
   - Red, Green, Blue
   - 1 → 2 → 3 → 4 → 5+
Q. Do the traces below satisfy this formula?

Red
Q. Do the traces below satisfy this formula?

Red

Satisfied because Red is on at some point

Implicit F misconception
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]

×

R
G
B

1 → R
G
B
2 → R
G
B
3 → R
G
B
4 → R
G
B
5+ → R
G
B
Q. Do the traces below satisfy this formula?

\textbf{Red until Blue} \quad \boxed{\text{[ strong until ]}}

\[ \begin{array}{c}
\text{R} \quad \text{G} \quad \text{B} \\
1 \quad 2 \quad 3 \quad 4 \quad 5+
\end{array} \]

\[ \begin{array}{c}
\text{R} \quad \text{G} \quad \text{B} \\
1 \quad 2 \quad 3 \quad 4 \quad 5+
\end{array} \]

\[ \begin{array}{c}
\text{R} \quad \text{G} \quad \text{B} \\
1 \quad 2 \quad 3 \quad 4 \quad 5+
\end{array} \]

\[ \begin{array}{c}
\text{R} \quad \text{G} \quad \text{B} \\
1 \quad 2 \quad 3 \quad 4 \quad 5+
\end{array} \]
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]

\[ \text{Red until Blue} \]

\[
\begin{array}{c c c c c}
1 & 2 & 3 & 4 & 5+ \\
\text{R} & \text{G} & \text{B} & \text{R} & \text{G} \\
\text{G} & \text{B} & \text{R} & \text{G} & \text{B} \\
\text{G} & \text{B} & \text{R} & \text{G} & \text{B} \\
\text{G} & \text{B} & \text{R} & \text{G} & \text{B} \\
\text{G} & \text{B} & \text{R} & \text{G} & \text{B} \\
\end{array}
\]
Q. Do the traces below satisfy this formula?

**Red until Blue**

[ strong until ]
Q. Do the traces below satisfy this formula?

Red until Blue

[ strong until ]
Q. Do the traces below satisfy this formula?

**Red until Blue**

[ strong until ]

1 2 3 4 5+
Q. Do the traces below satisfy this formula? 

**Red until Blue**

[ strong until ]

**X**

Satisfied because Blue may stay off

*Even among researchers!*

Weak U misconception
Part 2:
LTL to English
Q. Translate to English

{Red until Blue} and {always Red}
Q. Translate to English

{Red until Blue} and {always Red}

"Red is always on"
Q. Translate to English

{Red until Blue} and {always Red}

✗ "Red is always on"
Q. Translate to English

{Red until Blue} and {always Red}

"Red is always on"

"Red is always on and Blue is eventually on"
Q. Translate to English

\{\text{Red until Blue}\} \text{ and } \{\text{always Red}\}

- "Red is always on"

- "Red is always on and Blue is eventually on"
Q. Translate to English

{Red until Blue} and {always Red}

× "Red is always on"

✓ "Red is always on and Blue is eventually on"

"This statement can never be satisfied"
Q. Translate to English

\{Red until Blue\} and \{always Red\}

× "Red is always on"

✓ "Red is always on and Blue is eventually on"

× "This statement can never be satisfied"
Q. Translate to English

{Red until Blue} and {always Red}

"This statement can never be satisfied"
Q. Translate to English

{Red until Blue} and {always Red}

When Blue turns on, Red **must** be off

Exclusive U  misconception

✗ "This statement can never be satisfied"
Q. Translate to English

{eventually Red} implies {always Blue}
Q. Translate to English

{eventually Red} implies {always Blue}

"if Red is ever on, then Blue is always on"
Q. Translate to English

{eventually Red} implies {always Blue}

✓ "if Red is ever on, then Blue is always on"
Q. Translate to English

{eventually Red} implies {always Blue}

✓ "if Red is ever on, then Blue is always on"

"Red is on at some point, after which Blue is on"
Q. Translate to English

\{\text{eventually Red}\} \implies \{\text{always Blue}\}

✓ "if Red is ever on, then Blue is always on"

✗ "Red is on at some point, after which Blue is on"
Q. Translate to English

{eventually Red} implies {always Blue}

"Red is on at some point, after which Blue is on"
Q. Translate to English

{eventually Red} implies {always Blue}

Red **will** turn on (also, a precedence issue)

**Bad Prop** misconception

"Red is on at some point, after which Blue is on"
Part 3:
English to LTL
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Impossible!
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Impossible!
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

× Impossible!

{eventually Red} and {always {Red => always !Red}}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Impossible!

'{eventually Red} and {always {Red => always !Red}}'}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Impossible!

{eventually Red} and {always {Red => after {always !Red}}}

{eventually Red} and {always {Red => after {always !Red}}}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

❌ Impossible!

❌ \{\text{eventually Red}\} \land \{\text{always } \{\text{Red} \Rightarrow \text{always } \neg \text{Red}\}\}\}

✔ \{\text{eventually Red}\} \land \{\text{always } \{\text{Red} \Rightarrow \text{after } \{\text{always } \neg \text{Red}\}\}\}\}
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

\[\text{x} \{\text{eventually Red}\} \text{ and } \{\text{always } \{\text{Red } \Rightarrow \text{ always } !\text{Red}\}\}\]
Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

An implication constrains the next state

Bad State Index misconception

✗ \{\text{eventually Red}\} \text{ and } \{\text{always } \{\text{Red }\Rightarrow \text{ always } \neg\text{Red}\}\}$
All Done!
Simple formulas, yet subtle issues and expert blind spots
Quiz Q's Based on 3 Instruments

- Trace Satisfaction
- LTL to English
- English to LTL

Example Question: Is the formula always (Engine or Light) satisfied by this trace?

Example Question: Is the formula (Red) until (Blue) satisfied by this trace?

Example Answer: Yes, because either the engine (smoke) or the headlight is on in each state.

Does the example make sense to you?*
- Yes
- No (please explain)

Q: Is the formula satisfied by this trace?
- Yes
- No

[cs.brown.edu/~bgreenma/ltl-instruments.pdf]
Code Book for Analysis

Bad Prop

Implicit F

Bad State Index

Implicit G

Bad State Quantification

Other Implicit

Exclusive U

Weak U

Coding Rubric in paper, past versions in artifact
Software: Quizius

Class-sourcing to discover misconceptions

1. Answer Top Q's

Question
The above sentence should describe a set of traces over the variables x1, x2, etc. Encode it formally in LTL. Please ignore superficial mistakes like typos, and do not use external tools like Spin to help you.

2. Submit New Q's

Write a Question
We are asking you to describe, in English, a set of traces that are interesting, tricky or surprising to encode in LTL. Please try to give an English description that is no bigger than it needs to be in
What Next?
What Next?

1. Teach Better

our instruments can help!
What Next?

1. Teach Better

our instruments can help!

... but learners are everywhere not just in classrooms
What Next?

1. Teach Better

our instruments can help!
What Next?

1. Teach Better
   our instruments can help!

2. Build Tools
   guard against misconceptions
What Next?

1. Teach Better
   our instruments can help!

2. Build Tools
   guard against misconceptions

3. Design Logics
   Alloy 6  Electrum
   our findings have inspired changes
Thank You!

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