

Misconceptions in Finite-Trace and Infinite-Trace Linear Temporal Logic (**LTL**)



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Giuseppe De Giacomo Shriram Krishnamurthi Marco Montali Tim Nelson Milda Zizyte

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RQ. **In what ways** are temporal logics difficult to use?

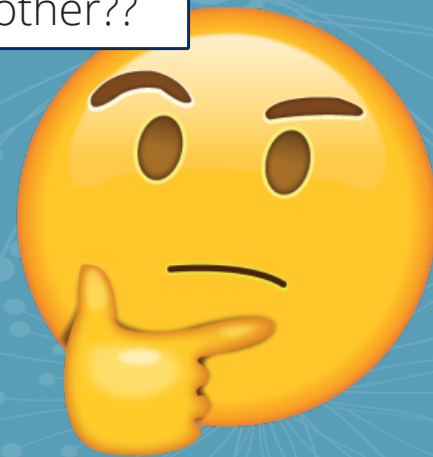
LTL

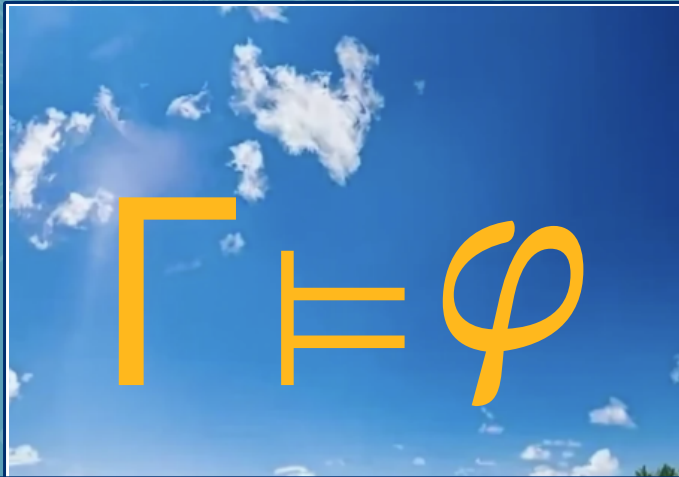
LTLf

+3 years of studies



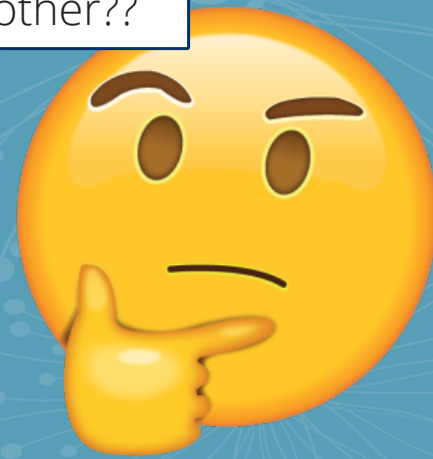
Why bother??





Logics have precise semantics!

Why bother??





Quiz time!



Format:

1 question,
1-3 possible answers,
you decide yes/no



Question

✓ Possible Answer 1

✗ Possible Answer 2

✓ Possible Answer 3



Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

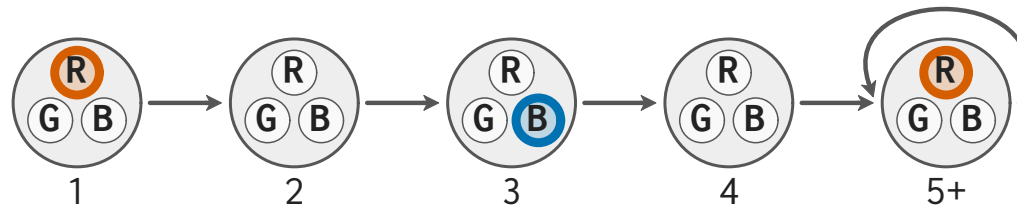
[**F** = eventually]



Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

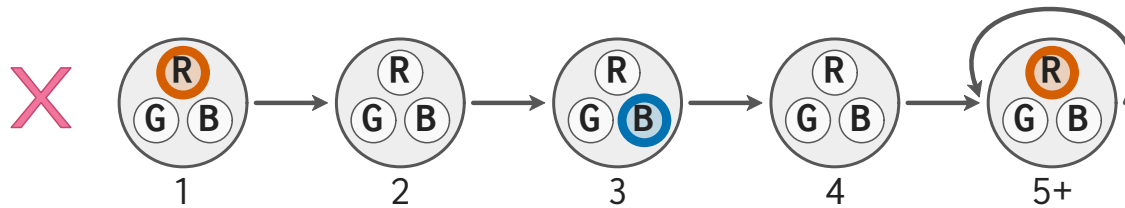
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Q. Do the traces below satisfy this LTL formula?

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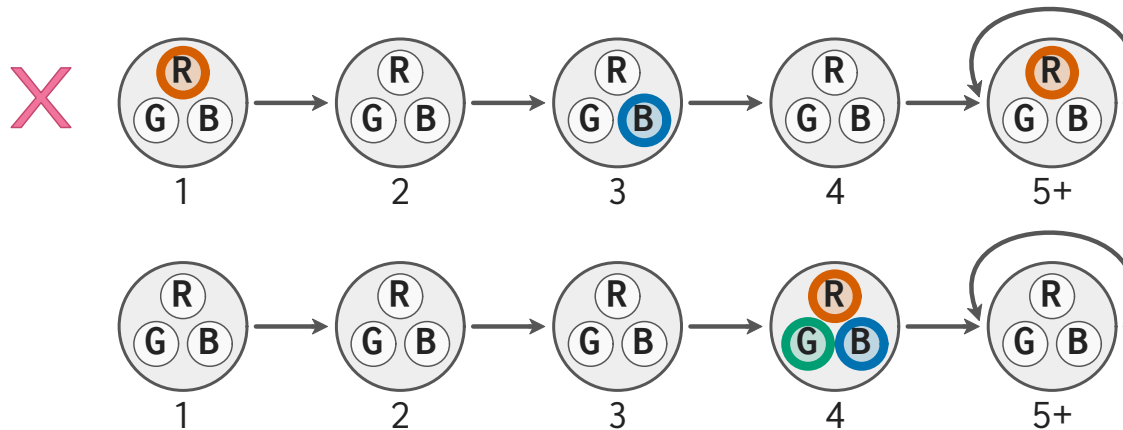
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Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

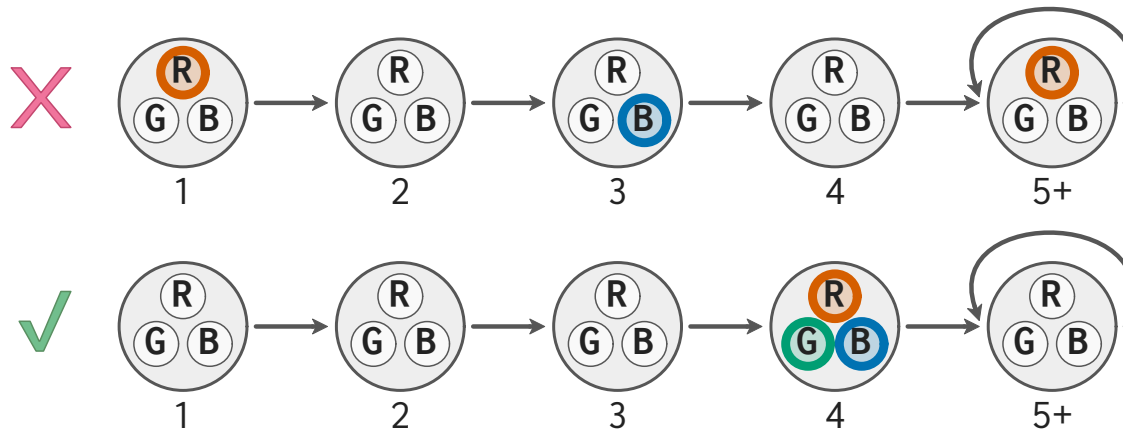
[**F** = eventually]



Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

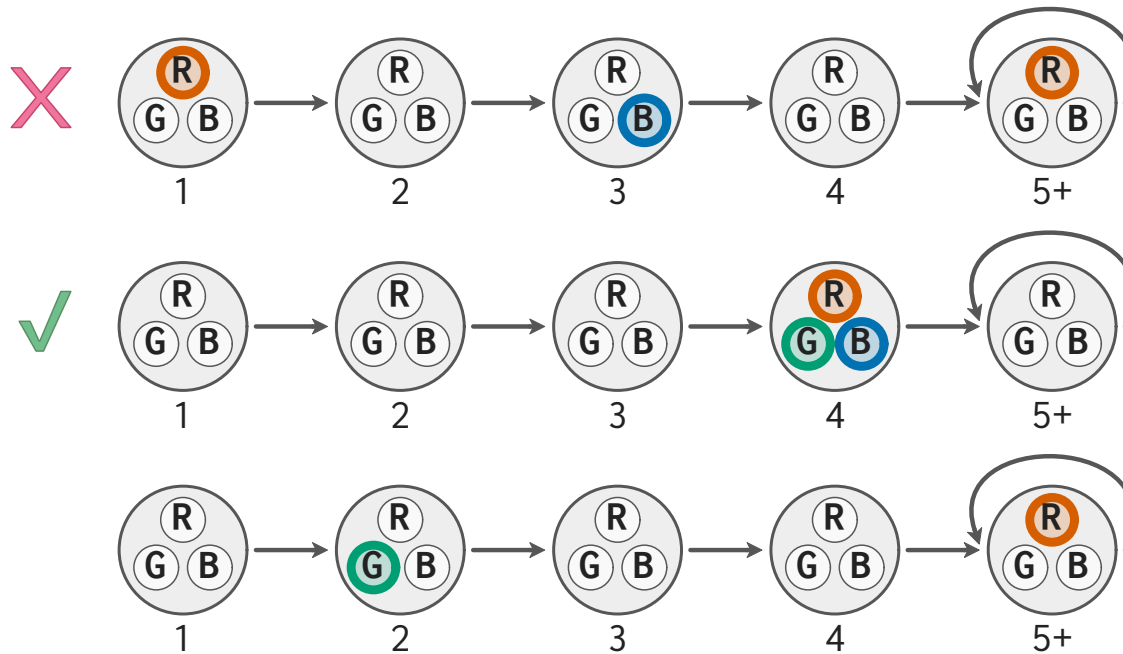
[**F** = eventually]



Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

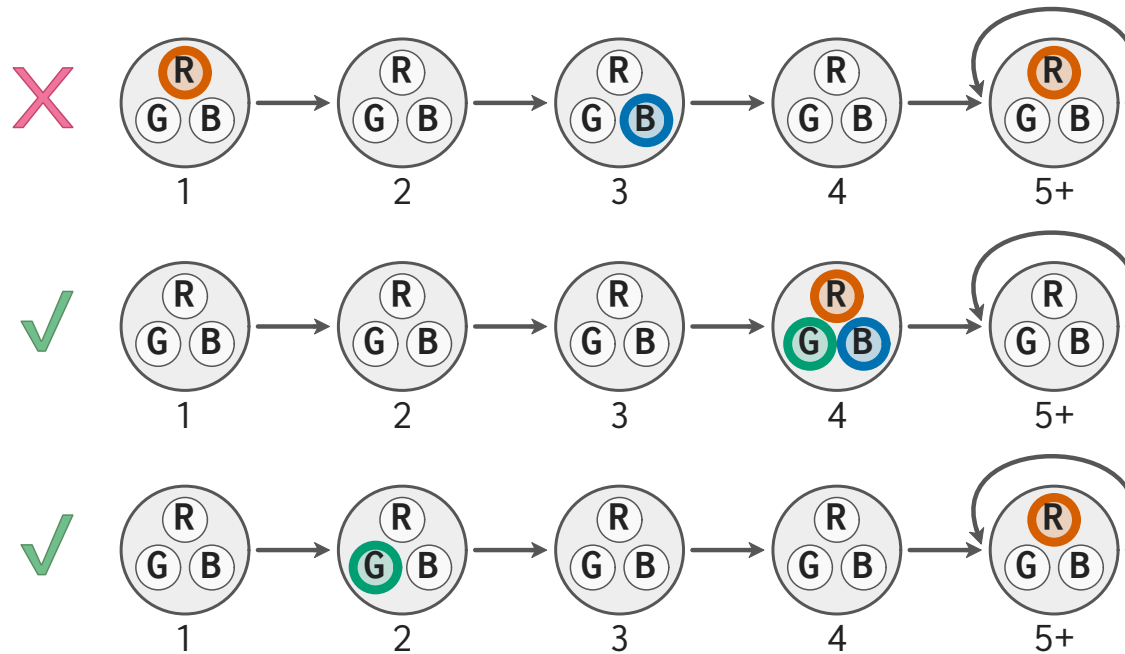
[**F** = eventually]



Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

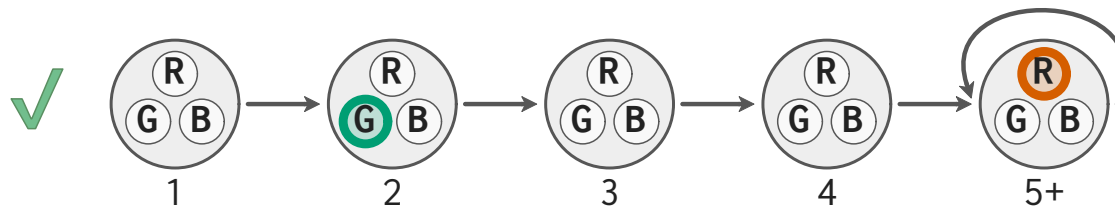
[**F** = eventually]



Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

[**F** = eventually]

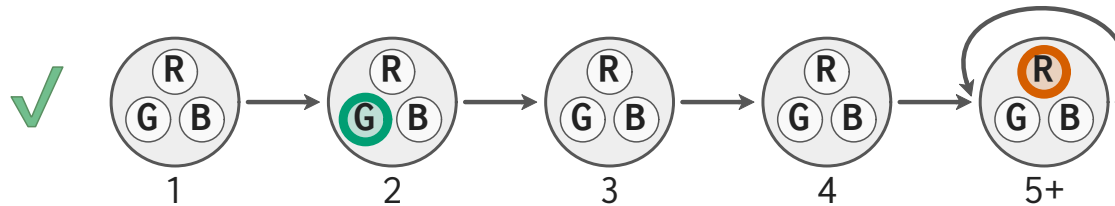


Q. Do the traces below satisfy this LTL formula?

F(Red) and F(Green)

[**F** = eventually]

Not satisfied, because Green comes before Red
Bad Prop misconception





Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]



Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

$\neg X(X(X(\text{Green})))$

Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

X !X(X(X(Green)))

Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

X !X(X(Green))

!F(Green & X(Green) & X(X(Green)))

Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

X $\neg X(X(X(\text{Green})))$

✓ $\neg F(\text{Green} \ \& \ X(\text{Green}) \ \& \ X(X(\text{Green})))$

Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

X $\neg X(X(X(\text{Green})))$

✓ $\neg F(\text{Green} \ \& \ X(\text{Green}) \ \& \ X(X(\text{Green})))$

$G(\text{Green} \ \& \ X(\text{Green}) \ ==> \ X(X(\neg \text{Green})))$

Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

X !X(X(X(Green)))

✓ !F(Green & X(Green) & X(X(Green)))

✓ G(Green & X(Green) ==> X(X(!Green)))

Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

X !X(X(X(Green)))

Q. Translate to LTL

Green cannot stay on for 3 states in a row

[**X** = next state]

X !X(X(X(Green)))

Issue 1: **X(X(X(-)))** constrains 1 state

Issue 2: property must hold for all states

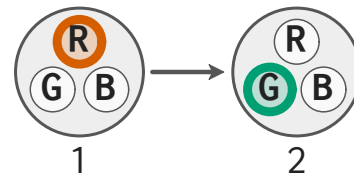
Spreading X, Implicit G misconception



How about LTLf?

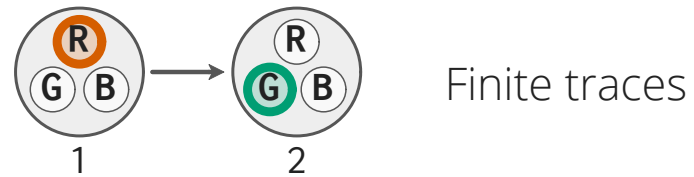


LTLf



Finite traces

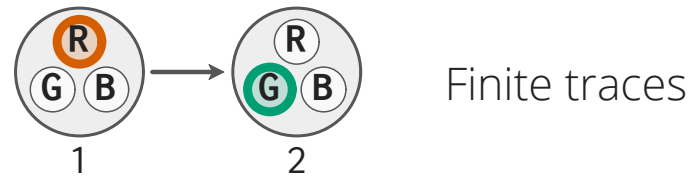
LTLf



X ~ next state, which **must** exist

Xw ~ next state, if it exists

LTLf



X ~ next state, which **must** exist

Xw ~ next state, if it exists

G ~ always (within trace bounds)

F ~ eventually



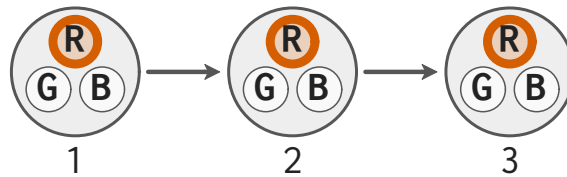
Q. Do the traces below satisfy this LTLf formula?

$G(\text{Red}) \ \& \ Xw(Xw(!\text{Red}))$



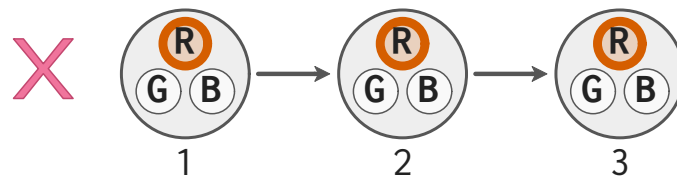
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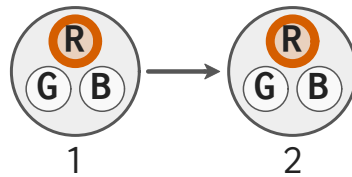
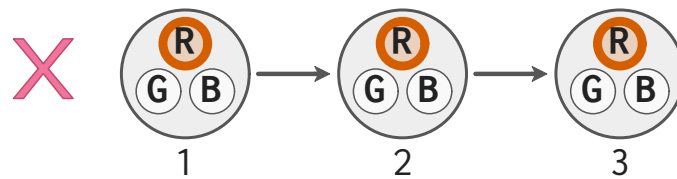
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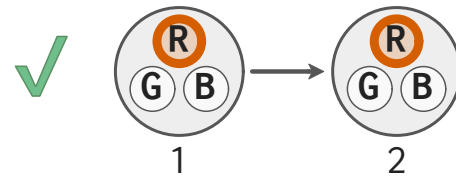
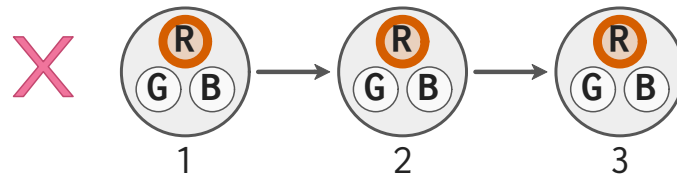
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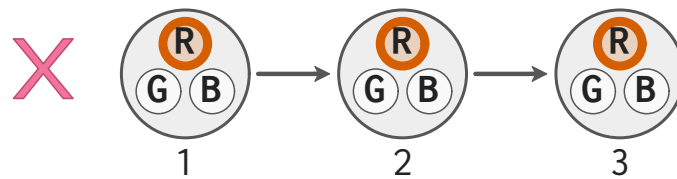
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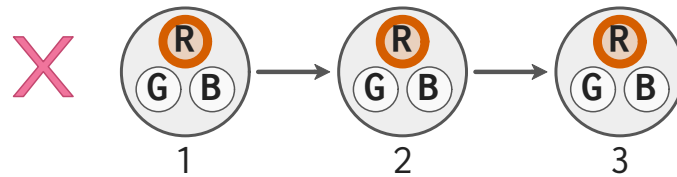
Q. Do the traces below satisfy this LTL formula?

$G(\text{Red}) \ \& \ Xw(Xw(!\text{Red}))$




Q. Do the traces below satisfy this LTL formula?

$G(\text{Red}) \ \& \ Xw(Xw(!\text{Red}))$




Formula accepts no traces of **length > 2**

Length misconception



Q. Translate to LTLf
Blue is on in at least two states



Q. Translate to LTLf
Blue is on in at least two states

$F(\text{Blue} \ \& \ X(F(\text{Blue})))$

Q. Translate to LTLf
Blue is on in at least two states

✓ $F(\text{Blue} \ \& \ X(F(\text{Blue})))$

Q. Translate to LTLf
Blue is on in at least two states

✓ $F(\text{Blue} \ \& \ X(F(\text{Blue})))$

Q. Translate to LTLf
Blue is on in at least two states

✓ $F(\text{Blue} \ \& \ X(F(\text{Blue})))$

Works for LTL too!

Q. Are these equations correct? [for all terms **e**]

LTL LTLf

$$\mathbf{!X(e)} \quad == \quad \mathbf{X(!e)}$$

Q. Are these equations correct? [for all terms **e**]

$$!X(e) == X(!e)$$

LTL

LTLf



Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
$\neg X(e) \quad == \quad X(\neg e)$	✓	✗

Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
$\neg X(e) \quad == \quad X(\neg e)$	✓	✗
$F(e) \quad == \quad e \ \ \ \ X(F(e))$		

Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
$\neg X(e) \quad == \quad X(\neg e)$	✓	✗
$F(e) \quad == \quad e \ \ \ X(F(e))$	✓	

Q. Are these equations correct? [for all terms **e**]

		LTL	LTLf
$\neg X(e)$	$\equiv X(\neg e)$	✓	✗
$F(e)$	$\equiv e \vee X(F(e))$	✓	✓

Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
$\neg X(e) == X(\neg e)$	✓	✗
$F(e) == e \vee X(F(e))$	✓	✓
$F(G(e)) == G(F(e))$		

Q. Are these equations correct? [for all terms **e**]

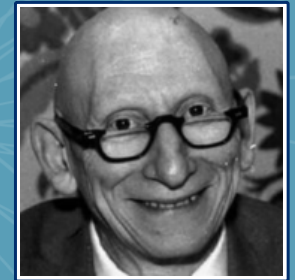
	LTL	LTLf
$\neg X(e) == X(\neg e)$	✓	✗
$F(e) == e \vee X(F(e))$	✓	✓
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Q. Are these equations correct? [for all terms **e**]

	LTL	LTLf
$\neg X(e) == X(\neg e)$	✓	✗
$F(e) == e \vee X(F(e))$	✓	✓
$F(G(e)) == G(F(e))$	✗	✓

All Done!

"One cannot proceed from the **informal** to the formal
by purely formal means"

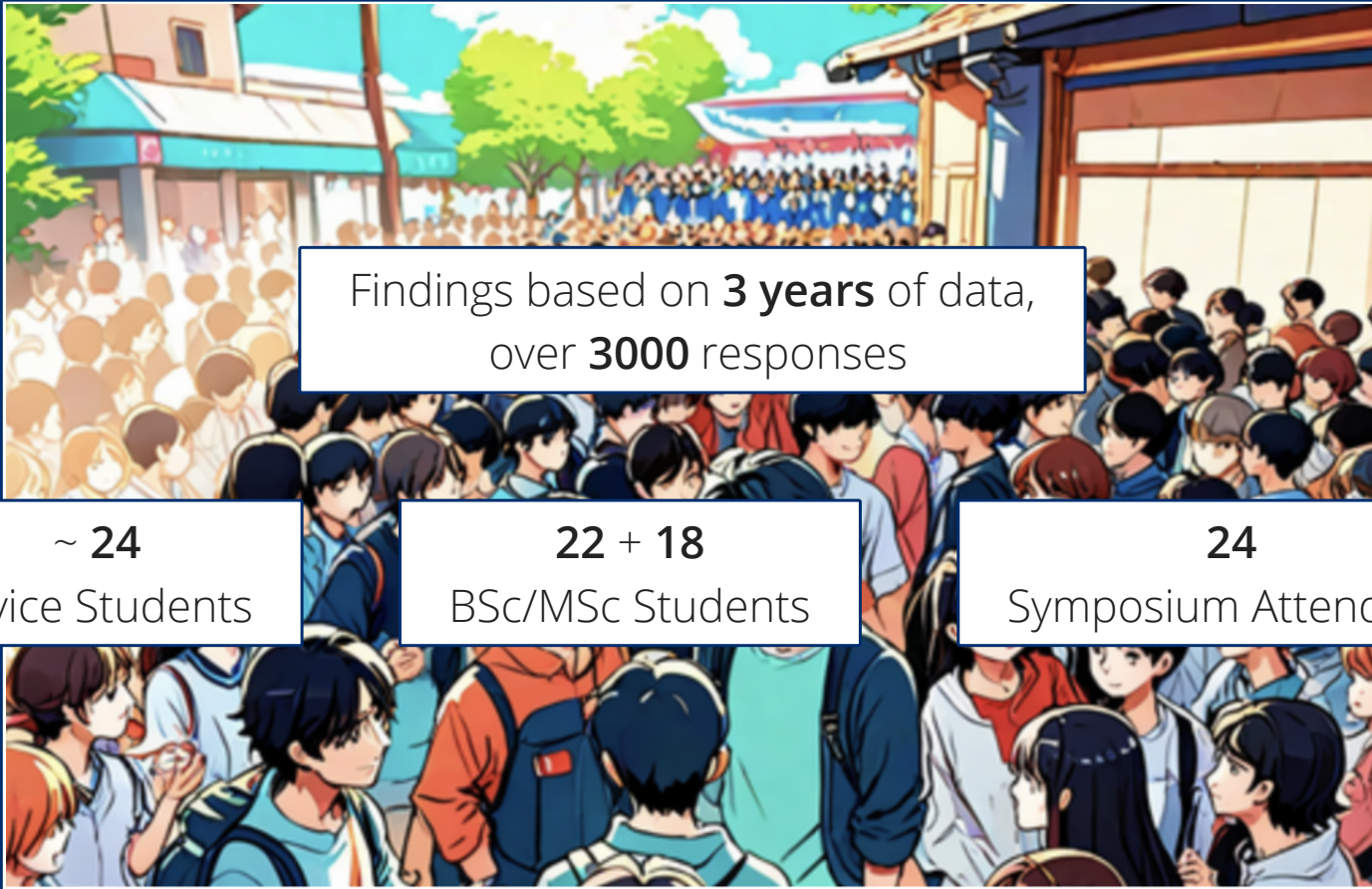


Perlis

Misconceptions get in the way!



Findings based on **3 years** of data,
over **3000** responses



Findings based on **3 years** of data,
over **3000** responses

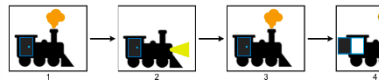
~ 24
Novice Students

22 + 18
BSc/MSc Students

24
Symposium Attendees

3 Survey Instruments

Example Question: Is the formula
always (Engine or Light)
satisfied by this trace?



Example Answer: Yes, because either the engine
headlight is on in each state.

Does the example make sense to you?*

Yes

No (please explain)

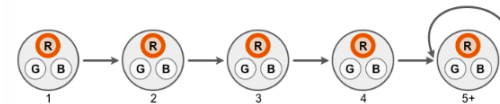
Example Question:

$G (X (Red))$

Example Answer:

- *LTL description:* The Red light is on in every state.
- *LTLf description:* Every state must be followed by Red on. No finite traces satisfy the formula.

satisfied by this trace?*



Yes

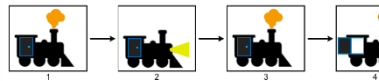
No

3 Survey Instruments

Q. Formats:

- ▶ LTL --> English
- ▶ English --> LTL
- ▶ Trace Matching
- ▶ Explain Mismatches
- ▶ Check Equations

Example Question: Is the formula
always (Engine or Light)
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Example Answer: Yes, because either the engine
headlight is on in each state.

Does the example make sense to you?*

- Yes
- No (please explain)

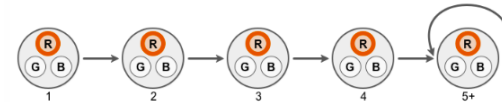
Example Question:

$G(X(\text{Red}))$

Example Answer:

- *LTL description:* The Red light is on in every state.
- *LTL description:* Every state must be followed by Red on. No finite traces satisfy the formula.

satisfied by this trace?*

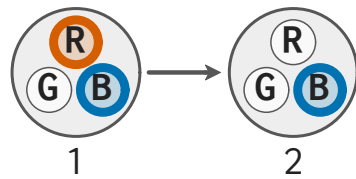


Yes

No

Protocol

Instruments

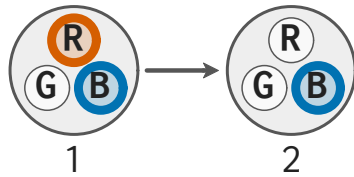


Catalog

Lightweight to Reuse

Protocol

Instruments



Catalog

The background of the slide is a solid blue color. It features four decorative starburst or radial patterns. Each pattern consists of a central circle with many thin lines radiating outwards to smaller circles of varying sizes, creating a sunburst or star-like effect. The patterns are arranged in a roughly 2x2 grid, with the top two being larger and the bottom two being smaller.

Catalog

14 categories of LTL and LTLf Errors

Catalog

14 categories of LTL and LTLf Errors

Length

Last

Cycle G

Implicit Prefix

Trace Split U

Spreading X

Bad Prop

Bad State Index

Implicit F

Implicit G

Other Implicit

Exclusive U

Bad State Quantification

Weak U

Catalog

14 categories of LTL and LTLf Errors

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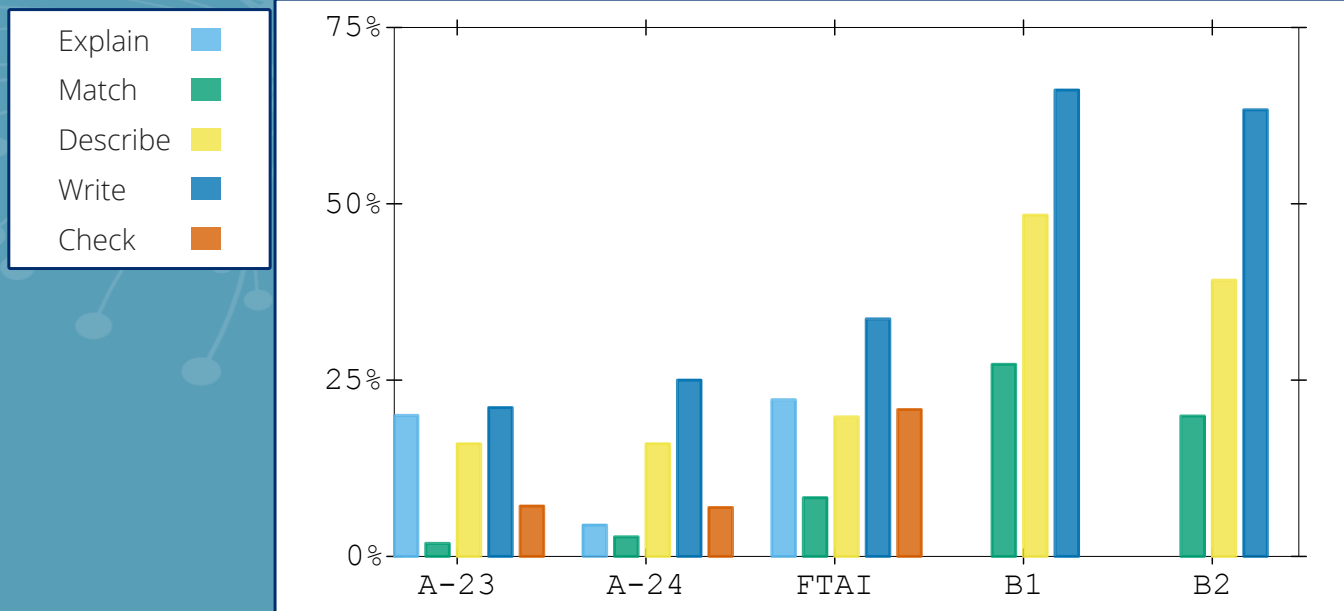
Other Implicit

Exclusive U

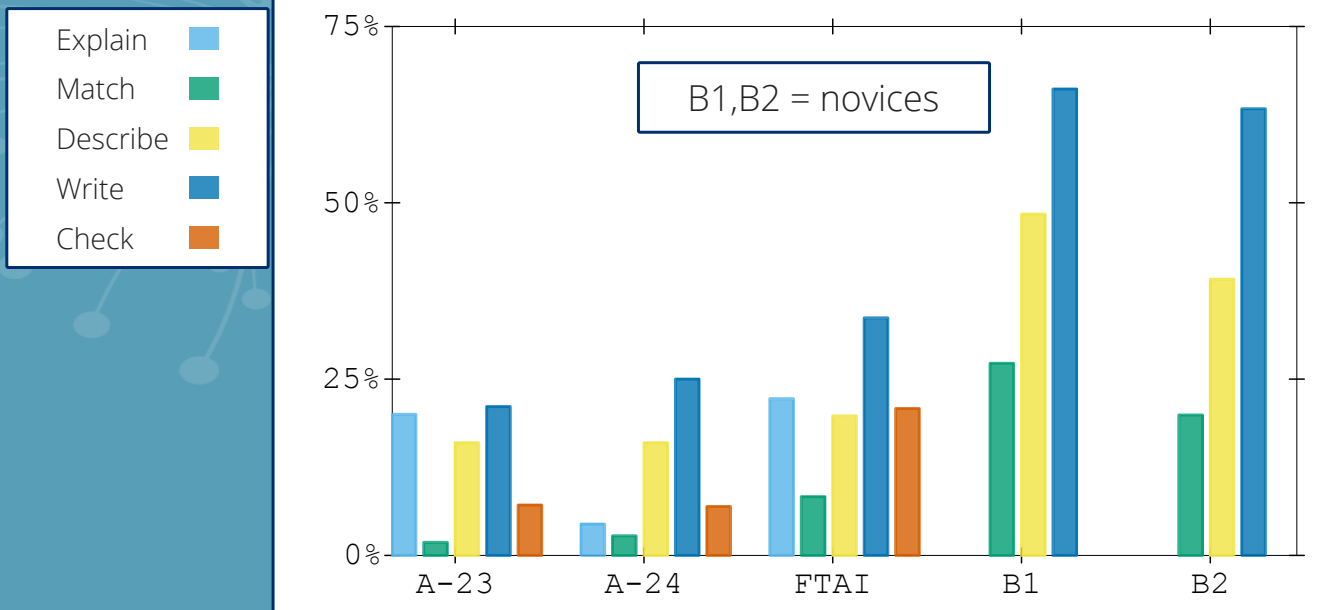
Bad State Quantification

Weak U

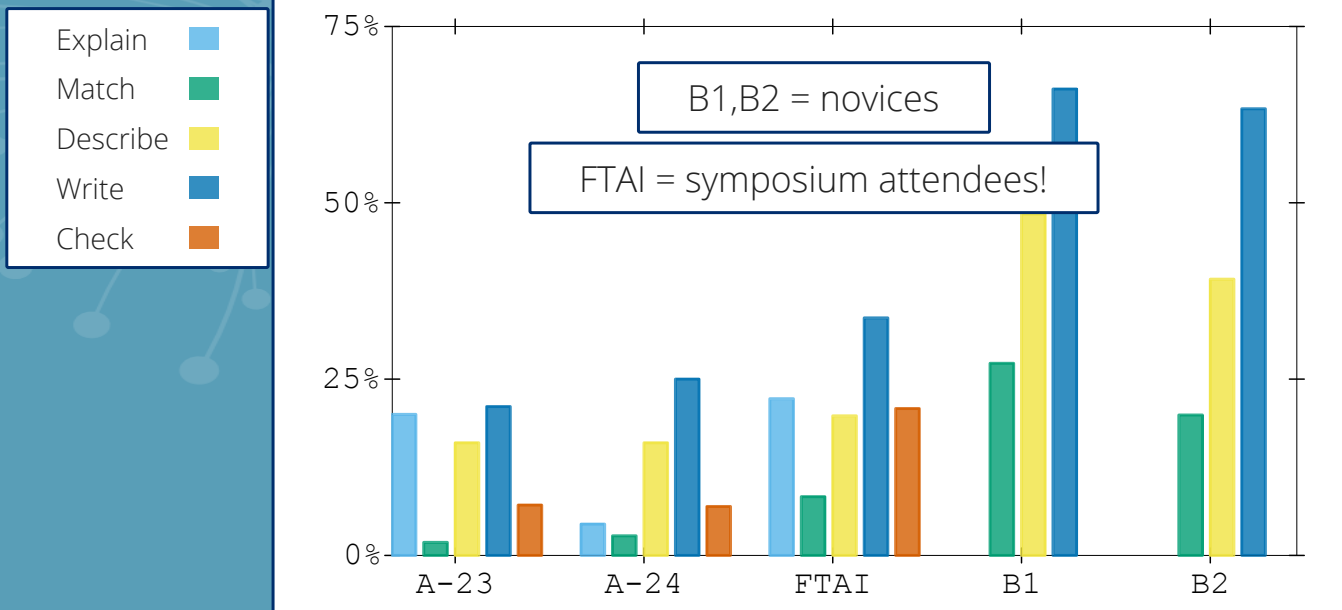
Total Incorrect Responses



Total Incorrect Responses



Total Incorrect Responses



Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length					
Last					
Bad Prop					
Bad State Index					
Bad State Quantification					
Cycle G					
Implicit F					
Implicit G					
Implicit Prefix					
Other Implicit					
Weak U					
Exclusive U					
Trace-Split U					
Spreading X					
Precedence					
Reasonable Variant					
Unlabeled					

Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length	17	7	8		
Last	1	5	-		
Bad Prop	-	8	6		
Bad State Index	2	9	5		
Bad State Quantification	-	1	2		
Cycle G	-	2	2		
Implicit F	10	13	7		
Implicit G	5	8	2		
Implicit Prefix	8	4	8		
Other Implicit	1	-	3		
Weak U	2	2	3		
Exclusive U	-	3	3		
Trace-Split U	-	-	3		
Spreading X	-	-	-		
Precedence	-	1	1		
Reasonable Variant	2	-	-		
Unlabeled	13	2	13		

Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length	17	7	8		
Last	1	5	-		
Bad Prop	-	8	6		
Bad State Index	2	9	5		
Bad State Quantification	-	1	2		
Cycle G	-	2	2		
Implicit F	10	13	7		
Implicit G	5	8	2		
Implicit Prefix	8	4	8		
Other Implicit	1	-	3		
Weak U	2	2	3		
Exclusive U	-	3	3		
Trace-Split U	-	-	3		
Spreading X	-	-	-		
Precedence	-	1	1		
Reasonable Variant	2	-	-		
Unlabeled	13	2	13		

Errors in Responses

	A-23	A-24	FTAI	B1	B2
Length	17	7	8	-	-
Last	1	5	-	-	-
Bad Prop	-	8	6	28	29
Bad State Index	2	9	5	19	25
Bad State Quantification	-	1	2	16	16
Cycle G	-	2	2	-	-
Implicit F	10	13	7	12	16
Implicit G	5	8	2	37	31
Implicit Prefix	8	4	8	-	8
Other Implicit	1	-	3	-	5
Weak U	2	2	3	15	11
Exclusive U	-	3	3	8	9
Trace-Split U	-	-	3	-	2
Spreading X	-	-	-	17	6
Precedence	-	1	1	2	7
Reasonable Variant	2	-	-	-	-
Unlabeled	13	2	13	15	35

Contributions

- LTLf + LTLf Instruments
- Catalog of Errors
- Data from 4 populations
- **Specific insights** into how LTL + LTLf can be tricky



Contributions

- ▶ LTLf + LTLf Instruments
- ▶ Catalog of Errors
- ▶ Data from 4 populations
- ▶ **Specific insights** into how LTL + LTLf can be tricky



==> Better teaching, tools, and logics

* Bonus

<https://ltl-tutor.xyz>

The screenshot shows a web browser window with the title "LTL Tutor" and the URL "https://ltl-tutor.xyz/exercise/generate". The page content includes a navigation bar with "Tutor Dashboard", "LTL Syntax", "Generate Exercise", "Instructor Dashboard", "Profile", and "Log Out". Below the navigation bar, the page is titled "Exercise" and contains the question "Does this trace satisfy the following LTL formula?". The LTL formula is $(! (F p))$. The trace is represented by two states, each containing the proposition $!p \ \& \ a \ \& \ !d$, connected by a double-headed arrow. Below the trace, there are two radio buttons for "Yes" and "No". At the bottom of the question area, there are two buttons: "Check Answer" and "Next Question".

* Bonus

<https://ltl-tutor.xyz>

The image displays two overlapping browser windows from the LTL Tutor website. The left window shows a question: "Does this trace satisfy the formula $(\neg (F p))$?" with a trace $!p \ \& \ a \ \& \ !d \leftrightarrow !p \ \& \ a \ \&$ and radio buttons for "Yes" and "No". The right window shows the feedback for an incorrect answer. It states: "That's not correct 😞 Don't worry, keep trying! The correct answer is highlighted in green (i.e. $(X (p \rightarrow (X a)))$)". It explains that the user's selection is more permissive than the correct answer and shows a trace diagram: $!p \rightarrow p \rightarrow !a \rightarrow 1 \leftrightarrow 1$. Below this, it shows an "Alt Trace: $!p;p;!a;cycle\{1;1\}$ " and a Venn diagram where a green circle represents the "Correct answer" and a larger red circle represents "Your answer", with the green circle being a subset of the red one.

The background of the slide is a solid blue color. It features four large, light-blue starburst or radial patterns, one in each quadrant. Each starburst consists of a central circle with many thin lines radiating outwards to smaller circles of varying sizes, creating a network-like or atomic structure.

The End

`blg@cs.utah.edu`



Q. Translate to LTL
Green is on in the final state



Q. Translate to LTL

Green is on in the final state

$F(G(\text{green}))$

Q. Translate to LTL

Green is on in the final state

X $F(G(\text{green}))$

Q. Translate to LTL

Green is on in the final state

X $F(G(\text{green}))$

inexpressible

Q. Translate to LTL

Green is on in the final state

X $F(G(\text{green}))$

✓ inexpressible

Q. Translate to LTL

Green is on in the final state

X $F(G(\text{green}))$

Q. Translate to LTL

Green is on in the final state

X $F(G(\text{green}))$

Last misconception



Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...



Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...

$G(\text{blue} \ \& \ X(\! \text{blue}))$

Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...

X $G(\text{blue} \ \& \ X(\! \text{blue}))$

Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...

X $G(\text{blue} \ \& \ X(\!blue))$

$G(\text{blue} \ ==> \ X(\!blue) \ \& \ X(X(\text{blue}))) \ \& \ \text{blue}$

Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...

X $G(\text{blue} \ \& \ X(\! \text{blue}))$

✓ $G(\text{blue} \ ==> \ X(\! \text{blue}) \ \& \ X(X(\text{blue}))) \ \& \ \text{blue}$

Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...


X $G(\text{blue} \ \& \ X(\! \text{blue}))$

Q. Translate to LTL

Blue is on / off / on ... in state 0 / 1 / 2 ...

X $G(\text{blue} \ \& \ X(\! \text{blue}))$

Cycle G misconception



Q. Translate to LTL
Red is on exactly once

Q. Translate to LTL

Red is on exactly once

$F(\text{red} \ \& \ X(G(\! \text{red})))$

Q. Translate to LTL
Red is on exactly once

X $F(\text{red} \ \& \ X(G(\! \text{red})))$

Q. Translate to LTL

Red is on exactly once

X $F(\text{red} \ \& \ X(G(\! \text{red})))$

$\! \text{red} \ U \ (\text{red} \ \& \ X(G(\! \text{red})))$

Q. Translate to LTL

Red is on exactly once

X $F(\text{red} \ \& \ X(G(\! \text{red})))$

✓ $\! \text{red} \ U \ (\text{red} \ \& \ X(G(\! \text{red})))$

Q. Translate to LTL
Red is on exactly once

X $F(\text{red} \ \& \ X(G(\! \text{red})))$

Q. Translate to LTL
Red is on exactly once

X $F(\text{red} \ \& \ X(G(\! \text{red})))$

Implicit Prefix misconception



Q. Translate to LTL

Blue is on in at least two states



Q. Translate to LTL

Blue is on in at least two states

$F(\text{blue}) \cup F(\text{blue})$

Q. Translate to LTL

Blue is on in at least two states

X $F(\text{blue}) \cup F(\text{blue})$

Q. Translate to LTL

Blue is on in at least two states

X $F(\text{blue}) \cup F(\text{blue})$

$! \text{blue} \cup (\text{blue} \ \& \ F(X(\text{blue})))$

Q. Translate to LTL

Blue is on in at least two states

X $F(\text{blue}) \cup F(\text{blue})$

✓ $!\text{blue} \cup (\text{blue} \ \& \ F(X(\text{blue})))$

Q. Translate to LTL

Blue is on in at least two states

X $F(\text{blue}) \cup F(\text{blue})$

Q. Translate to LTL

Blue is on in at least two states

X $F(\text{blue}) \cup F(\text{blue})$

Trace Split U misconception

