Misconceptions in Finite-Trace and Infinite-Trace Linear Temporal Logic (**LTL**)

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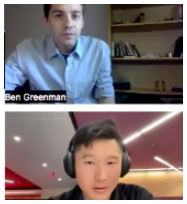
RQ. **In what ways** are temporal logics difficult to use?

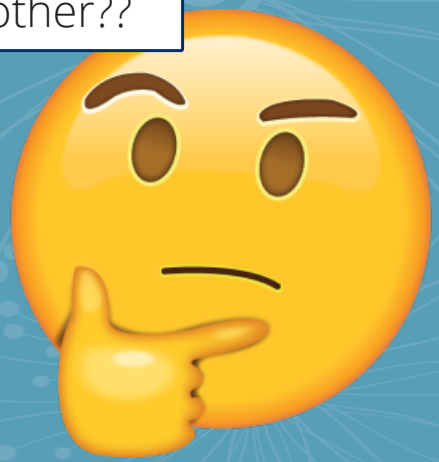
LTL **LTLf**

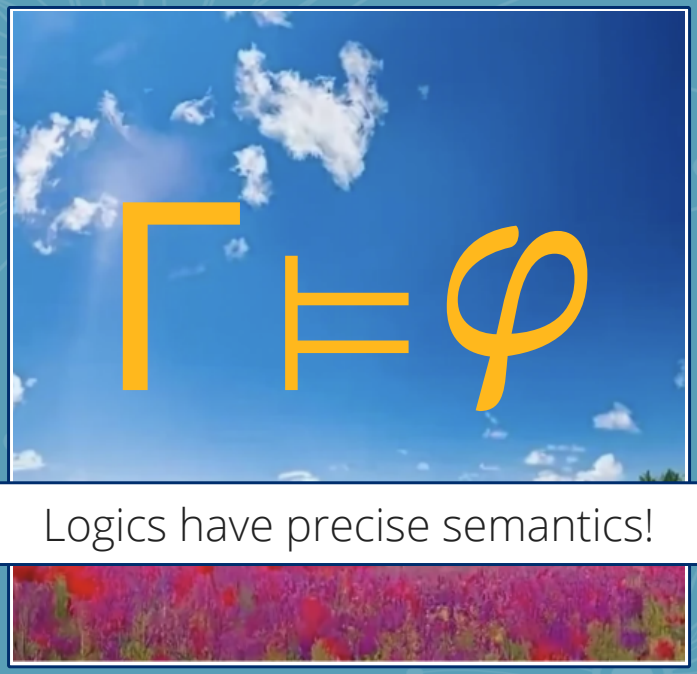
+3 years of studies



qualtrics

Why bother??





Logics have precise semantics!

Why bother??

Quiz time!

Question

Format:

1 question,
1-3 possible answers,
you decide yes/no

✓ ✗

✓ Possible Answer 1

✗ Possible Answer 2

✓ Possible Answer 3

Q. Do the traces below satisfy this LTL formula?

`F(Red) and F(Green)`

[**F** = eventually]

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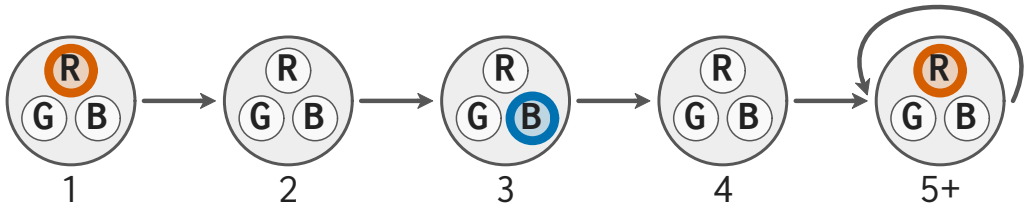

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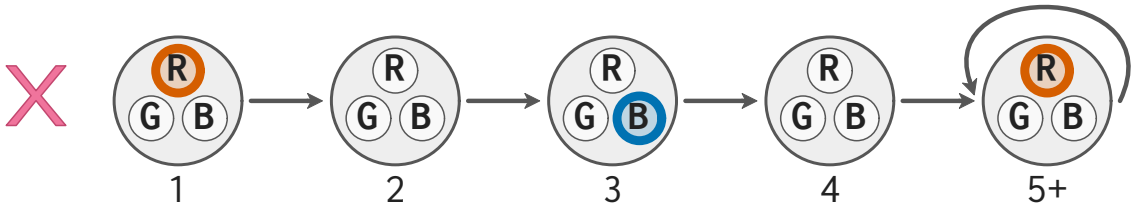

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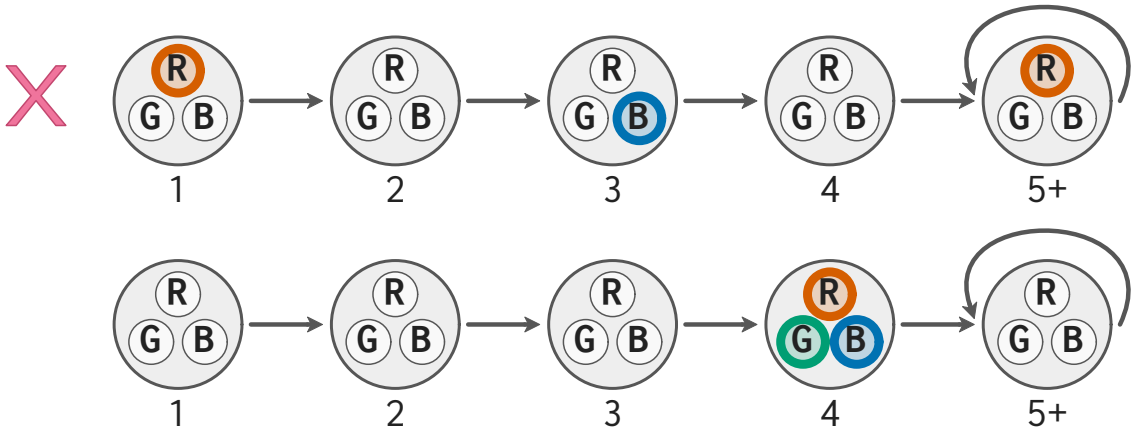

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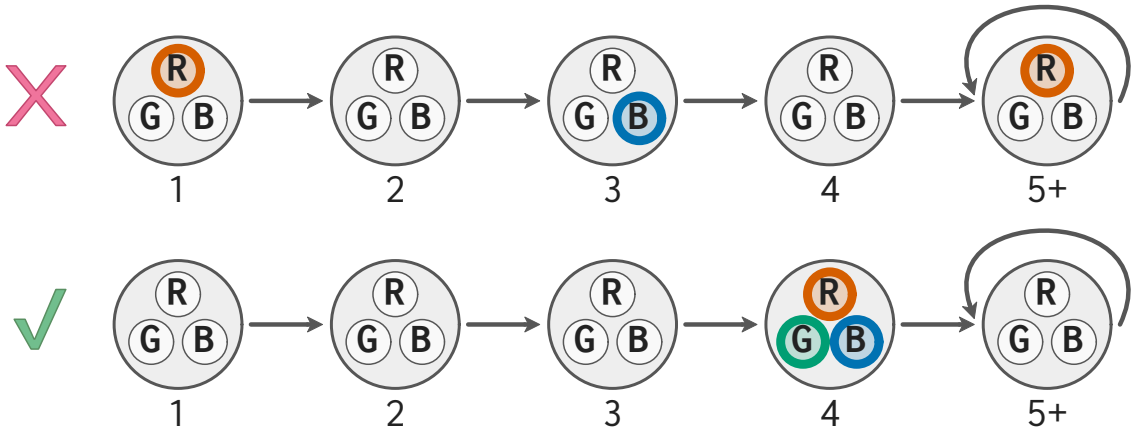

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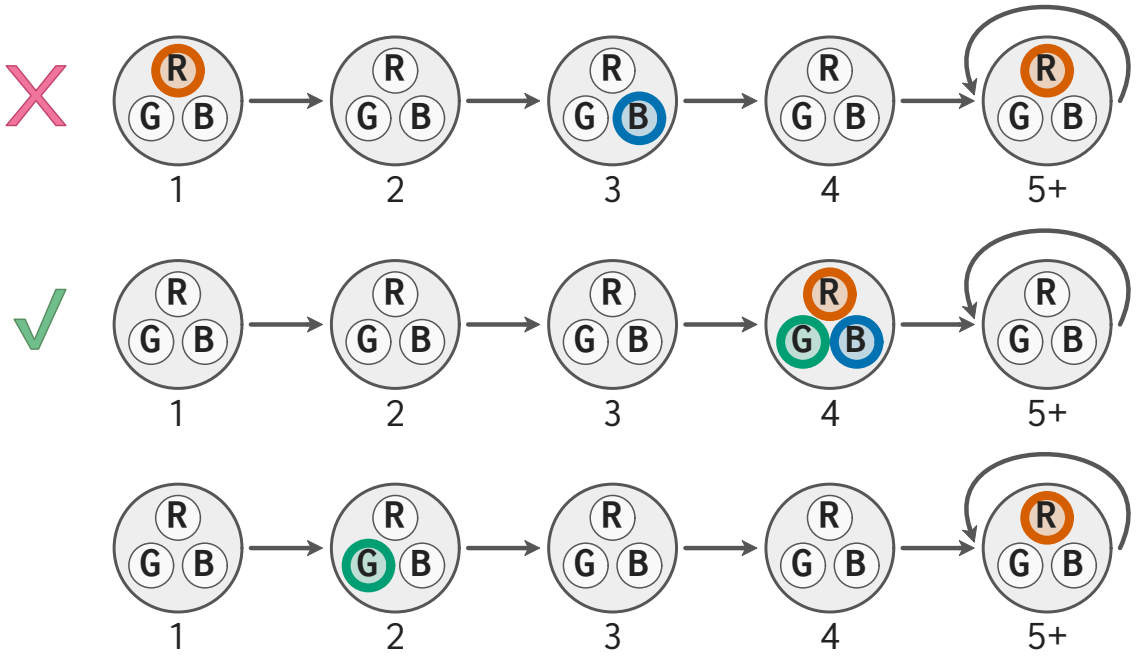

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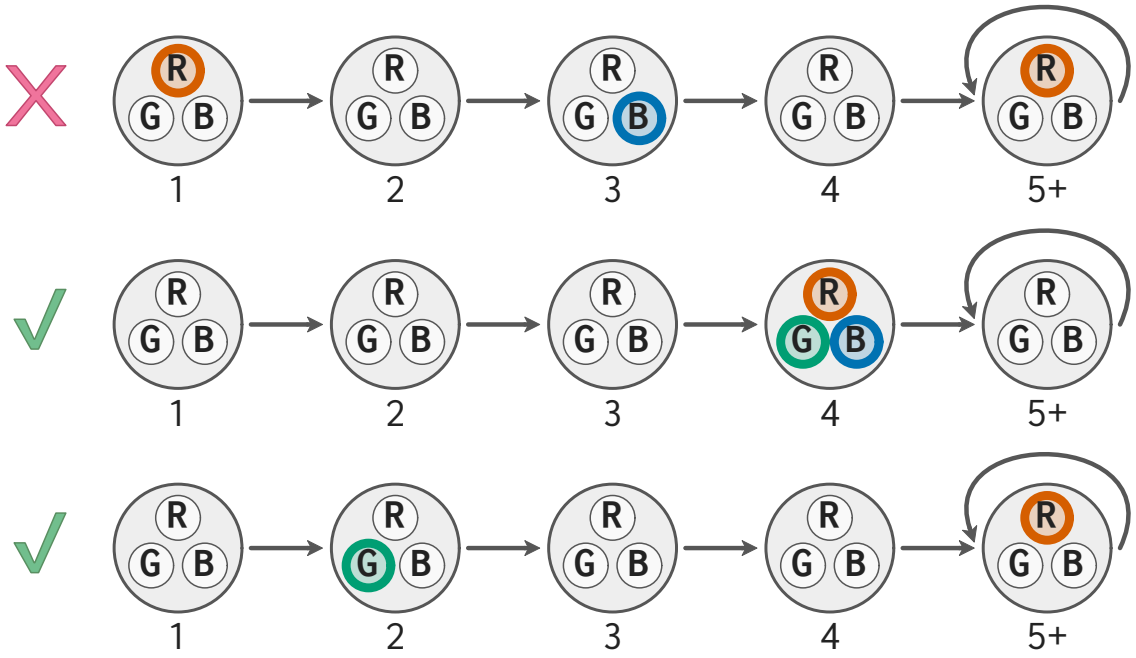

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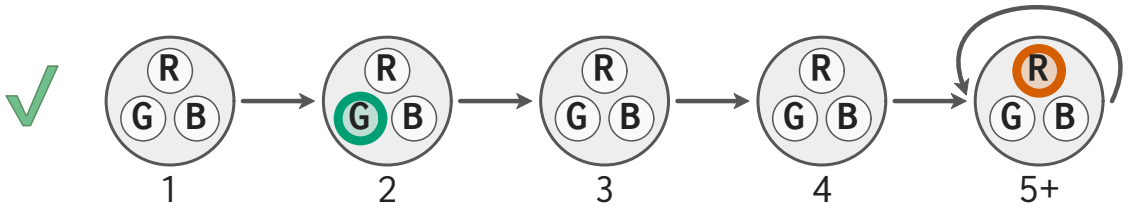
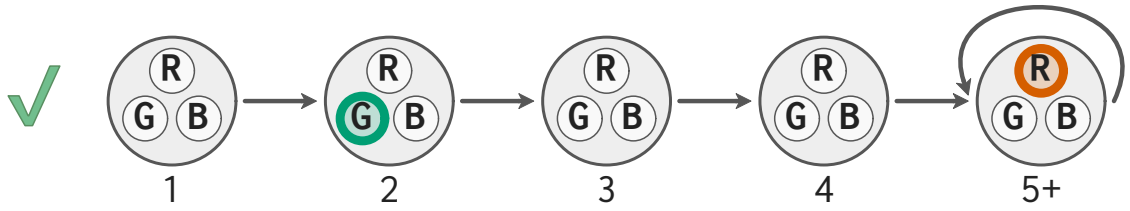


Q. Do the traces below satisfy this LTL formula?

`F(Red) and F(Green)` [**F** = eventually]

Not satisfied, because Green comes before Red

`Bad Prop` misconception



Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

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!X(X(X(Green)))
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```
!F(Green & X(Green) & X(X(Green)))
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✗ `!X(X(X(Green)))`

✓ `!F(Green & X(Green) & X(X(Green)))`

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✓ `!F(Green & X(Green) & X(X(Green)))`

`G(Green & X(Green) ==> X(X(!Green)))`

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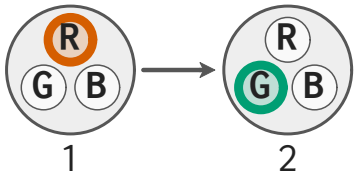
Issue 1: **`X(X(X(-)))`** constrains 1 state

Issue 2: property must hold for all states

`Spreading X, Implicit G` misconception

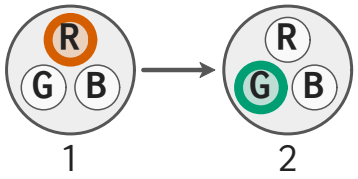
How about LTLf?

LTLf



Finite traces

LTLf

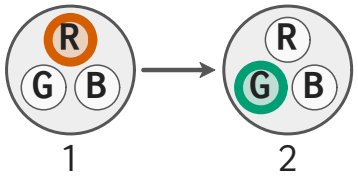


Finite traces

X ~ next state, which **must** exist

Xw ~ next state, if it exists

LTLf



Finite traces

X ~ next state, which **must** exist

Xw ~ next state, if it exists

G ~ always

F ~ eventually

(within trace bounds)

Q. Do the traces below satisfy this LTLf formula?

`G(Red) & Xw(Xw(!Red))`

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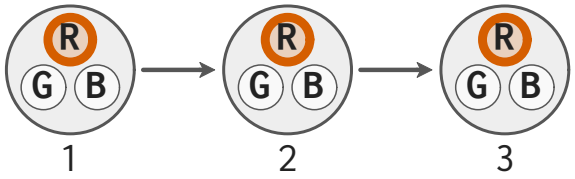

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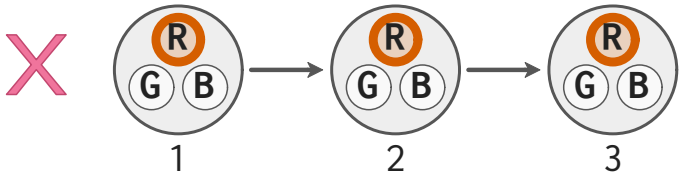

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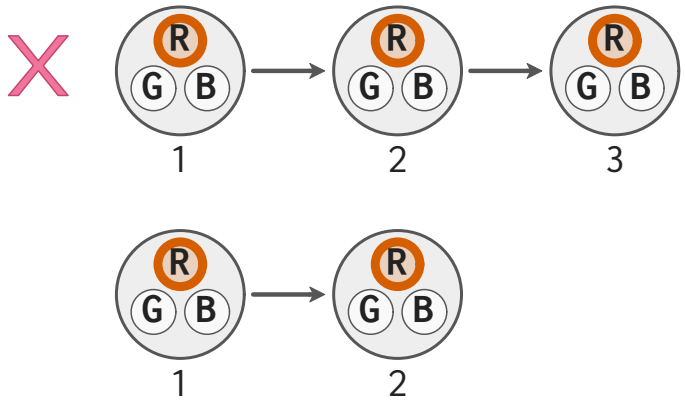

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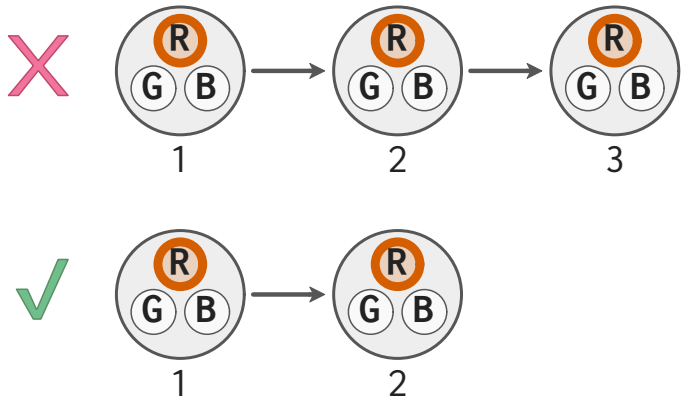

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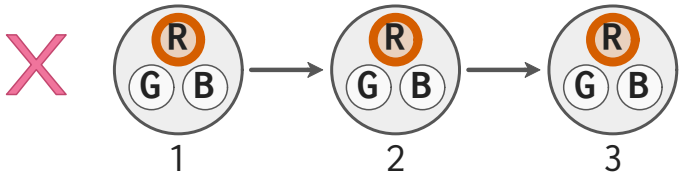

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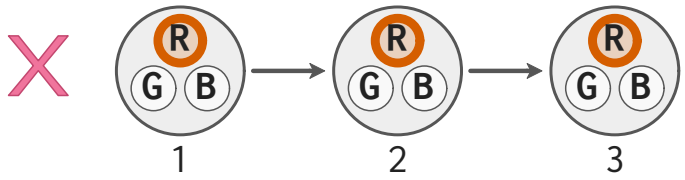

Formula accepts no traces of length > 2

`Length` misconception

Q. Translate to LTLf

Blue is on in at least two states

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```
F(Blue & X(F(Blue)))
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Works for LTL too!

Q. Are these equations correct? [for all terms **e**]

LTL LTLf

```
!X(e)   == X(!e)
```

Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
`!X(e) == X(!e)`	✓	

Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
`!X(e) == X(!e)`	✓	✗

Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
`!X(e) == X(!e)`	✓	✗
`F(e) == e || X(F(e))`		

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`!X(e) == X(!e)`	✓	✗
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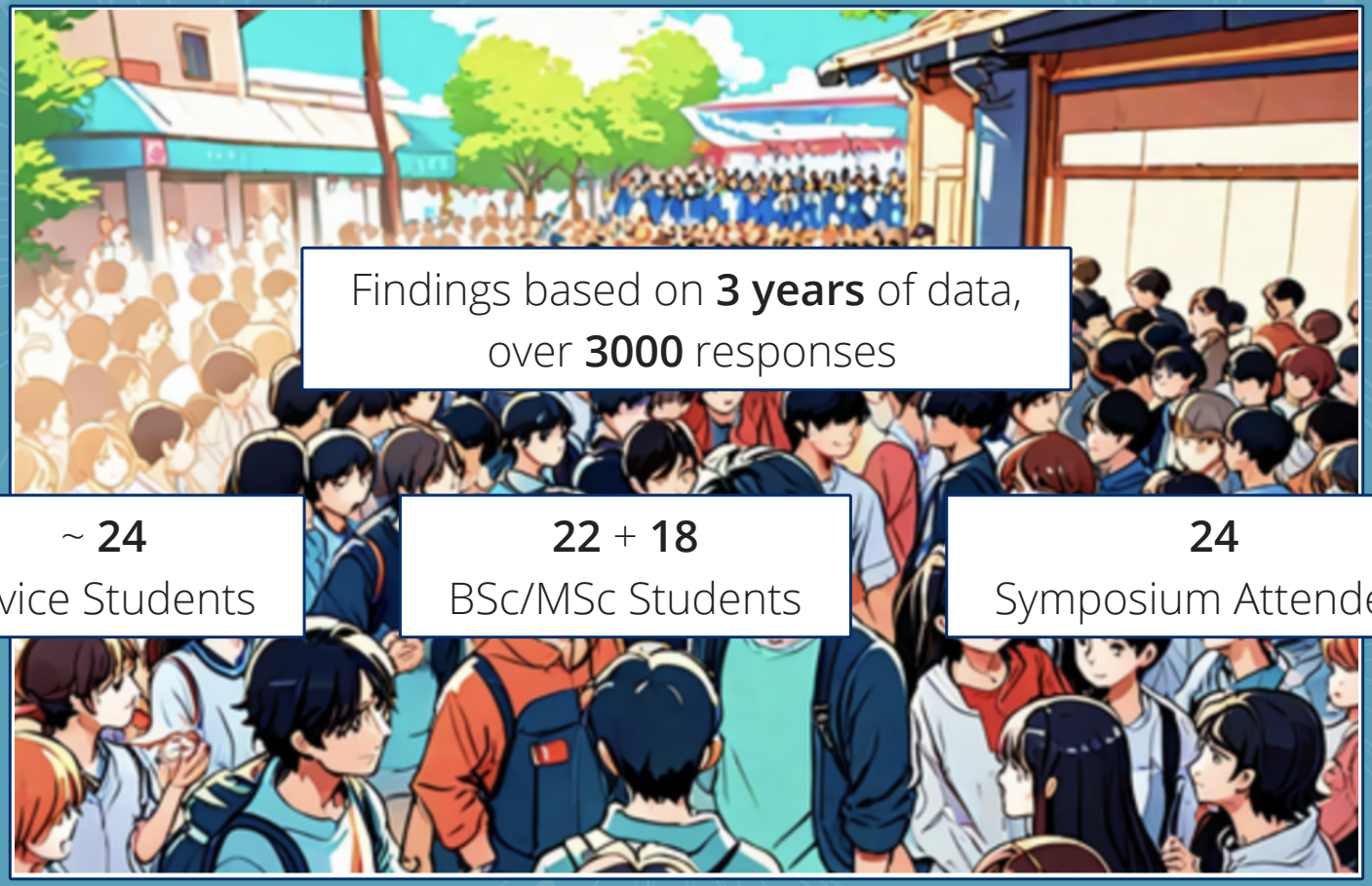
All Done!

"One cannot proceed from the informal to the formal by purely formal means"

Perlis

Misconceptions get in the way!


Findings based on **3 years** of data,
over **3000** responses



Findings based on **3 years** of data,
over **3000** responses

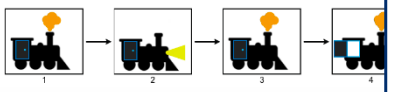
~ **24**
Novice Students

22 + **18**
BSc/MSc Students

24
Symposium Attendees

3 Survey Instruments

Example Question: Is the formula
`always ( Engine or Light )`
satisfied by this trace?



Example Answer: Yes, because either the engi
headlight is on in each state.

Does the example make sense to you?*

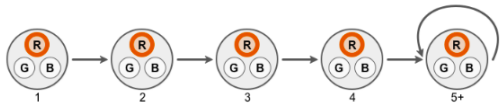
- Yes
- No (please explain)

Example Question:
`G(X(Red))`

Example Answer:

- *LTL description*: The Red light is on in every st
  state.
- *LTLf description*: Every state must be followed
  Red on. No finite traces satisfy the formula.

satisfied by this trace?*

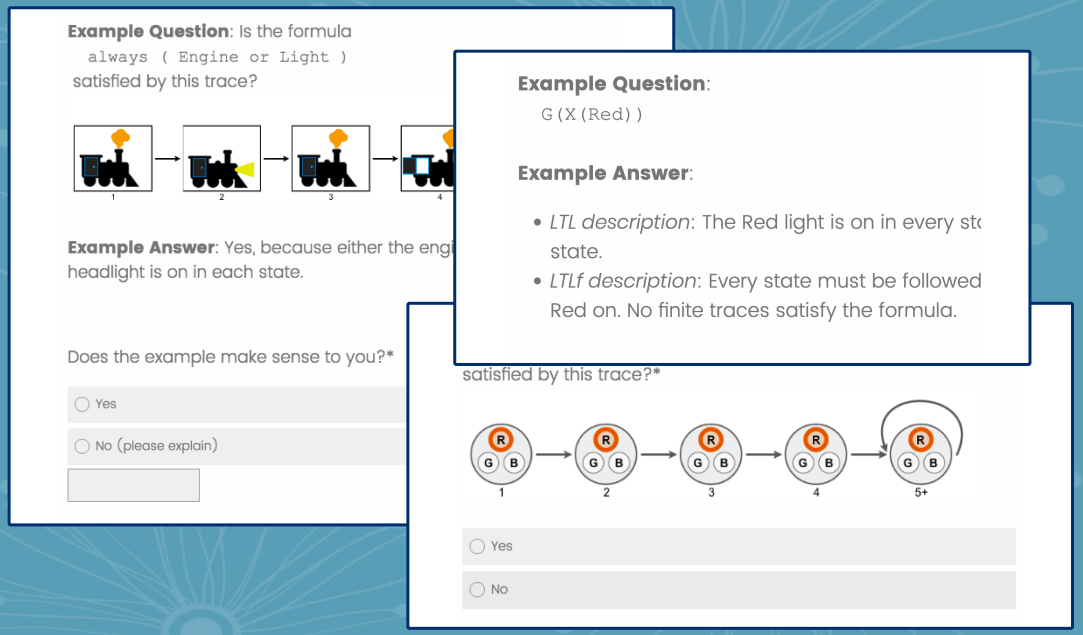


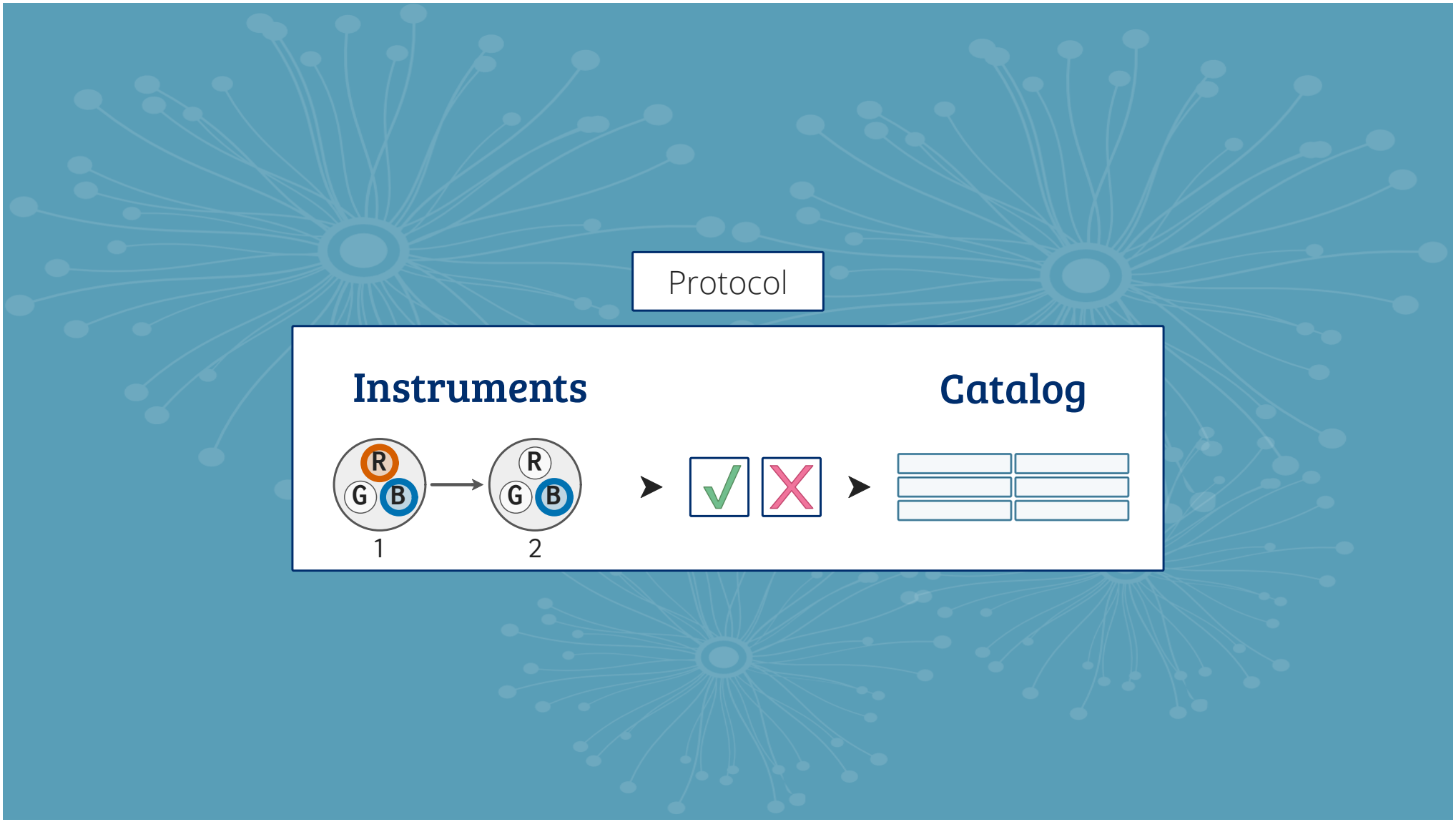
- Yes
- No

3 Survey Instruments

Q. Formats:

- LTL --> English
- English --> LTL
- Trace Matching
- Explain Mismatches
- Check Equations

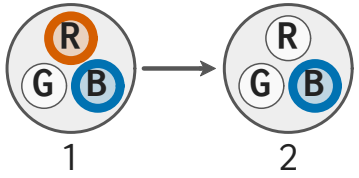



Protocol
Instruments
Catalog
R
G
B
1
R
G
B
2

Lightweight to Reuse

Protocol

Instruments



Catalog

Catalog

14 categories of LTL and LTLf Errors

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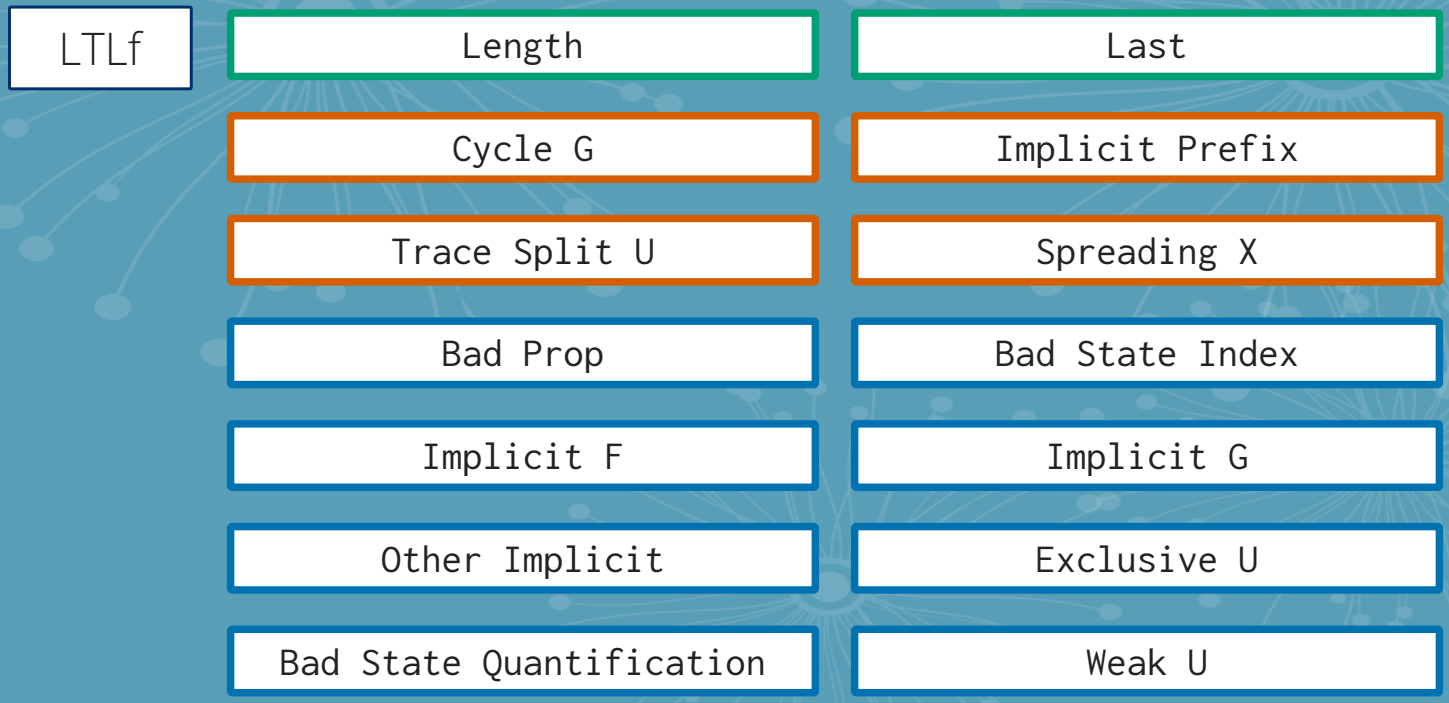
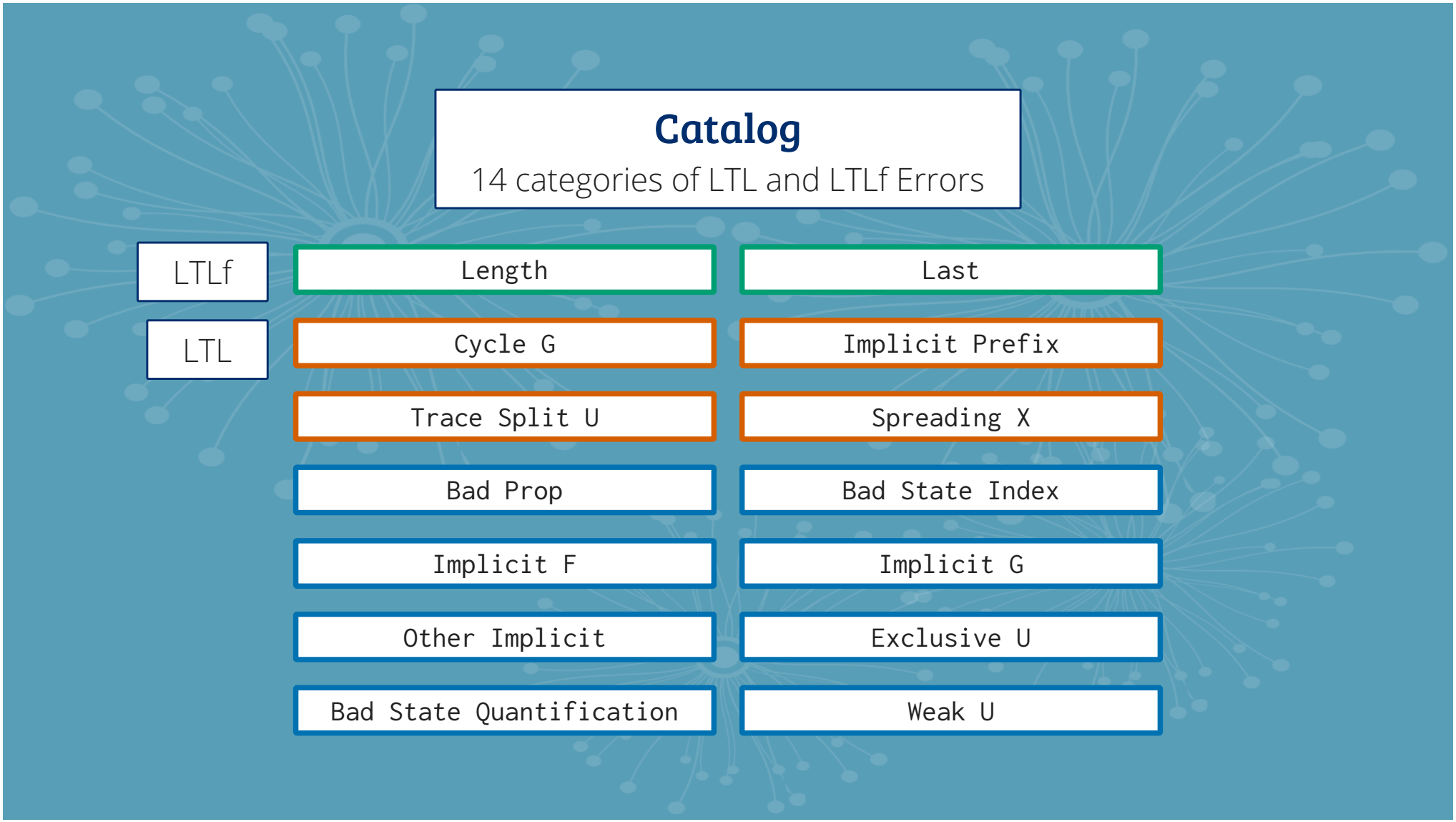
14 categories of LTL and LTLf Errors

- Length
- Last
- Cycle G
- Implicit Prefix
- Trace Split U
- Spreading X
- Bad Prop
- Bad State Index
- Implicit F
- Implicit G
- Other Implicit
- Exclusive U
- Bad State Quantification
- Weak U

Catalog

14 categories of LTL and LTLf Errors

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14 categories of LTL and LTLf Errors
LTLf
LTL
Length
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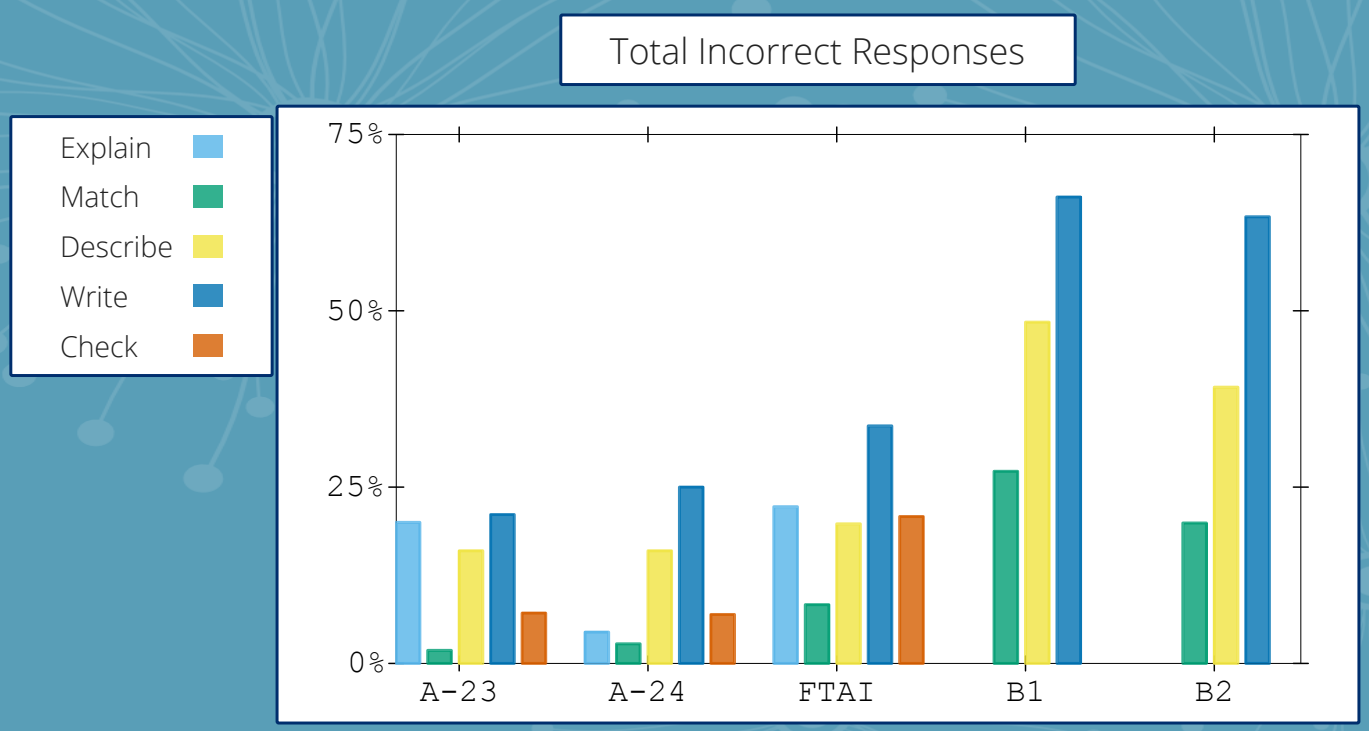
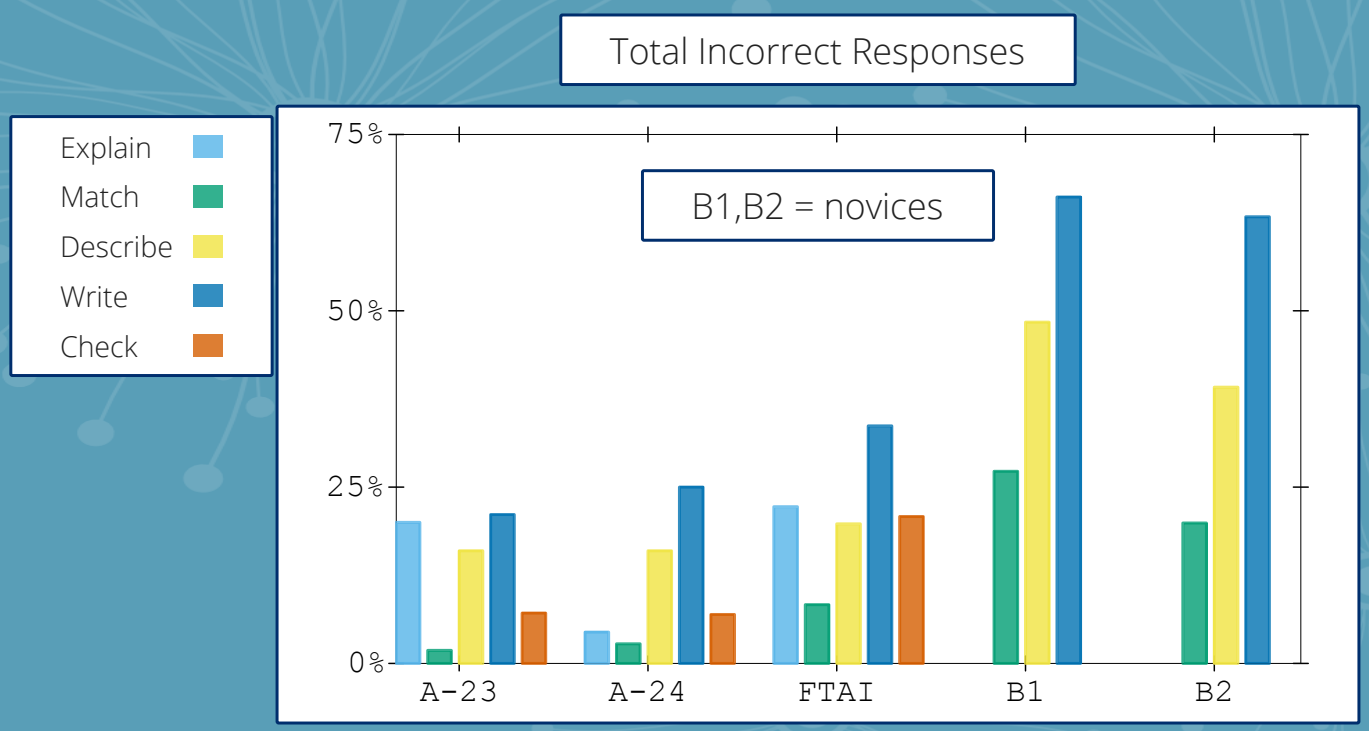

Total Incorrect Responses
Explain
Match
Describe
Write
Check
75%
50%
25%
0%
A-23
A-24
FTAI
B1
B2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B1,B2 = novices
A-23
A-24
FTAI
B1
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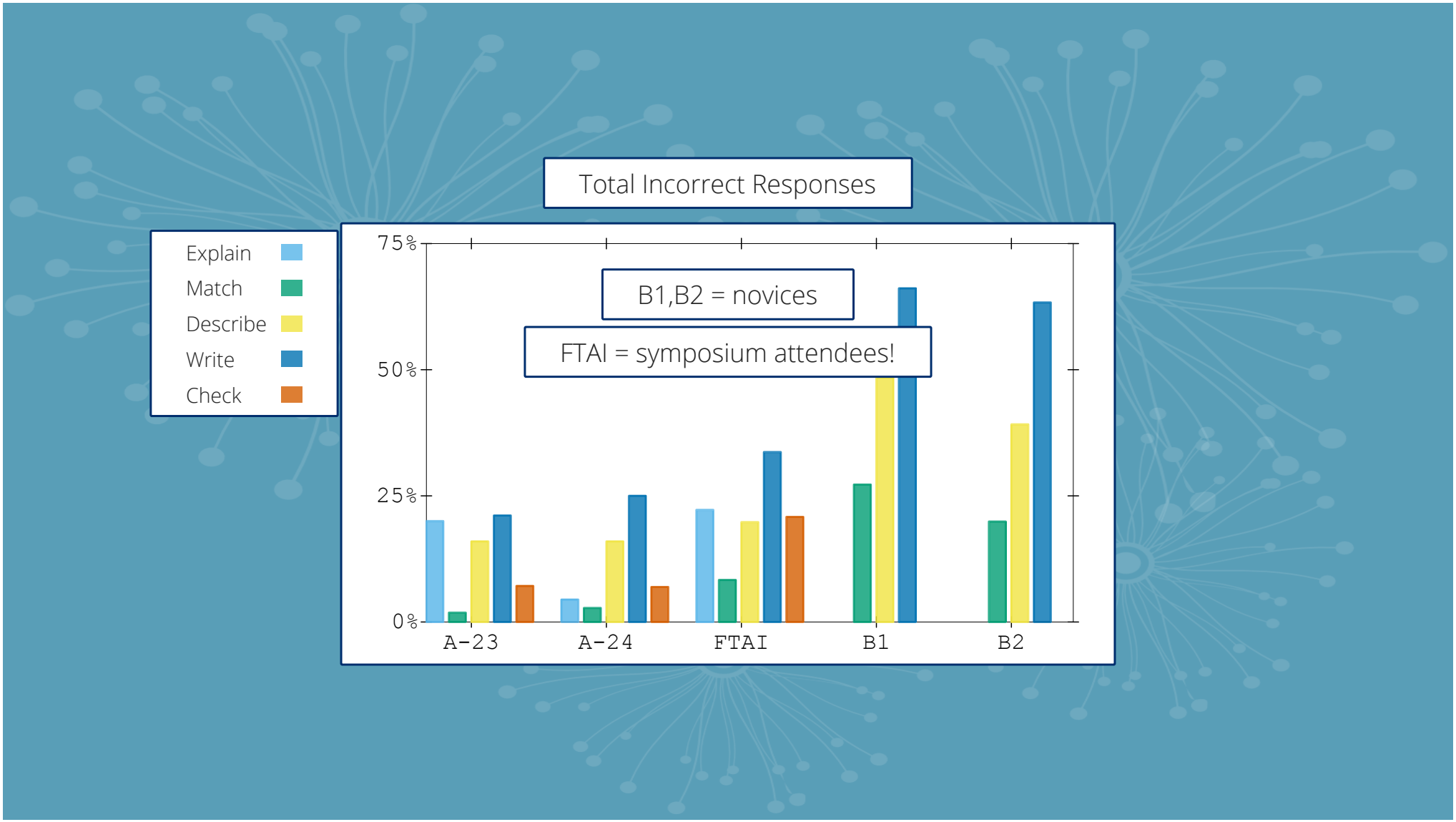
Total Incorrect Responses
Explain
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Write
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75%
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25%
0%
B1,B2 = novices
FTAI = symposium attendees!
A-23
A-24
FTAI
B1
B2

Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length					
Last					
Bad Prop					
Bad State Index					
Bad State Quantification					
Cycle G					
Implicit F					
Implicit G					
Implicit Prefix					
Other Implicit					
Weak U					
Exclusive U					
Trace-Split U					
Spreading X					
Precedence					
Reasonable Variant					
Unlabeled					

Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length	**17**	7	8		
Last	1	5	–		
Bad Prop	–	8	6		
Bad State Index	2	9	5		
Bad State Quantification	–	1	2		
Cycle G	–	2	2		
Implicit F	**10**	**13**	7		
Implicit G	5	8	2		
Implicit Prefix	8	4	8		
Other Implicit	1	–	3		
Weak U	2	2	3		
Exclusive U	–	3	3		
Trace-Split U	–	–	3		
Spreading X	–	–	–		
Precedence	–	1	1		
Reasonable Variant	2	–	–		
Unlabeled	**13**	2	**13**		

Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length	**17**	7	8		
Last	1	5	–		
Bad Prop	–	8	6		
Bad State Index	2	9	5		
Bad State Quantification	–	1	2		
Cycle G	–	2	2		
Implicit F	**10**	**13**	7		
Implicit G	5	8	2		
Implicit Prefix	8	4	8		
Other Implicit	1	–	3		
Weak U	2	2	3		
Exclusive U	–	3	3		
Trace-Split U	–	–	3		
Spreading X	–	–	–		
Precedence	–	1	1		
Reasonable Variant	2	–	–		
Unlabeled	**13**	2	**13**		

Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length	**17**	7	8	–	–
Last	1	5	–	–	–
Bad Prop	–	8	6	**28**	**29**
Bad State Index	2	9	5	**19**	**25**
Bad State Quantification	–	1	2	**16**	**16**
Cycle G	–	2	2	–	–
Implicit F	**10**	**13**	7	**12**	**16**
Implicit G	5	8	2	**37**	**31**
Implicit Prefix	8	4	8	–	8
Other Implicit	1	–	3	–	5
Weak U	2	2	3	**15**	**11**
Exclusive U	–	3	3	8	9
Trace-Split U	–	–	3	–	2
Spreading X	–	–	–	**17**	6
Precedence	–	1	1	2	7
Reasonable Variant	2	–	–	–	–
Unlabeled	**13**	2	**13**	**15**	**35**

Contributions

- LTLf + LTLf Instruments
- Catalog of Errors
- Data from 4 populations
- **Specific insights** into how LTL + LTLf can be tricky

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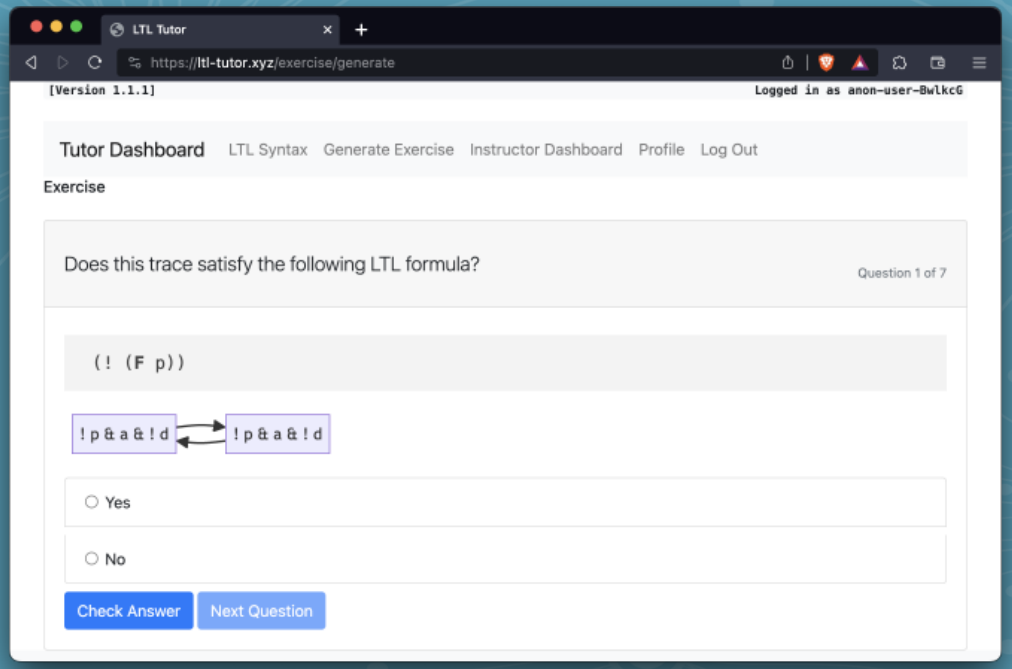
==> Better teaching, tools, and logics

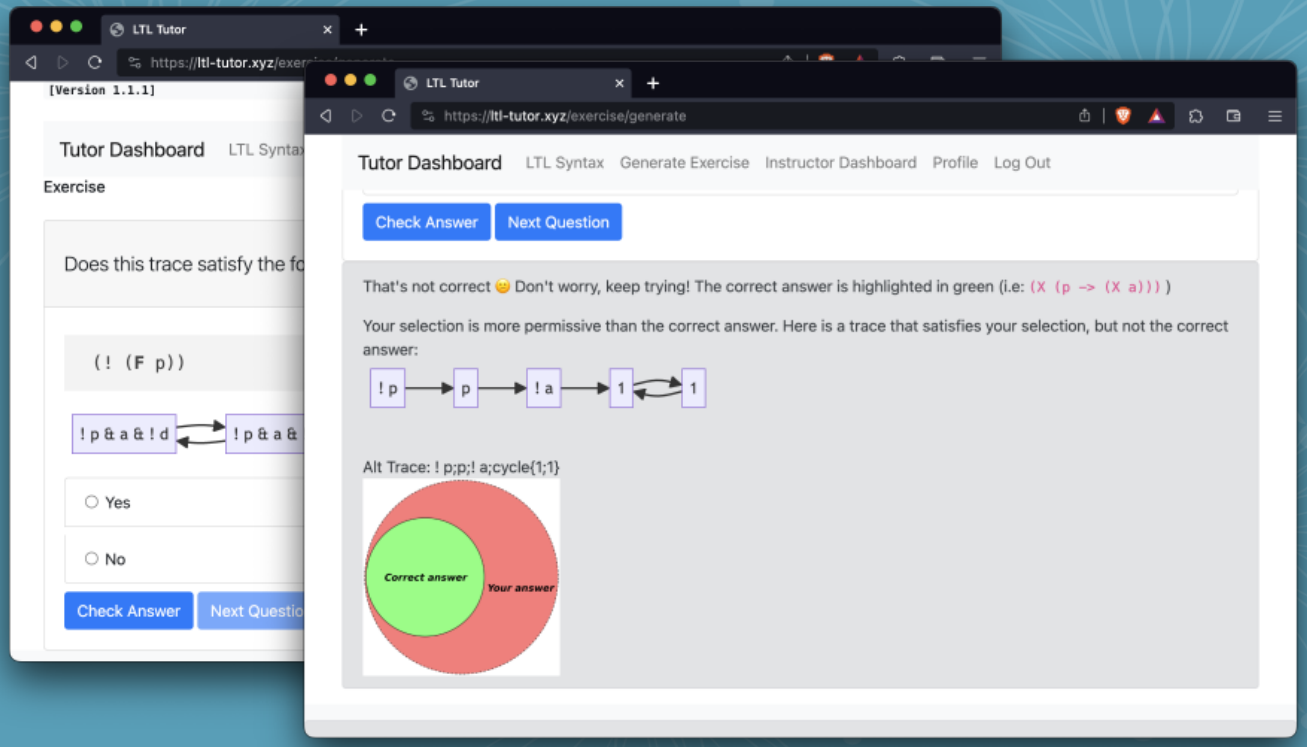
*** Bonus**

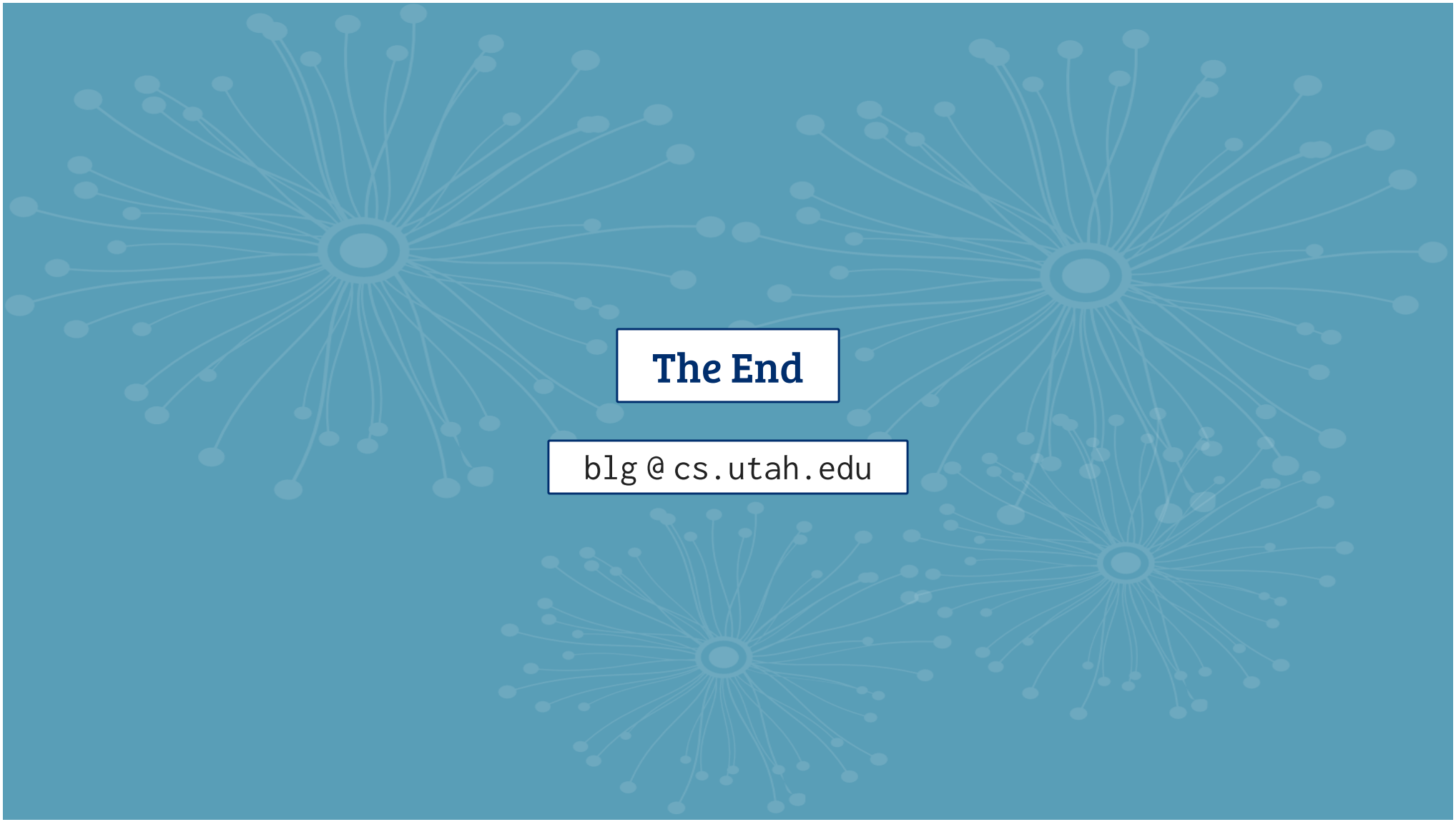
`https://ltl-tutor.xyz`



* Bonus

https://ltl-tutor.xyz




The End
blg@cs.utah.edu

Q. Translate to LTL

Green is on in the final state

Q. Translate to LTL

Green is on in the final state

`F(G(green))`

Q. Translate to LTL

Green is on in the final state

`F(G(green))`

Q. Translate to LTL

Green is on in the final state

`F(G(green))`

`inexpressible`

Q. Translate to LTL

Green is on in the final state

✗ `F(G(green))`

✓ `inexpressible`

Q. Translate to LTL

Green is on in the final state

`F(G(green))`

Q. Translate to LTL

Green is on in the final state

✗ `F(G(green))`

`Last` misconception

Q. Translate to LTL

Blue is on / off / on … in state 0 / 1 / 2 …

Q. Translate to LTL

Blue is on / off / on … in state 0 / 1 / 2 …

```
G(blue & X(!blue))
```

Q. Translate to LTL

Blue is on / off / on … in state 0 / 1 / 2 …

```
G(blue & X(!blue))
```

Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...

```
G(blue & X(!blue))
```

```
G(blue ==> X(!blue) & X(X(blue))) & blue
```

Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...

✗ `G(blue & X(!blue))`

✓ `G(blue ==> X(!blue) & X(X(blue))) & blue`

Q. Translate to LTL

Blue is on / off / on … in state 0 / 1 / 2 …

```
G(blue & X(!blue))
```

Q. Translate to LTL

Blue is on / off / on … in state 0 / 1 / 2 …

✗ `G(blue & X(!blue))`

`Cycle G` misconception

Q. Translate to LTL

Red is on exactly once

Q. Translate to LTL

Red is on exactly once

```
F(red & X(G(!red)))
```

Q. Translate to LTL

Red is on exactly once

```
F(red & X(G(!red)))
```

Q. Translate to LTL

Red is on exactly once

 `F(red & X(G(!red)))`

`!red U (red & X(G(!red)))`

Q. Translate to LTL

Red is on exactly once

✗ `F(red & X(G(!red)))`

✓ `!red U (red & X(G(!red)))`

Q. Translate to LTL

Red is on exactly once

```
F(red & X(G(!red)))
```

Q. Translate to LTL

Red is on exactly once

✗ `F(red & X(G(!red)))`

`Implicit Prefix` misconception

Q. Translate to LTL

Blue is on in at least two states

Q. Translate to LTL

Blue is on in at least two states

```
F(blue) U F(blue)
```

Q. Translate to LTL

Blue is on in at least two states

`F(blue) U F(blue)`

Q. Translate to LTL

Blue is on in at least two states

```
F(blue) U F(blue)
```

```
!blue U (blue & F(X(blue)))
```

Q. Translate to LTL

Blue is on in at least two states

✗ `F(blue) U F(blue)`

✓ `!blue U (blue & F(X(blue)))`

Q. Translate to LTL

Blue is on in at least two states

`F(blue) U F(blue)`

Q. Translate to LTL

Blue is on in at least two states

`F(blue) U F(blue)`

`Trace Split U` misconception